Right Triangle Trigonometry

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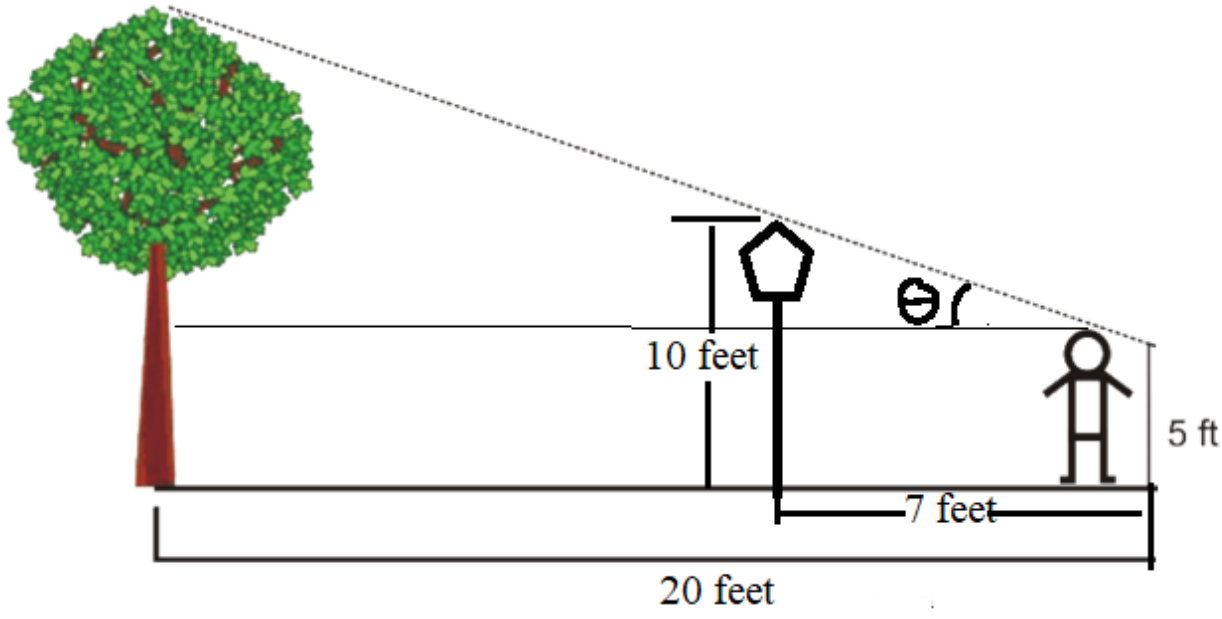
Introduction to Trigonometric Functions



Suppose you are 5 feet tall, and standing 7 feet away from a 10 foot sign.
20 feet away from you, and directly behind the sign, is a tree.

