

**THE BAD PLACE(S)**

**Discontinuities:** A discontinuity of  $f$  is a value of  $x$ ,  $x = a$ , such that the graph of  $f$  is not continuous. In other words, this is a **bad place** in which the graph of  $f$  will break, disappear momentarily, or shoot off into infinity (or negative infinity).

For rational functions, the discontinuities are where the denominator is equal to zero.

List the discontinuities of  $f(x) = \frac{x-4}{(x-1)(x-3)}$

**Classifying Discontinuities:** For rational functions, all discontinuities are **zeros** of the denominator:

1. **Removable Discontinuity:** A zero that cancels (*these are bad places*)
2. **Non-Removable Discontinuity:** A zero that doesn't cancel. (*these are really bad places*)
  1. For rational functions, the  $y$ -value will go to \_\_\_\_\_.
  2. These will be **vertical asymptotes** in rational functions.

Example: Classify the discontinuities of  $f(x) = \frac{(x+4)(x-1)}{(x-1)(x+3)}$

**Vertical asymptotes.** The vertical asymptotes are vertical lines,  $x = a$ , which the graph of  $f$  approaches as  $x \rightarrow a^-$  or  $x \rightarrow a^+$ .

**Steps to finding vertical asymptotes:**

1. **Factor numerator and denominator**
2. **Cancel common factors:** these give you the removable discontinuities
3. **Set remaining factors of the denominator = 0**
4. **Solve:** these are your vertical asymptotes

Exercise: Find the vertical asymptotes for  $f(x) = \frac{(x+4)(x-1)}{(x-1)(x+3)}$ .

**Horizontal Asymptote** : A horizontal asymptote of  $f$  is a value of  $y$  that  $f(x)$  approaches as  $x \rightarrow \infty$  or  $x \rightarrow -\infty$

**Steps to finding Horizontal Asymptotes:**

1. Find leading term of numerator
2. Find leading term of denominator
3. As  $x \rightarrow \infty$ ,  $\frac{\text{leading term of numerator}}{\text{leading term of denominator}} \rightarrow$ 
  - a.  $x \rightarrow \infty, \frac{1}{x^n} \rightarrow 0$
  - b.  $x \rightarrow \infty, x^n \rightarrow \infty$
  - c.  $x \rightarrow \infty, \frac{ax^n}{bx^n} \rightarrow \frac{a}{b}$

**Exercise:**

Find the horizontal asymptotes for

1.  $f(x) = \frac{x^4 + 3x - 1}{x^2 + 100}$

2.  $f(x) = \frac{x^2 + 3x - 10}{x^4 + 1}$

3.  $f(x) = \frac{x^4 + 3x - 1}{x^4 + 100}$

4.  $f(x) = \frac{(x-1)^2(x+2)}{(x-3)(x+1)}$