4 pole Band Stop Butterworth

$$P_h(s) = \frac{s^2}{s^2+\sqrt{2}\,s+1}$$ 2 pole Butterworth High Pass

$$\omega_0 = \sqrt{\omega_L \omega_h}$$

$$W = \omega_L - \omega_h$$

$$R_h(s) = P_h(s)\Big|_{s=(s^2+\omega_0^2)/Ws} = \frac{(s^2+\omega_0^2)^2}{(s^2+\omega_0^2)^2+\sqrt{2}\,Ws(s^2+\omega_0^2)+W^2s^2}$$

Same denominator as Band Pass

$$H(z) = R_h(s)\Big|_{s=\frac{z-1}{z+1}} = \frac{\left(\frac{z-1}{z+1}\right)^4+2\left(\frac{z-1}{z+1}\right)^2\omega_0^2+\omega_0^4}{\text{Deno.}} = \frac{(z-1)^4+2\omega_0^2(z-1)^2(z+1)^2+\omega_0^4(z+1)^4}{\text{Demo}}$$

$$(z-1)^4 = z^4 \quad -4z^3 \quad +6z^2 \quad -4z \quad +1$$

$$2\omega_0^2(z-1)^2(z+1)^2 = 2\omega_0^2 z^4 \quad -4\omega_0^2 z^2 \quad +2\omega_0^2$$

$$\omega_0^4(z+1) = \omega_0^4 z^4 \quad +4\omega_0^4 z^3 \quad +6\omega_0^4 z^2 \quad +4\omega_0^4 z \quad +\omega_0^4$$

$$H(z) = \frac{(1+2\omega_0^2+\omega_0^4)z^4+(-4+4\omega_0^4)z^3+(6-4\omega_0^2+6\omega_0^4)z^2+(-4+4\omega_0^4)z+(1+2\omega_0^2+\omega_0^4)}{\text{Demo}}$$

$$y(n) = \frac{1}{A}\Big[(1+2\omega_0^2+\omega_0^4)x(n)+(-4+4\omega_0^4)x(-1)+(6-4\omega_0^2+6\omega_0^4)x(-2)+(-4+4\omega_0^4)x(-3)+(1+2\omega_0^2+\omega_0^4)x(-4)$$

$$-By(-1)-Cy(-2)-Dy(-3)-Ey(-4)\Big]$$