Similar Triangles

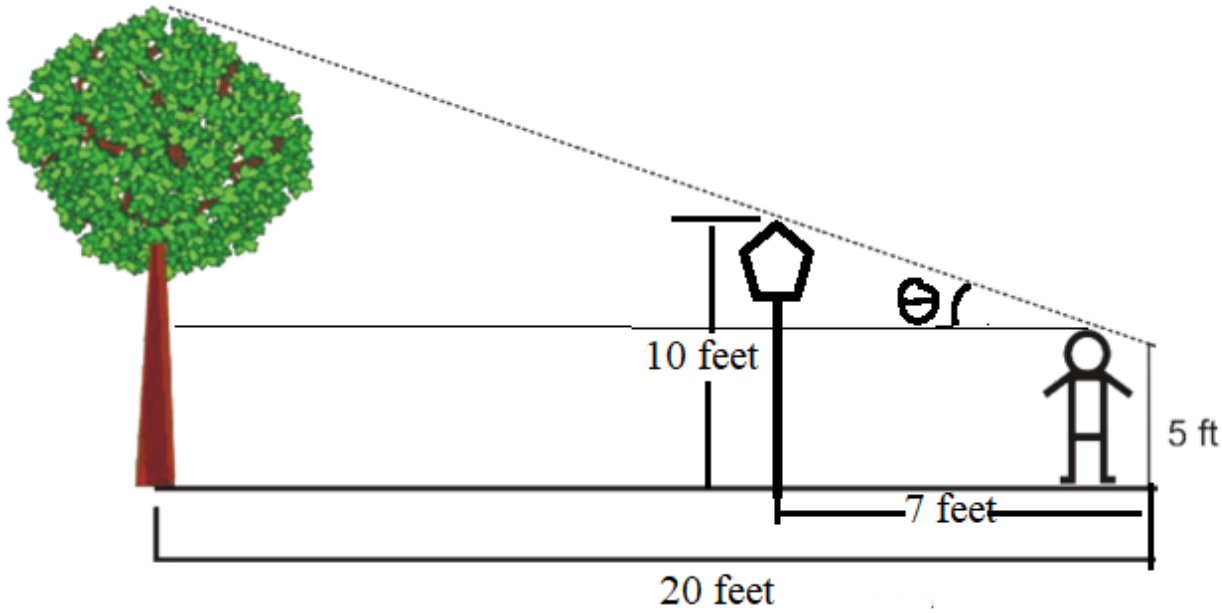


Figure: Find the similar triangles

You could use similarity of triangles to compute the height of that tree.

In the figure above you can see that similarity ratio is $\frac{7}{5}$, which gives us the proportion:

The Height of the Tree

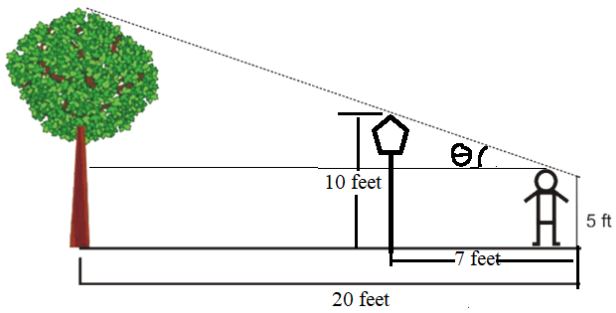


Figure: Find the similar triangles

$$\frac{7}{5} = \frac{20}{x}$$

Cross-multiplying, we get

$$\frac{100}{7} = x$$

The height of the tree is

$$5 + \frac{100}{7} \approx 19.29ft.$$

Angles of Elevation

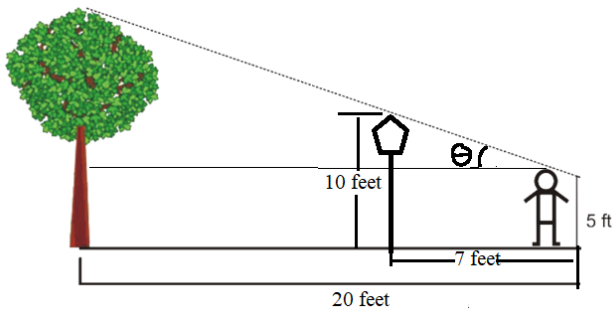


Figure: Find the similar triangles

This method only works if we are able to measure the height of that object and our distance to it, both of which are unlikely and inefficient.
So how do we compute the height of the tree, if we don't have such an object?

