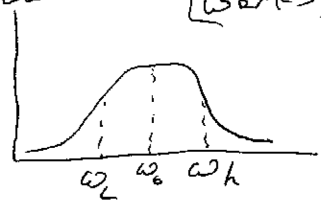
Band Pass 4th order Butterworth [works]



$f_s = 1.0$

$$\omega_A = \tan\left(\frac{2\pi f_L \, {}^1\!/\!f_s}{2}\right) = \tan(\pi f_L)\ ; \quad 0 < f_L < 0.5$$

$$\omega_B = \tan(\pi f_h) \quad f_L < f_h < 0.5$$

$$\omega_0 = \sqrt{\omega_L \omega_h} \quad \text{center logarithmic} \Rightarrow \omega_0 = \sqrt{\omega_A \omega_B}$$

$$W = \omega_h - \omega_L \Rightarrow W = \omega_B - \omega_A$$

f_s = Sample freq

f_L = HighPass Freq f_c/f_s

f_h = LowPass Freq f_c/f_s

$$P_n(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

$$R_n(s) = P_n(s)\Big|_{s = \frac{s^2+\omega_0^2}{Ws}} = \frac{W^2 s^2}{(s^2+\omega_0^2)^2 + \sqrt{2}\, W s (s^2+\omega_0^2) + W^2 s^2}$$

$$= \frac{W^2 s^2}{s^4 + 2s^2\omega_0^2 + \omega_0^4 + \sqrt{2}\, W s^3 + \sqrt{2}\, W \omega_0^2 s + W^2 s^2}$$

$$= \frac{W^2 s^2}{s^4 + (2\omega_0^2 + W^2) s^2 + \sqrt{2}\, W s^3 + \sqrt{2}\, W \omega_0^2 s + \omega_0^4}$$

$$= \frac{W^2 s^2}{s^4 + \underbrace{\sqrt{2}\, W}_{a_1} s^3 + \underbrace{(2\omega_0^2 + W^2)}_{a_2} s^2 + \underbrace{\sqrt{2}\, W \omega_0^2}_{a_3} s + \underbrace{\omega_0^4}_{a_4}}$$

$$H(z)\Big|_{s=\frac{(z-1)}{(z+1)}} = \frac{W^2 (z-1)^2 (z+1)^2}{(z-1)^4 + a_1 (z-1)^3 (z+1) + a_2 (z-1)^2 (z+1)^2 + a_3 (z-1)(z+1)^3 + a_4 (z+1)^4}$$

$(z-1)^4$	$=$	z^4	$-4z^3$	$+6z^2$	$-4z$	$+1$
$a_1(z-1)^3(z+1)$	$=$	$a_1 z^4$	$-2a_1 z^3$		$+2a_1 z$	$-a_1$
$a_2(z-1)^2(z+1)^2$	$=$	$a_2 z^4$		$-2a_2 z^2$		a_2
$a_3(z-1)(z+1)^3$	$=$	$a_3 z^4$	$+2a_3 z^3$		$-2a_3 z$	$-a_3$
$a_4(z+1)$	$=$	$a_4 z^4$	$+4a_4 z^3$	$+6a_4 z^2$	$+4a_4 z$	$+a_4$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{W^2[z^4 - 2z^2 + 1]}{\underbrace{(1+a_1+a_2+a_3+a_4)}_{A} z^4 + \underbrace{(-4-2a_1+2a_3+4a_4)}_{B} z^3 + \underbrace{(6-2a_2+6a_4)}_{C} z^2 + \underbrace{(-4+2a_1-2a_3+4a_4)}_{D} z + \underbrace{(1-a_1+a_2-a_3+a_4)}_{E}}$$

$$H(z) = \frac{W^2[1 - 2z^{-2} + z^{-4}]}{A + Bz^{-1} + Cz^{-2} + Dz^{-3} + Ez^{-4}}$$

$$Y(z)[A + Bz^{-1} + Cz^{-2} + Dz^{-3} + Ez^{-4}] = X(z)[W^2 - 2W^2 z^{-2} + W z^{-4}]$$

$$y(n) = \frac{1}{A}\left[W^2 x(n) - 2W^2 x(n-2) + W^2 x(n-4) - B y(n-1) - C y(n-2) - D y(n-3) - E y(n-4)\right]$$