AP Calculus BC Homework Name:

1) Find the arc length of the function $f(x)=2x^{\frac{3}{2}}$ from $x=0$ to $x=3$.

Solution: The formula for the integral $\int_0^3 \sqrt{1+(f'(x))^2}\,dx$ so we need to find

$f'(x)=2(\frac{3}{2}x^{\frac{1}{2}})=3\sqrt{x}$. So the arc length is given by

$$\int_0^3 \sqrt{1+(3\sqrt{x})^2}\,dx=\int_0^3 \sqrt{1+9x}\,dx$$

Letting $u = 1 + 9x$, then $du = 9dx$. $\frac{1}{9}\int_1^{28} u^{\frac{1}{2}}du=\frac{2}{27}u^{\frac{3}{2}}\Big|_1^{28}=\frac{2}{27}(28\sqrt{28}-1)=$

2) Let $r(t)=(3t,\cos(\pi t))$.

a) Find $\frac{dy}{dx}$.

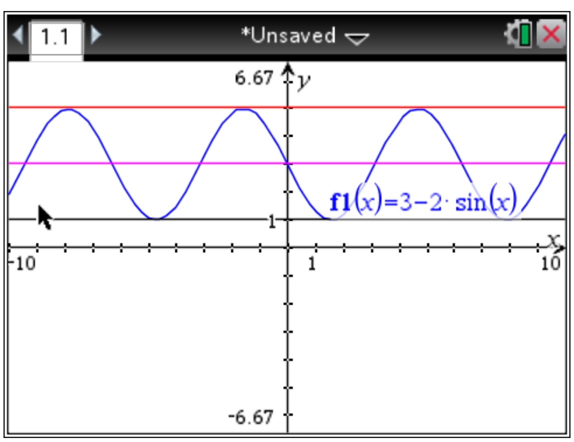
b) Find $\frac{d^2y}{dx^2}$.

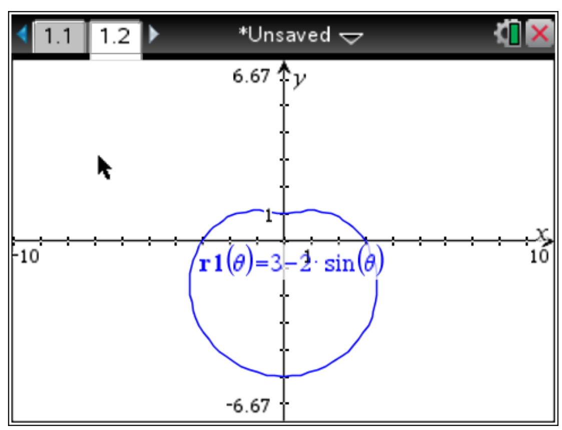
Solution:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-\pi\sin(\pi t)}{3}$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}(\frac{dy}{dt})}{\frac{dx}{dt}}=\frac{-\pi^2\cos(\pi t)}{9}$$

3) Graph the polar curve $r=3-2\sin(\theta)$.





$$\frac{1}{2}\int_0^{2\pi}(3-2\sin(\theta))^2\,d\theta=\int_{-\frac{\pi}{2}}^{\pi}(3-2\sin(\theta))^2\,d\theta$$

Written by Bailey

5) Use partial fractions to solve the integral

$$\int \frac{4x+37}{x^2+11x+28}dx$$

Solution:

$$\frac{4x+37}{x^2+11x+28} = \frac{A}{x+7} + \frac{B}{x+4}$$

$$4x+37 = A(x+4)+B(x+7)$$

$$x=-4 \to B=7 \quad x=-7 \to A=-3$$

$$\to \int \frac{4x+37}{x^2+11x+28}dx = \int -\frac{3}{x+7}dx + \int \frac{7}{x+4}dx$$

$$= -3\ln|x+7| + 7\ln|x+4| + C$$

6) Use trigonometric substitution to solve the integral

$$\int \sqrt{36-x^2}\,dx$$

Solution:

Letting $x = 6\sin(\theta) \to dx = 6\cos(\theta)$, we get

$$\int \sqrt{36-x^2}\,dx = \int \sqrt{36-36sin^2(\theta)}\,6\cos(\theta)d\theta$$

$$= \int 36\cos^2(\theta)d\theta$$

$$= 18\int 1+\cos(2\theta)d\theta$$

$$= 18\theta + 18\frac{\sin(2\theta)}{2} + C$$

$$= 18\theta + 18\sin(\theta)\cos(\theta) + C$$

$$= 18\arcsin(\frac{x}{6}) + 18(\frac{x}{6})(\frac{\sqrt{36-x^2}}{6}) + C$$

7) Use trigonometric identities to solve the integral

$$\int \sin^5(x)dx$$

Solution:

$$\int \sin^5(x)dx = \int (1-\cos^2(x))^2\sin(x)dx$$

$$= \int [1-2\cos^2(x)+\cos^4(x)]\sin(x)dx$$

$$= -\int 1-2u^2+u^4du$$

$$= -u + 2\frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= -\cos(x) + \frac{2}{3}\cos^3(x) - \frac{1}{5}\cos^5(x) + C$$

8) Use integration by parts to solve the integral

$$\int 7x^3\cos(3x)dx$$

Solution: $\frac{7}{3}x^3\sin(3x)+\frac{21}{9}x^2\cos(3x)-\frac{42}{27}x\sin(3x)-\frac{42}{81}\cos(3x)+C$

Sign	U	Dv
+	$7x^3$	$\cos(3x)$
-	$21x^2$	$\frac{1}{3}\sin(3x)$
+	$42x$	$-\frac{1}{9}\cos(3x)$
-	42	$-\frac{1}{27}\sin(3x)$
+	0	$\frac{1}{81}\cos(3x)$

9) Let R be the region between $f(x)=-\sqrt{36-x^2}$ and $g(x)=\sqrt{36-x^2}$. Find the volume of the solid with cross-sections perpendicular to R and the x-axis are squares.

Solution:

$$2\int_0^6 (2\sqrt{36-x^2})^2dx=1152$$

10) Let R be the region bounded by the graphs of $f(x)=e^x$ and $g(x)=1-x^2$, $x=0$ and $x=1$.

a) Find the area of R.

$$\int_0^1 e^x-(1-x^2)dx=e-\frac{5}{3}$$

b) Find the volume of the solid generated by revolving R around the x-axis.

$$\pi\int_0^1 (e^x)^2-(1-x^2)^2\,dx=\frac{\pi}{2}e^2-\frac{31\pi}{30}$$

c) Find the volume of the solid generated by revolving R around the y-axis.

$$2\pi\int_0^1 x(e^x-(1-x^2)dx=\frac{3\pi}{2}$$

12) Let $y+\cos(y)=x+1$.

a) Find the equation for the tangent line at the origin.

$$\frac{dy}{dx}=\frac{1}{1-\sin(y)}$$

$$y-0=1(x-0)$$

$$y=x$$

b) Is the graph of the curve defined by the equation concave up, concave down, or neither at the origin. Justify your answer.

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\frac{1}{1-sin(y)}$$

$$=-(1-\sin(y))^{-2}(-\cos(y))\frac{dy}{dx}$$

$$=\frac{\cos(y)}{(1-\sin(y))^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,0)}=1$$