Similarity Ratio

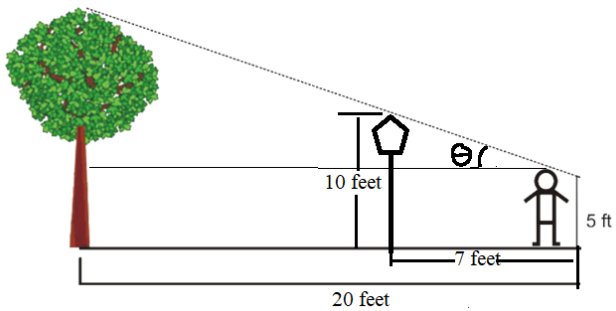
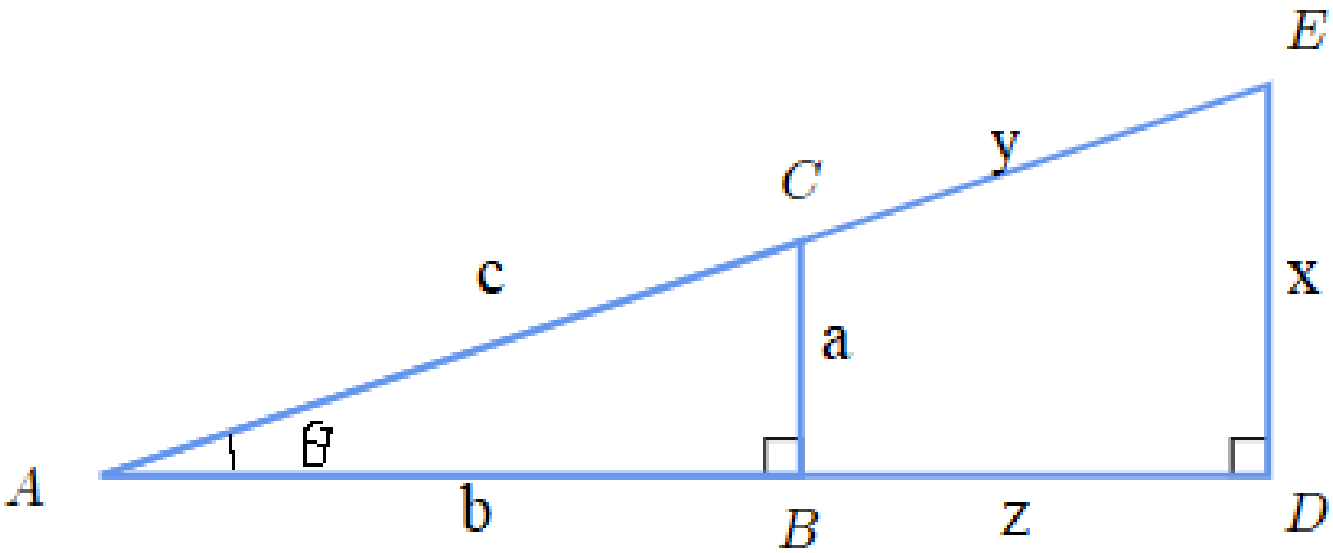


Figure: Find the similar triangles

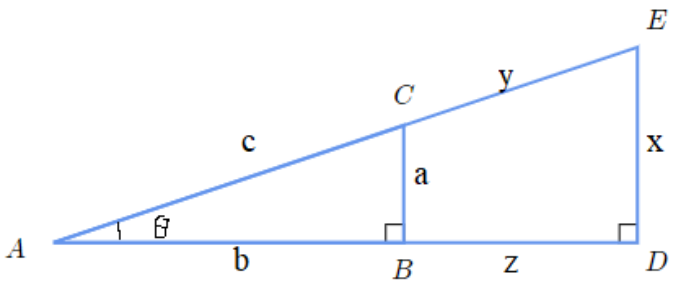
Height is always perpendicular.
We will always be dealing with a right triangle.
The similarity ratio is completely determined by the **angle of elevation** , θ.

Ratio of Angle is Well-Defined

In fact, we can measure the height of any other object that is on that line.

