

f, f', f'' Review Sheet

1. **critical point**

- a. f occurs at c if and only if $f'(c) = 0$ or $f'(c)$ does not exist.
- b. SO THE GRAPH of f' MUST NOT EXIST OR HIT THE X-AXIS

2. f is **increasing**

- a. $x < y$ implies $f(x) < f(y)$.
- b. $f'(x) > 0$.
- c. THE GRAPH OF f' MUST BE ABOVE THE X-AXIS

3. f is **decreasing**

- i. $x < y$ implies $f(x) > f(y)$.
- ii. $f'(x) < 0$
- iii. THE GRAPH OF f' MUST BE BELOW THE X-AXIS

4. f has a **local maximum**

- a. $f'(x)$ changes from positive to negative
- b. THE GRAPH OF f' CROSSES FROM ABOVE TO BELOW THE X-AXIS.

5. f has a **local minimum** if and only if the following are true:

- a. $f'(x)$ changes from negative to positive
- b. THE GRAPH OF f' CROSSES FROM BELOW TO ABOVE THE X-AXIS

6. You are asked to find a value $f(a)$

- a. Look at prompt for a given value, $f(b)$.
- b. Use the FUNDAMENTAL THEOREM OF CALCULUS

$$f(b) - f(a) = \int_a^b f'(t) dt$$

- c. You will need to find the integral of f' by adding the SIGNED areas between the graph and the x -axis.
- d. REMEMBER IF $b < a$ THEN THE SIGN SWITCHES!!!!

7. **absolute maximum** or **absolute minimum**

- a. Plug in relative max or min AND endpoints back into f
- b. USE FTC

8. Tangent line equation at $x = a$: $y - f(a) = f'(a)(x - a)$

- a. To find $f'(a)$, look at the y -value on the graph of f' .
- b. To find $f(a)$, use FTC or see if you were given the value in the prompt.

9. An **inflection point** of f

- a. $f''(x)$ changes sign
- b. THE GRAPH OF $f'(x)$ changes direction
- c. $f(x)$ changes concavity

d. The tangent line crosses the graph at the point of tangency.

10. f is **concave up** is equivalent to the following

- a. f'' is positive
- b. THE GRAPH OF f' is increasing
- c. f is changing at a increasing rate.
- d. the tangent line to f is BELOW the graph

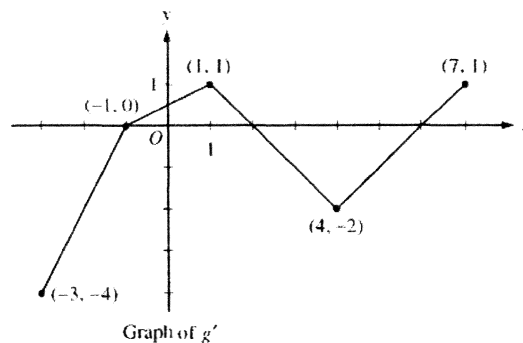
11. f is **concave down** is equivalent to the following

- a. f'' is negative
- b. THE GRAPH OF f' is decreasing
- c. f is changing at a decreasing rate.
- d. the tangent line to f is ABOVE the graph

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Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.



- (a) Find the x -coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.
 - (b) Find the absolute maximum value of g on the interval $-3 \leq x \leq 7$. Justify your answer.
 - (c) Find the average rate of change of $g(x)$ on the interval $-3 \leq x \leq 7$.
 - (d) Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?
-

- (a) g' changes from increasing to decreasing at $x = 1$;
 g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at
 $x = 1$ and $x = 4$.

- (b) The only sign change of g' from positive to negative in
the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x) dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$

$$g(2) = 5$$

$$g(7) = 5 + \int_2^7 g'(x) dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \leq x \leq 7$ is $\frac{15}{2}$.

(c)
$$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

(d)
$$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a
value c with the stated properties because g' is not
differentiable for at least one point in $-3 < x < 7$.

$$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$$

$$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$$

$$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$$

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Question 4

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

- (a) Find the x -coordinate of the critical point of f . Determine whether the point is a relative maximum, a relative minimum, or neither for the function f . Justify your answer.
- (b) Find all intervals on which the graph of f is concave down. Justify your answer.
- (c) Given that $f(1) = 2$, determine the function f .

- (a) $f'(x) = 0$ at $x = 4$
 $f'(x) > 0$ for $0 < x < 4$
 $f'(x) < 0$ for $x > 4$
 Therefore f has a relative maximum at $x = 4$.

$$3 : \begin{cases} 1 : x = 4 \\ 1 : \text{relative maximum} \\ 1 : \text{justification} \end{cases}$$

- (b) $f''(x) = -x^{-3} + (4 - x)(-3x^{-4})$
 $= -x^{-3} - 12x^{-4} + 3x^{-3}$
 $= 2x^{-4}(x - 6)$
 $= \frac{2(x - 6)}{x^4}$
 $f''(x) < 0$ for $0 < x < 6$

$$3 : \begin{cases} 2 : f''(x) \\ 1 : \text{answer with justification} \end{cases}$$

The graph of f is concave down on the interval $0 < x < 6$.

