

1 Writing Logarithmic Equations

If you remember from chapter 2.6, we learned that if f and g are inverse functions, then

$$f(x) = y \text{ if and only if } g(y) = x$$

Applying that to exponentials and logarithmic functions, we get

$$b^x = y \text{ if and only if } \log_b(y) = x$$

For example, below you can see 3 exponential equations (left) and their corresponding logarithmic equation (right). The base, value, and exponent have been identified.

$$2^3 = 8 \Leftrightarrow \log_2(8) = 3$$

base = 2 exponent = 3 value = 8

$$4^{\frac{1}{2}} = 2 \Leftrightarrow \log_4(2) = \frac{1}{2}$$

base = 4 exponent = $\frac{1}{2}$ value = 2

$$3^{-1} = \frac{1}{3} \Leftrightarrow \log_3\left(\frac{1}{3}\right) = -1$$

base = 3 exponent = -1 value = $\frac{1}{3}$

Check for Understanding 1. Write the exponential equation corresponding to each logarithmic equation.

(a) $\log_3(27) = 3$

(b) $\log_{\frac{1}{9}}(81) = -2$

(c) $\log_{\sqrt{2}}(8) = 6$

(d) $\log_5(25) = 2$

Check for Understanding 2. Write the logarithmic equation corresponding to each exponential equation.

(a) $2^5 = 32$

(b) $4^0 = 1$

(c) $4^{\frac{1}{2}} = 2$

(d) $3^{-2} = \frac{1}{9}$

Questions and Answers:

Answers:

Check for Understanding 1:

(a) $3^3 = 27$ (b) $\left(\frac{1}{9}\right)^{-2} = 81$ (c) $\sqrt{2^6} = 8$ (d) $5^2 = 25$

Check for Understanding 2:

(a) $\log_2(32) = 5$ (b) $\log_4(1) = 0$ (c) $\log_4(2) = \frac{1}{2}$ (d) $\log_3\left(\frac{1}{9}\right) = -2$

2 Solving Logarithmic Equation

Now that we know how to translate between logarithmic equations and exponential equations, we can start to solve logarithmic equations. Suppose you are asked to evaluate

$$\log_5(125) = x$$

First, you would want to translate into an exponential equation:

$$5^x = 125$$

Since we know that $(5)(5)(5) = 125$, then $5^3 = 125$, so $x = 3$.

Example 1. Evaluate the logarithmic expression:

$$\log_3\left(\frac{1}{9}\right)$$

(Step 1) Write the logarithmic expression into a logarithmic equation

$$\log_3\left(\frac{1}{9}\right) = x$$

(Step 1) Convert the logarithmic equation into an exponential equation

$$3^x = \frac{1}{9}$$

(Step 2) Solve for x .

Since $3^2 = 9$ then $3^{-2} = \frac{1}{9}$. So $x = -2$.

$$\log_3\left(\frac{1}{9}\right) = -2$$

Check for Understanding 3. Evaluate the logarithmic expressions below:

(a) $\log_4(16)$

(b) $\log_{16}(4)$

(c) $\log_2\left(\frac{1}{8}\right)$

(d) $\log_3(3)$

(e) $\log_{10}(1)$

Check for Understanding 3:

(a) 2 (b) $\frac{1}{2}$ (c) -3 (d) 1 (e) 0