Super Power(Puff) Series Worksheet

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1. Let $f(x) = \ln(\sqrt{1-x})$.

   a. Write the first four terms and the general term for the power series of $g(x) = \dfrac{1}{1-x}$.

   b. Write the first four nonzero terms and the general term for the MacLaurin series of the function defined by $h(x) = \int_0^x g(t)dt$ and express the function, *h*, explicitly in terms of *x*.

   c. Use part (b) to write the first four nonzero terms of the MacLaurin series of $f(x)$.

d. Find $f'(0)$, $f''(0)$ and $f^{(6)}(0)$ by analyzing the MacLaurin series only (no taking derivatives)

e. Find the interval of convergence for the MacLaurin series for $f(x)$.

f. Use the second degree MacLaurin polynomial to estimate $\ln(\sqrt{\frac{1}{2}})$. Given that $|f'''(x)| < 8$ for all x, $-\frac{1}{2} \le x \le \frac{1}{2}$, use LaGrange's theorem to show that it could not be -0.05.

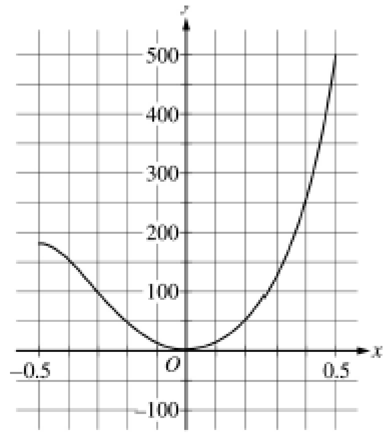
2. Let $f(x) = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \cdots + (-1)^n \frac{x^{n+1}}{n!} + \cdots$

   a. Find the interval of convergence for $f(x)$.

   b. Write the first four nonzero terms and the general term for the MacLaurin Series of $g(x) = \frac{f(x)}{x}$ and use this to describe $g(x)$ and $f(x)$ as a function of x.

c. Use the MacLaurin series to determine whether $f(x)$ has a local maximum, a local minimum, or neither at $x = 0$.

d. Use the explicit function you found in part (b) to write the first four terms of the MacLaurin series for $h(x) = \sin(x) + x\cos(x)$



Graph of $f^{(5)}$

3. Let *f* and *g* be functions given by $f(x) = xe^{x^3}$ and $g(x) = \int_0^x f(t)dt$. The graph of $f^{(5)}(x)$ the fifth derivative of *f* (*x*) is shown above for $-\frac{1}{2} \leq x \leq \frac{1}{2}$.

   a. Use the following steps to write the first four nonzero terms and the general term for the MacLaurin series for *f* (*x*).

      i. Write the first four nonzero terms and the general term for e^x

      ii. Write the first four nonzero terms and the general term for e^{x^3}

      iii. Write the first four nonzero terms and the general term for xe^{x^3}

   b. Write the first four nonzero terms and the general term for the MacLaurin series for *g*(*x*).

   c. Find the value of $g'(0), g''(0)$ and $g^{(5)}(0)$

d. Use the fact that $g(x)=\int_0^x f(t)dt$ to find and upper bound on $g^{(6)}(x)$ on $-\frac{1}{2}\le x\le\frac{1}{2}$.

e. Use LaGrange Error bound and the fifth degree MacLaurin polynomial for g to show that $g(\frac{1}{2})\neq 0.1$ by following the steps below.

i. Write the fifth degree MacLaurin polynomial for g, $P_5(x)$.

ii. Evaluate $P_5(\frac{1}{2})$

iii. Use your information in part (d) to find the LaGrange Error Bound.

iv. Explain why $g(\frac{1}{2})\neq 0.1$.

4. The Taylor series about $x = 2$ for a function f is given by the

$$f^{(n)}(2) = \frac{(-1)^{n+1}(n+1)}{5^n(n-1)^2}$$

for $n \geq 2$. The graph has a horizontal tangent line at $x = 2$ and $f(2) = 6$.

a. Write the first four terms and the general term for the Taylor series of f about $x = 2$.

b. Determine whether f has a local maximum, a local minimum, or neither at $x = 2$.

c. Find the interval of convergence for f about $x = 2$.

d. Write the first four nonzero terms and the general term for the Taylor series of $g(x) = \int_0^x f(t)dt$.

e. Write the first four terms and the general term for the Taylor series of $h(x) = f(x^2 + 2)$. What is the center for the Taylor series for *h*?

f. Determine whether *h* has a local maximum, a local minimum, or neither at $x = 0$.

g. Use the third degree Taylor polynomial about $x = 2$ to approximate $f(1)$ and use alternating series error bound to show that your approximation is accurate to 3 decimal places.

5. Euler's Identity
(The most beautiful equation in the world … and the equation which decorated our wedding invitation)

a. Write the first four nonzero terms and the general term for the MacLaurin series for $\cos(x)$.

b. Write the first four nonzero terms and the general term for the MacLaurin series for $\sin(x)$ and $i\sin(x)$, where $i^2 = -1$ is the complex number you know as the "square root of -1"

c. Write the first four nonzero terms and general terms for the MacLaurin series of $\cos(x) + i\sin(x)$.

d. Write the first four nonzero terms and the general term for the MacLaurin series of e^{ix} and describe the relationship between the series in part (d) and part (c).

e. Compute $e^{i\pi}$ and $e^{i\pi} + 1$.