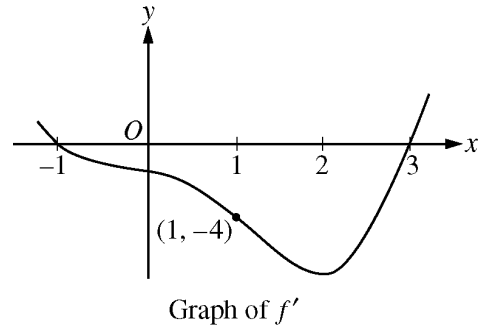
- (c) $f(x) = 2 + \int_1^x (4t^{-3} - t^{-2}) dt$
 $= 2 + \left[-2t^{-2} + t^{-1} \right]_{t=1}^{t=x}$
 $= 3 - 2x^{-2} + x^{-1}$

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

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Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f' , the derivative of f , is shown above. The graph of f' crosses the x -axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



- (a) Write an equation for the line tangent to the graph of g at $x = 1$.
- (b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.
- (c) The second derivative of g is $g''(x) = e^{f(x)}[(f'(x))^2 + f''(x)]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.
- (d) Find the average rate of change of g' , the derivative of g , over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$
 $g'(x) = e^{f(x)} f'(x)$, $g'(1) = e^{f(1)} f'(1) = -4e^2$
 The tangent line is given by $y = e^2 - 4e^2(x - 1)$.

3 : $\begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$

(b) $g'(x) = e^{f(x)} f'(x)$
 $e^{f(x)} > 0$ for all x
 So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g''(-1) = e^{f(-1)} [(f'(-1))^2 + f''(-1)]$
 $e^{f(-1)} > 0$ and $f'(-1) = 0$
 Since f' is decreasing on a neighborhood of -1 , $f''(-1) < 0$. Therefore, $g''(-1) < 0$.

2 : $\begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

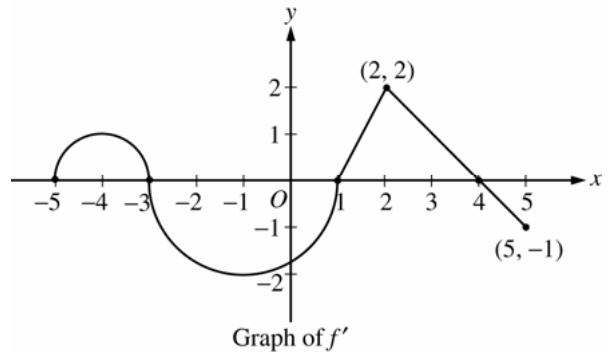
(d) $\frac{g'(3) - g'(1)}{3 - 1} = \frac{e^{f(3)} f'(3) - e^{f(1)} f'(1)}{2} = 2e^2$

2 : $\begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

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Question 4

Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.



- (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

- (a) $f'(x) = 0$ at $x = -3, 1, 4$
 f' changes from positive to negative at -3 and 4 .
 Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : { 1 : x-values
 1 : justification

- (b) f' changes from increasing to decreasing, or vice versa, at $x = -4, -1$, and 2 . Thus, the graph of f has points of inflection when $x = -4, -1$, and 2 .

2 : { 1 : x-values
 1 : justification

- (c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : { 1 : intervals
 1 : explanation

- (d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

3 : { 1 : identifies $x = 1$ as a candidate
 1 : considers endpoints
 1 : value and explanation

$$f(-5) = 3 + \int_1^{-5} f'(x) dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(x) dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

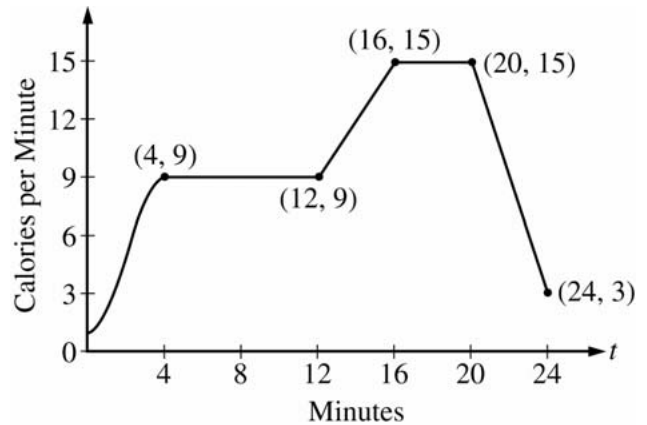
The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

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Question 4

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function

f . In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \leq t \leq 4$ and f is piecewise linear for $4 \leq t \leq 24$.



