Pop Quiz

1) Write the three properties that define a metric d: X --> Real numbers

a) d(x,y) ______ 0 and d(x,y) _____ 0 if and only if ___ = ___.

b) d(x,y) = _______ for all x,y in X.

c) For all x,y,z in X,

d(x,y) + d(y,z) _______ _________

2) Describe and draw the $B_1(0,0)$ under the metric d((a,b),(x,y)) = |x-a|+|y-b|

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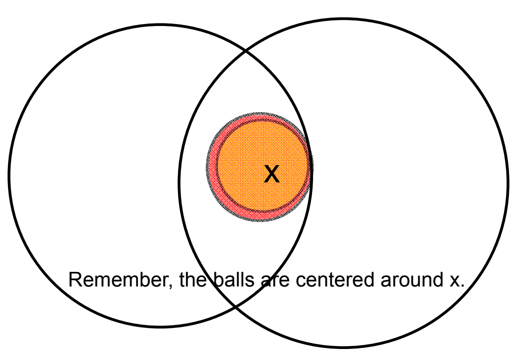
If A and B are open in X, then A∩B is open in X.

Proof: We need to show that for any $x \in A \cap B$, there exists r > 0, such that $B_r(x) \subset A \cap B$.

We know that there exists $r_1 > 0$ such that

$B_{r1}(x) \subset A$ and there exists $r_2 > 0$ such that

$B_{r2}(x) \subset B$. Therefore, if we let $r = 1/2\ (\min(r_1, r_2))$ then $B_r(x) \subset A \cap B$.



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Now let's try together.

Let $\{A_\alpha\}$ be an arbitrary collection of open sets.

Then $\cup_\alpha A_\alpha$ is an open set.

Proof: Let x be in __________.

Then x is in _______ for some ______.

Since A_α is open for all α, then there exists,

r > 0, such that ______ ____ ______.

By definition of union, $B_r(x) \subset$ _____ implies

$B_r(x) \subset$_________. Therefore, ______ is

open, by definition.

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DEFINITION: closed

A set C is closed in a metric space X if and only if X\C is open.

Definition: Limit Point

Definition 6 (limit point). .
Let X, d be a metric space and $A \subset X$.
We say $a \in A$ is a *limit point* if for all $r > 0$,

$$B_r(a) \cap A \neq \varnothing$$

if all open balls containing a have nontrivial intersection with A.

* your book requires for the point in the intersection to be different from a. Some books call this an accumulation point.

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Example: 0 is a limit point of the set

$A = \{1/n \mid n \in \mathbb{N}\}$ under the usual metric.

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Definition 7 (boundary point). . We say, $a \in A$ is a *boundary point* of A if and only if for all $r > 0$,

$$B_r(a) \cap A \neq \varnothing$$
$$B_r(a) \cap A^c \neq \varnothing$$

every ball of radius, r, about a has nonempty intersection with A and it's complement.

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3. Exercises

Problem 1. Show that every set is closed and open in the discrete topology.

Problem 2. Show that every point in Y is a limit point of Y.

Problem 3. Let $\bar{Y}$ be the the set of all limit points of Y.
Show that Y is closed if and only if $Y = \bar{Y}$

Problem 4. Show that $(A^o)^o = A^o$.

Problem 5. Show that $\bar{\bar{A}} = \bar{A}$.

Problem 6. Show that the arbitrary union of open sets is open.

Problem 7. Show that $\varnothing$ and X are open sets.

Problem 8. Show that the finite intersection of open sets is open.

Problem 9. Show that the finite intersection of closed sets is closed.

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