# 2 Pole high pass Butterworth IIR

## Bilinear Transformation

Start with BW low pass equation and let $s = \frac{1}{s}$

$$\text{LP Butterworth} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$\text{HP Butterworth} = \frac{s^2}{1 + \sqrt{2}s + s^2}$$

Let $\hat{Q}(s) = Q\big|_{s = s/W_A} = \dfrac{s^2}{W_A^2 + \sqrt{2}\,W_A s + s^2}$ where $W_A = \tan(\pi f_c')$, $f_c' = f_c/f_s$

We have an equation for a highpass BW

$$H(z) = \frac{Y(z)}{X(z)} = \hat{Q}(s)\Big|_{s=\frac{z-1}{z+1}} = \frac{(z-1)^2}{(z+1)^2 W_A + \sqrt{2}W_A(z-1)(z+1) + (z-1)^2}$$

We need $z$ in $-n$ so $\frac{z^{-2}}{z^{-2}} \cdot H(z) = \frac{z^{-2}}{z^{-2}} \frac{Y(z)}{X(z)}$ solve for $z$.

$$\frac{Y(z)}{X(z)} = \frac{z^2 - 2z + 1}{(z^2 + 2z + 1)W_A^2 + \sqrt{2}\,W_A(z^2 - 1) + (z^2 - 2z + 1)}$$

$$\begin{array}{lclll}
(z^2 + 2z + 1)W_A & \Rightarrow & W_A^2 z^2 & + 2W_A^2 z & + W_A^2 \\
\sqrt{2}W_A(z^2 - 1) & \Rightarrow & \sqrt{2}W_A z^2 & 0 & -\sqrt{2}W_A \\
z^2 - 2z + 1 & \Rightarrow & z^2 & -2z & + 1 \\
 & & \Downarrow & \Downarrow & \Downarrow \\
 & & A & B & C
\end{array}$$

$$\frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} + z^{-2}}{A \underbrace{\cancel{-2(W_A^2 - 1)}}_{B} z^{-1} + Cz^{-2}}$$

$$Y(z)\left[A - Bz^{-1} + Cz^{-2}\right] = X(z)\left[1 - 2z^{-1} + z^{-2}\right]$$

$$y(n) = \frac{1}{A}x(n) - \frac{2}{A}x(n-1) + \frac{1}{A}x(n) + \frac{B}{A}y(n-1) - \frac{C}{A}y(n-2)$$