- (a) Find $f'(22)$. Indicate units of measure.
- (b) For the time interval $0 \leq t \leq 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval $6 \leq t \leq 18$ minutes.
- (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \leq t \leq 18$.

(a) $f'(22) = \frac{15 - 3}{20 - 24} = -3$ calories/min/min

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \frac{15 - 9}{16 - 12} = \frac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\frac{3}{4}t^2 + 3t$ and

$f''(t) = -\frac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \frac{3}{2}$.

(c) $\int_6^{18} f(t) dt = 6(9) + \frac{1}{2}(4)(9 + 15) + 2(15)$
 $= 132$ calories

(d) We want $\frac{1}{12} \int_6^{18} (f(t) + c) dt = 15$.

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\frac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c .

So $c = 4$ to get an average of 15 calories/min.

1 : $f'(22)$ and units

4 : $\begin{cases} 1 : f' \text{ on } (0, 4) \\ 1 : \text{shows } f' \text{ has a max at } t = 2 \text{ on } (0, 4) \\ 1 : \text{shows for } 12 < t < 16, f'(t) < f'(2) \\ 1 : \text{answer} \end{cases}$

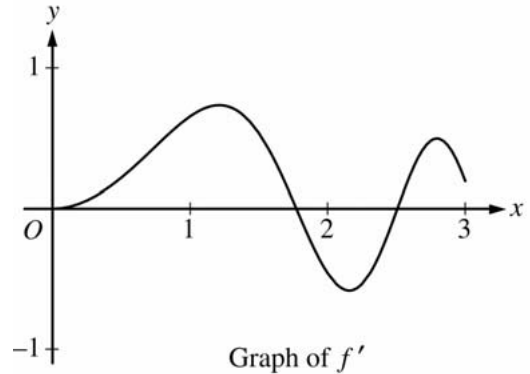
2 : $\begin{cases} 1 : \text{method} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{setup} \\ 1 : \text{value of } c \end{cases}$

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Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.



- (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
- (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \dots$
 On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$$f(0) = 5$$

$$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x) dx = 5.67911$$

$$f(3) = f(0) + \int_0^3 f'(x) dx = 5.57893$$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

(c) $f(2) = f(0) + \int_0^2 f'(x) dx = 5.62342$

$$f'(2) = e^{-0.5} \sin(4) = -0.45902$$

$$y - 5.623 = (-0.459)(x - 2)$$

2 : { 1 : answer
 1 : reason

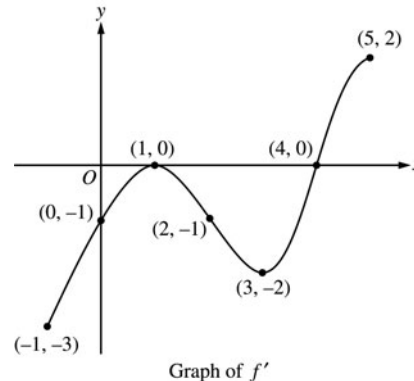
3 : { 1 : identifies $\sqrt{\pi}$ and 3 as candidates
 - or -
 indicates that the graph of f increases, decreases, then increases
 1 : justifies $f(\sqrt{\pi}) > f(3)$
 1 : answer

4 : { 2 : $f(2)$ expression
 1 : integral
 1 : including $f(0)$ term
 1 : $f'(2)$
 1 : equation

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Question 4

The figure above shows the graph of f' , the derivative of the function f , on the closed interval $-1 \leq x \leq 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.



- (a) Find the x -coordinate of each of the points of inflection of the graph of f . Give a reason for your answer.
- (b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \leq x \leq 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \leq x \leq 5$? Show the analysis that leads to your answers.
- (c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

- (a) $x = 1$ and $x = 3$ because the graph of f' changes from increasing to decreasing at $x = 1$, and changes from decreasing to increasing at $x = 3$.

$$2 : \begin{cases} 1 : x = 1, x = 3 \\ 1 : \text{reason} \end{cases}$$

- (b) The function f decreases from $x = -1$ to $x = 4$, then increases from $x = 4$ to $x = 5$. Therefore, the absolute minimum value for f is at $x = 4$. The absolute maximum value must occur at $x = -1$ or at $x = 5$.

$$4 : \begin{cases} 1 : \text{indicates } f \text{ decreases then increases} \\ 1 : \text{eliminates } x = 5 \text{ for maximum} \\ 1 : \text{absolute minimum at } x = 4 \\ 1 : \text{absolute maximum at } x = -1 \end{cases}$$

$$f(5) - f(-1) = \int_{-1}^5 f'(t) dt < 0$$

Since $f(5) < f(-1)$, the absolute maximum value occurs at $x = -1$.

- (c) $g'(x) = f(x) + xf'(x)$
 $g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$
 $g(2) = 2f(2) = 12$

$$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$$

Tangent line is $y = 4(x - 2) + 12$