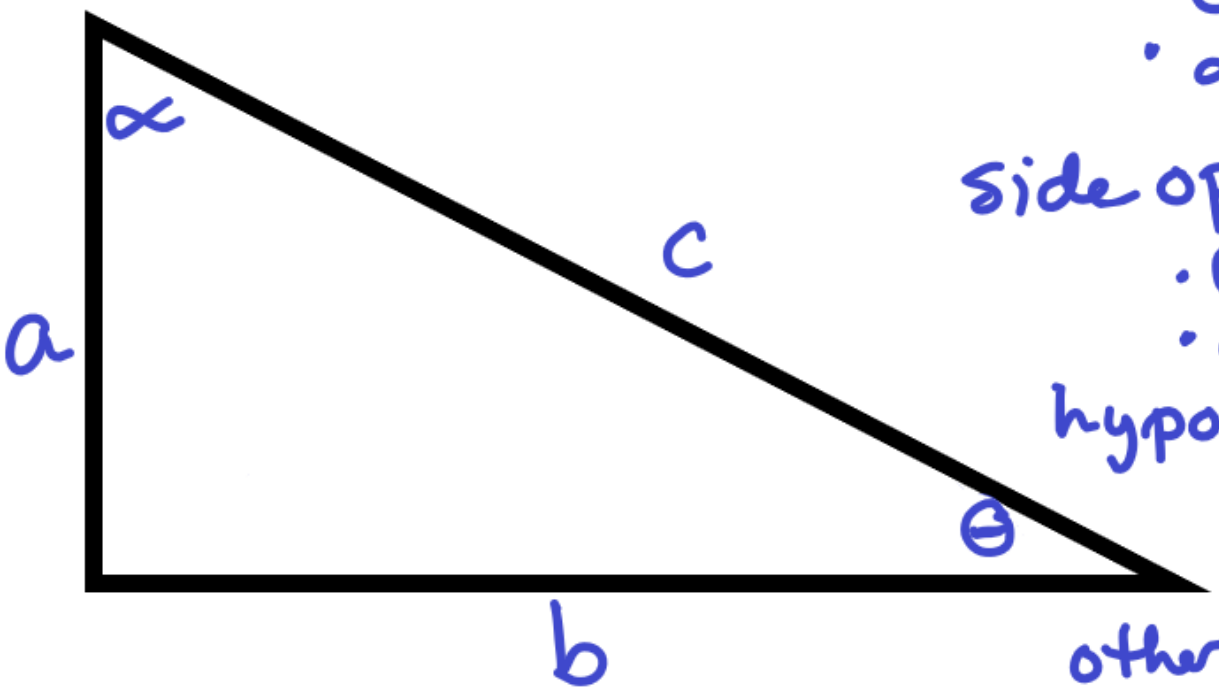


The Birth of Trigonometric Functions



We can define functions whose input is an angle measure and whose output is a similarity ratio.
These functions are called *trigonometric functions*.

Vocabulary with Right Triangles



Side adjacent to
- θ is b
- α is a

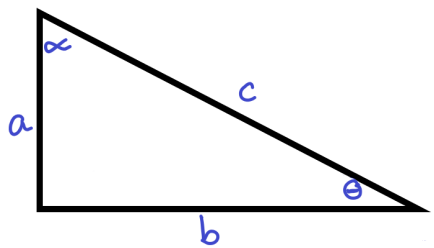
Side opposite of:
- θ is a
- α is b

hypotenuse is:
- c
- opposit right angle

other information
- θ + α = 90
- θ = "theta" α = "alpha"

Sine, Cosine, Tangent

In this lesson, we will study the three most common trigonometric function: sine, cosine, and tangent.



Definition of Sine, Cosine, and Tangent

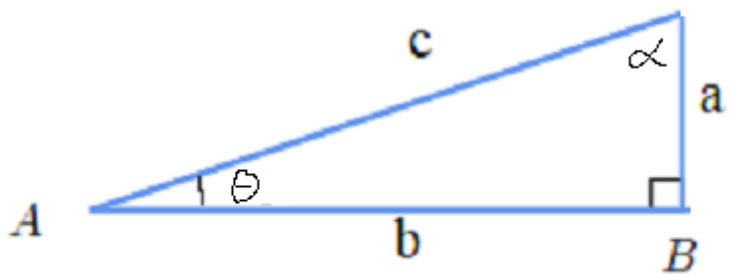
If θ is an angle measure, the sine of it is denoted $\sin(\theta)$, the cosine is denoted $\cos(\theta)$, and the tangent is denoted $\tan(\theta)$ and are defined:

- $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

Check For Understanding 1

Check for Understanding

Write the following in terms of a, b and c.

