On the Horizontal Axis Wind Turbine’s Efficiency

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Abstract: It is known for a long time in the wind power industry that horizontal axis wind turbines (HAWTs) have higher wind energy utilization rate, but a lot of operating wind farms find that the actual utilization rate of HAWTs is not able to reach the level claimed by manufacturers, resulting in the loss of many newly built wind farms. However the wind farms lack evidence to prove it. This paper discusses the defects in methods of calculating the efficiency of HAWTs, pointing out the theoretical causes of miscalculation, and by correcting the efficiency of a certain type of HAWT, proving its actual efficiency is far less than the calculated value. The purpose is to make would-be wind farm operators treat the utilization indicators provided by manufacturers rationally, avoiding unnecessary losses.

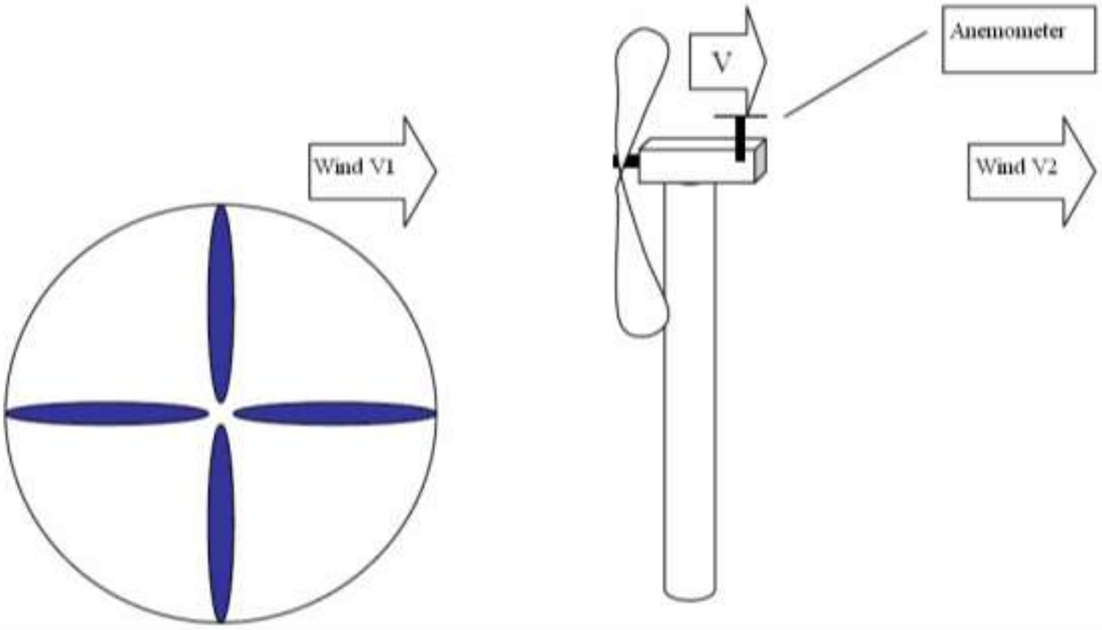

Figure 1 Schematic of horizontal axis wind turbine

1. Betz theory

a. Glossary

Solidity ratio: the ratio of total windward area of blades to swept area of the wind rotor.

Tip-speed ratio: the ratio of the blade tip linear speed to the wind speed.

b. Assumptions:

1) there is no cone angles, inclinations, deflections in a wind rotor
2) the wind is not viscous
3) wind turbine flow model can be simplified as a unit flow tube

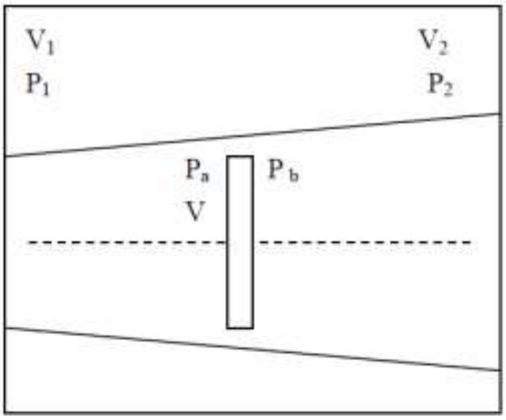

Figure 2 Schematic diagram of Betz theory

4) front and rear air static pressure of the wind rotor are equal P1 = P2
5) thrust acting on the wind rotor is even

c. Formula
Thrust T acting on the wind turbine is T=m(V1-V2), wherein V1 is the inflow velocity, V2 is the wind velocity at indefinite far after the air flows through the wind rotor, m = ρSV is the mass flow per unit of time.

