Linear Algebra
The Transpose of a Matrix

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Definition 1 (Transpose of a Matrix)**.** The transpose of a matrix $A = [a_{ij}]$ with a_{ij} as the entry in the i-th row and j-th column, is the matrix $A^T = [b_{ij}]$ where $b_{ij} = a_{ji}$. In other words, A^T switches the rows and columns.

For examples, see the matrices below:

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^T \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 4 & 3 \\ 8 & 2 & 6 \\ 7 & 8 & 3 \\ 4 & 9 & 6 \\ 7 & 8 & 1 \end{bmatrix} \qquad A^T \begin{bmatrix} 1 & 8 & 7 & 4 & 7 \\ 4 & 2 & 8 & 9 & 8 \\ 3 & 6 & 3 & 6 & 1 \end{bmatrix}$$

Exercise 1. Consider the matrix below:

$$A = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{-2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{pmatrix}$$

Write A^T in the space below.

Now compute the product $A^T A$ in the space below.

What do you notice? What does this say about the relative geometry of the columns of A? Write your answer in the space provided below.

Definition 2. A matrix A is *orthogonal* if and only if $A^T A = I$. Or in other words, its transpose is its inverse.

Definition 3. A matrix A is *symmetric* if and only if $A^T = A$. Or in other words, it is its own transpose.

Write an example of a symmetric $2x2$ matrix in the space provided. Make sure to check your work.

Definition 4. A matrix A is *skew-symmetric* if and only if $A^T = -A$. Or in other words, its the negative of its transpose.

Write an example of a skew-symmetric $2x2$ matrix in the space provided. Make sure to check your work.