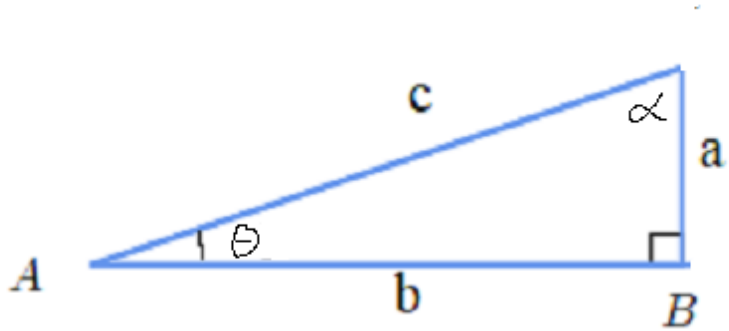


- Find $\sin(\theta)$
- Find $\cos(\theta)$
- Find $\tan(\theta)$
- Find $\sin(\alpha)$
- Find $\cos(\alpha)$
- Find $\tan(\alpha)$

Check for Understanding 1: Answers

Check for Understanding

Write the following in terms of a, b and c.



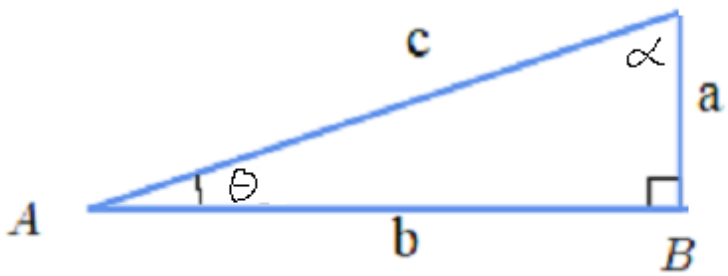
- Find $\sin(\theta) = \frac{a}{c}$
- Find $\cos(\theta) = \frac{b}{c}$
- Find $\tan(\theta) = \frac{a}{b}$

- Find $\sin(\alpha) = \frac{b}{c}$
- Find $\cos(\alpha) = \frac{a}{c}$
- Find $\tan(\alpha) = \frac{b}{a}$

Do you see any patterns?

Why they call it **Co**sine

If α and θ are complementary angles, $\alpha + \theta = 90^o$.



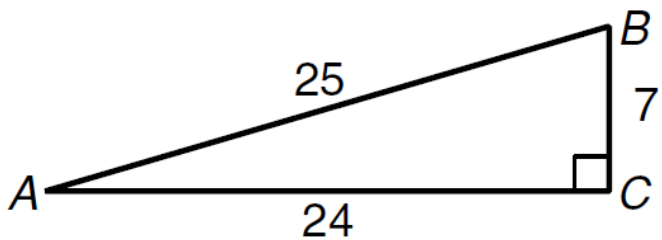
Cosine of θ is **Sine** of the α which is the **co**mplement of θ.

- Find $\cos(\alpha) = \frac{a}{c} = \sin(\theta)$
- Find $\cos(\theta) = \frac{b}{c} = \sin(\alpha)$
- Find $\tan(\theta) = \frac{a}{b} = \frac{1}{\tan(\alpha)}$

Check For Understanding 2

Check for Understanding

Evaluate the following:

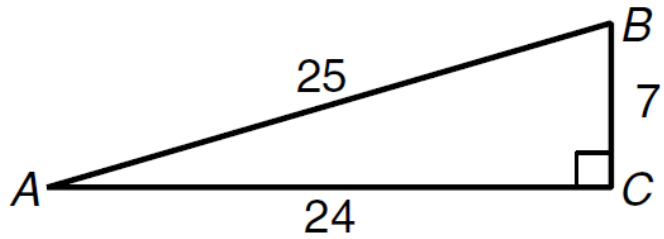


- Find $\sin(A)$
- Find $\cos(A)$
- Find $\tan(A)$
- Find $\sin(B)$
- Find $\cos(B)$
- Find $\tan(B)$

Check for Understanding 2: Answers

Check for Understanding

Write the following in terms of a, b and c.



- Find $\sin(A) = \frac{7}{25}$
- Find $\cos(A) = \frac{24}{25}$
- Find $\tan(A) = \frac{7}{24}$
- Find $\sin(B) = \frac{24}{25}$
- Find $\cos(B) = \frac{7}{25}$
- Find $\tan(B) = \frac{24}{7}$