Based on the pressure difference before and after the wind rotor, the thrust acting on the wind rotor can be expressed as T = S (Pa-P b), wherein Pa is the pressure in front of the wind rotor, and Pb is the air flow pressure behind the wind rotor.

According to Bernoulli’s equation, we can obtain:
 1/2ρV12 +P1= 1/2ρV2 +Pa
 1/2ρV22 +P2= 1/2ρV2 +Pb
 V=1/2(V1+V2)
Let V=V1(1-a), therefore V2=V1(1-2a)

V2/V1=(1-2a) is the ratio of the wind velocity at indefinite far after the air flows through the wind turbine to the incoming flow velocity,

a=(1-V2/V1)/2 is turbulence factor,

Therefore the maximum shaft power of a HAWT is
P=m(V12/2-V22/2)
P=2ρSV13 a(1-a)2

And the maximum shaft power is occurred at dp/da=0, i.e.
dp/da=2ρSV13(1-4a+3a2)=0, when a=1/3, i.e. (when V2/ V1=1/3)
Pmax=16/27(0.5ρSV13)
Cp=P/0.5ρSV13
Cpmax=16/27=0.593
$Cp=4a(1-a)^2$ （1）
a=(1-V2/V1)/2
When V2 / V1 is 1/2, i.e. a=1/4， Cp1/2 =0.563
WhenV2 / V1 is 2/3， i.e. a=1/6， Cp1/6 =0.463
When V2 / V1 is 7/10， i.e. a=3/20， Cp7/10 =0.434
When V2 / V1 is 15/20， i.e. a=5/40， Cp5/40 =0.383
When V2 / V1 is 8/10， i.e. a=1/10， Cp8/10 =0.324
When V2 / V1 is 9/10， i.e. a=1/20， Cp9/10 =0.18

From the above calculation results, we know that wind energy utilization rate depends on the size of value of the turbulence factor, i.e. the ratio of the wind velocity at indefinite far after the air flows through the wind turbine to the incoming flow velocity, and based on the law of conservation of energy, it can be understood as after the wind pass through the wind rotor, the wind speed drops from V1 to V2 because part of the wind energy has been absorbed by the wind rotor. With the wind speed V2 become continuously close to V1, the turbulence factor a would drop dramatically, resulting in the rapid drop in the value of Cp.

2. Blade element theory

The Betz theory obtains the theoretical Cp values merely from law of mass conservation, momentum and energy conservation theorems. However, the blade element theory divides a blade into many micro-segments (blade elements), and regards the various blade elements of the relative flow as separate two-dimensional flows. Aerodynamic and moments of blade profiles can be obtained based on airfoil theory, and then integral along the blade radius, then averaged along the azimuth, we can obtain the entire blade aerodynamic and torque. The deficiency of the blade element theory is that it ignores the mutual interference between the various blade elements. The results of blade element theory are more accurate than those of the Betz theory, since the former takes into account of the effects of axial turbulence on the velocity, but also take into account of loss of the tip.

However, due to the deficiency in application of blade element theory into design HAWTs, in the calculation process the drag on the blade during rotation can not be considered. Although aerodynamic drag on the HAWT airfoil design is not significant, the effect on the wind energy conversion efficiency Cp is significant. Therefore, the blade element theory calculation method is more suitable for blade design and calculation, not suitable for the direct calculation of the HAWT wind energy utilization Cp values. If the blade element theory method is used directly to calculate the wind utilization rate of HAWT, the results must be corrected.

In fact, for HAWTs, the blade solidity is chosen to be small in order to obtain higher tip-speed ratio, especially in the tip portion of a blade it is designed to be very narrow, and the results is less wind energy contacted with blades. Further, among the energy swept the blades, a large part create axial thrust bending the blades, and still less wind energy is converted into mechanical energy, which explains the reasons for the low efficiency of HAWTs. The following are two widely used methods to apply the blade element theory to the design of HAWTs.

Glauert method obtains energy equations and Cp formulas, while ignoring the loss of blade drag and tip loss on HAWT Cp values:

$b(1+b)\lambda2=a(1-a)$

$Cp=8/\lambda02\int0\lambda0b(1-a)\ \lambda3d\lambda=2b(1-a)\ \lambda02$

Wherein a is the axial turbulence factor, b is tangential turbulence factor, and λ is the tip-speed ratio.

To correct the tip loss in the Glauert method, in Wilson method, blade tip losses is considered, but not the resistance, and under such conditions the energy equation and Cp formula are:
$a(1-aF)=b(1+b)\ \lambda2$, wherein F is blade tip loss factor.
$dCp=\{(8/\lambda02)b(1-a)F\lambda3d\lambda\}$, wherein dCp is the wind energy utilization rates at different sections.

Currently Wilson method is widely used for designing blades for HAWTs and calculating wind energy utilization rate.

3. Cp value corrections

Figure 4 shows the power curve of a Goldwind 600 kw wind turbine (S43/600). During the operation of the wind turbine, the internal computer created the power curve based on the live samplings of the wind speeds and corresponding output. The wind speeds measured are those in the rear of the nacelle. The wind speeds measured is not V2 or V1, and it is smaller than V but can be regarded approximately as V. According to the definition of Cp, we get Cp = p / (0.5 * ru * S * v3). In theoretical calculation, v is the incoming flow velocity V1. However, in actual sampling of the computer, v is the velocity V measured by the anemoscope, and V is smaller than V1. Therefore the Cp values shall be corrected.

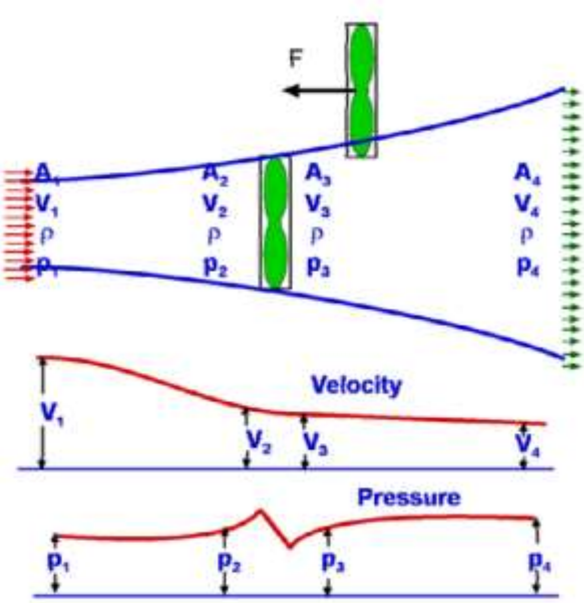

Figure 3 Airflow velocity, pressure distribution curve after the wind passing the wind rotor

The left figure shows the actual wind velocity and air pressure after the wind passing the wind rotor. V_1 is the incoming flow velocity, and V_2 and V_3 are wind velocity before and after the wind rotor respectively, and close to the V in Betz theory. V_4 is wind speed at indefinitely far, and is the wind speed V_2 in the Betz theory.

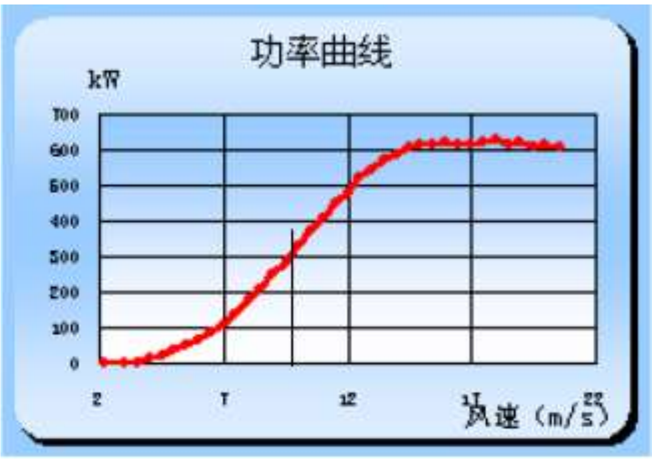

Figure 4 Power curve of Goldwind S43/600 wind turbine generated by computer wind speed samplings

The turbine's swept area is 1466 m^2, and from the power curve generated by computer live samplings, we can see that when the anemoscope measured a wind speed of 9.5 m / s, the output is 300 kw, and the Cp value is about 39%.

When the output is 300 kwh, the wind speed measured by the anemometer is 9.5 m, and the corresponding Cp value is 39%. The wind turbine converts wind energy to mechanical energy and to electricity, and the loss is believed to be 30%, i.e. the efficiency is 0.7. Therefore, the theoretical value of Cp is 0.39/0.7 =55.7%. According to Betz theory, when the theoretical value Cp is 55.7%, by the equation (1) a is calculated at 0.242. Therefore V1 = V / (1-a) = 9.5 / (1-0.24) = 12.53 m / s, that is, by the corrected inflow speed is 12.53 m/s, and then the corrected Cp is 16.97% based on the power curve. The Cp values decline by 67% after the first correction, and apparently the Cp values are significantly overestimated.

Based on the first corrected Cp value 16.97%, and the theoretical Cp value is 0.1697/0.7 = 24.2%. According to Betz theory, calculated by the formula (1) and we know a = 0.0698, thus V1 = V / (1-a) = 9.5 / (1-0.0698) = 10.2 m/s, i.e., after the correction inflow velocity is 10.2 m / s, and then based on the power curve, we obtain the second corrected Cp value of 31.4%, higher than the first corrected value.

Based on the second corrected Cp value 31.4%, and the theoretical Cp value is 0.314/0.7=44.86%. According to Betz theory, calculated by the formula (1) and we know a = 0.1575, thus V1 = V / (1-a) = 9.5 / (1-0.1575) = 11.28 m/s, i.e., after the correction inflow velocity is 11.28 m/s, and then based on the power curve, we obtain the third corrected Cp value of 23.3%, lower than the second corrected value.

Corrected time	Corrected a value	Corrected velocity	Corrected Cp value
0	0.0000000	0.000000	0.3900000
1	0.2420000	12.53298	0.1697141
2	0.6980000	10.21286	0.3136458
3	0.1575000	11.27596	0.2330344
4	0.1032000	10.59322	0.2810588
5	0.1333000	10.96112	0.2536978
6	0.1155000	10.74053	0.2696522
7	0.1257000	10.86584	0.2604305
8	0.1198000	10.79300	0.2657385
9	0.1231000	10.83362	0.2627608
10	0.1212000	10.81020	0.2644725
11	0.1223000	10.82374	0.2634806
12	0.1217000	10.81635	0.2640213
13	0.1220000	10.82005	0.2637509
14	0.1219000	10.81881	0.2638410

By repeatedly correction, the actual Cp value of S43/600 turbine is approximately 26.4%, but because in the correction process we assume that the wind speed measured behind the wind rotor close to V, while the V values in Betz theory is the wind speed in front of the wind rotor which is higher than that measured behind the wind rotor. Therefore, the real Cp value is slightly smaller than the corrected value. For example, assume the wind velocity measured behind the wind rotor is 90% of the V value, corrected by the above methods, V_1 is 11.62 m /s, and the ultimate Cp is 21.3%.

The reason why the corrected Cp value is far less than the theoretical value is that the application of blade element theory calculation assumptions differ from the actual situation greatly, in particularly ignoring the effects of air resistance on the Cp values. Therefore, blade element theory is more suitable for HAWTs blade design, not for calculating wind energy utilization rate of HAWTs.

In addition, regarding the efficiency of HAWTs with variable pitch, the purpose of pitching is to obtain maximum Cp values under any wind speeds and improving the mean Cp value under various wind speeds, not the extreme values. Therefore, the power curves of HAWTs with fixed pitch and variable pitch will converge at the maximum Cp value.