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Numerical Study of the Impact of Wake Surfing on Inland Bodies of Water

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Abstract

In many areas around the world, wake surfing has been cited as one of the major causes of lakeshore erosion and turbidity. This paper quantifies the impact related to turbidity and erosion with the use of computational fluid dynamics (CFD) of boat wakes in shallow water and the build-up of wind driven waves. The energy, type and direction of the boat's wake are described quantitatively and a table for predicting wind driven waves over varying fetches, depth and wind speeds is provided. The CFD simulation shows that if a wake surf boat is operated 200 ft from shore and in at least 10 ft of water, the environmental impact is minimal.

Keywords

CFD, Erosion, Waves, Wakes, Wind Driven Waves

1. Introduction

Wake surfing is a water sport in which a surfer rides a boat generated wake without the need for a direct connection to the boat. This contrasts with water skiing and wake boarding, which require the use of a tow rope to pull the rider. Wake surfing has rapidly gained popularity over the last few decades, causing the watercraft industry to introduce hulls specifically designed to optimize the wake shape and size for surfing. Typically, this is done by adding ballast such as water tanks to the back of the hull and utilizing special tabs to change the hull orientation. Wake surf boat operation can be segregated into two modes: surf mode and planning mode. Surf mode uses the ballast and/or tabs to create a wake capable of pulling a surfer without need of a tow rope (see **Figure 1**). Typical speeds range from 10 - 13 mph. Planning mode, defined further in Section 2, is a more efficient mode of operation for travelling longer distances and is the most widely used mode of operation for power boats. Typical planning speeds for recreational craft range from 20 to 60 mph.

Due to the increased frequency of wake surf boat activity in recent years, users and residents of inland bodies of water have raised the question of whether wake surfing is detrimental to the shoreline and bottom. The purpose of this study is to examine the actual impact of wake surf boats on the shoreline and water bottom using computational fluid dynamics (CFD).

CFD utilizes high performance computing to numerically solve the equations governing fluid flow. It has gained popularity in the marine industry for product development over the last 20 years due to the increasing availability of reasonably priced computing power. Many studies have been performed to validate CFD results and prove their usefulness for predicting real world performance. A well-known example for non-planning hulls such as large container ships is the 2016 Lloyd Register blind CFD workshop. Participants using various software packages were not allowed access to test results ahead of time, and all simulation results predicted speed within 4% of test data. The 2018 Multi Agency Craft Conference (MACC) generic prismatic planning hull (GPPH) simulation grand challenge is another case specific to planning hulls with a similar match between CFD and test data. Additionally, CFD has broad ranging applications in other fields such as the aerospace, automotive, and process industries.

This study starts with a discussion of linear wave theory and the wave generation power of different boat types. Then, CFD simulations using the OpenFOAM solver are performed on a popular wake surf boat to compare the impact of vessel weight, vessel speed, water depth, and distance from shore on wake propagation. Simulations are additionally performed using Siemens Star-CCM+ to solve for the interaction of wake surf boat propeller wash with various bottom depths. Finally, waves generated by the wind for various lake sizes and wind speeds are discussed and a CFD simulation is performed for validation. The purpose of the research is to accurately model the wake behind a planning craft and how it dissipates over time and distance.

Figure 1. Typical wake boat in surf mode.

2. Boat Generated Waves

The wake behind a boat is a combination of a group of wave patterns that combine into one train behind the craft. **Figure 2** shows an aerial view of the typical wake of a high-speed planning craft. The energy applied to the water includes a viscous effect that generates heat through hull friction and an inertial effect that generates the group of waves on the free surface of the water. The energy in the water is proportional to the power provided to the propeller. The general dimensions of power into the water are the vessels' resistance times the speed. The resistance components are frictional resistance and wave making resistance. The frictional resistance is the viscous effect of the water running over the surface of the boat. The drag generates a boundary layer that builds based on the length and speed of the boat. The wave making resistance is the remainder of the resistance that the propeller must overcome to allow the boat to travel at a particular speed.

The type of boat has an impact based on the normal operating speed of the boat. A small sailboat, kayak or canoe generates little or no wake because of the size of the boat as well as speed. These small boats are running at their hull speed or below depending on the wind speed or person paddling. The hull speed is generally estimated as the square root of the waterline length of the craft in nautical miles per hour. For statute miles per hour, we can multiply by 1.15. For example, a twelve-foot canoe has a hull speed of 3.5 knots or 4 miles per hour (mph). In the case of a 23-foot wake boat with a waterline length of 19.5 feet, the hull speed is 5 miles per hour. In a displacement type of hull, the hull will sink into the water as the speed increases. In the case of a large ship, the resistance increases dramatically at this speed and makes it economically impossible to go any faster than hull speed.

In the case of the motorboat, the hull has been designed as a planning craft that is able to plane on the surface of the water much like a waterski on the surface of the water. The boat has the power available to go beyond hull speed and begin to

Figure 2. Typical wave pattern of a planning craft.

plane where the center of gravity will begin to rise. The term is generally noted as semi-displacement speeds and finally planning speeds. The planning speed is generally defined as the point where the center of gravity is finally passed above the vertical center when the boat is at rest and the hull is supported by the hydrodynamic lift of the hull running over the surface of the water. The Froude number for planning craft is generally taken as the following.

$$F_{\nabla} = \frac{V}{\sqrt{g\nabla^{\frac{1}{3}}}} \tag{1}$$

where:

V: Speed in feet per second;

g: acceleration due to gravity = 32.17 feet/sec^2;

∇ : Volumetric Displacement = cubic feet.

The transition to planning speeds is estimated at a volumetric Froude number of 2. For a wake boat at a displacement of 7100 pounds, the full planning speed is 25 feet per second or 17 mph (27 kph). At speeds below 17 mph, the boat is semi-planning and is partially supported by buoyancy. The normal wake surfing speed is 12 mph (19.3 kph) which is midway between the displacement speed and planning speed with a Froude number of 1.41. The total power applied to the water is the engine power absorbed by the propeller. The wake of the propeller generates the thrust needed to propel the boat and overcome the resistance generated by the frictional and wave making components. The propeller efficiency is proportional to the thrust times speed divided by the torque times engine RPM. The thrust from the propeller is equal to the resistance of the boat plus a thrust deduction to overcome the added resistance caused by the high velocity water off the propeller that strikes the rudder and reduces the pressure under the hull increasing the trim of the boat. The thrust deduction is generally estimated at ten percent so the required propeller thrust can be estimated at approximately 110% of the estimated resistance of the boat.

The wave making resistance is proportional to the weight of the boat and the running trim angle. The applied force to the surface of the water is estimated by the following equation [1].

$$F_w = \Delta \tan(\tau) \tag{2}$$

where:

F_w: force applied to the free surface;

Δ: boats displacement in lbf;

τ: running trim angle at speed.

Figure 3 shows the boats' resistance curves in the free running and ballast conditions.

The free running displacement is estimated at 5500 lbs and the ballast conditions are estimated at 7100 lbs and 10,500 lbs. The data is assumed to represent an average wake boat operating in North America. The condition of interest is the wake surfing condition at 12 mph in the ballasted conditions to represent the largest wake developed by this type of craft. The speed versus engine RPM is shown in **Figure 4** and the engine power is shown in **Figure 5**. The energy in the wake is a combination of the kinetic and potential energy components.

The results of the ballasted condition at 7200 lbs (3264 kg) are:

Vessel Speed 12 mph (19.3 kph);

Total Resistance 1510 lbf (6.72 kN);

Engine RPM 2410;

Engine Power 104 horsepower (77.6 kW);

Overall Efficiency 46.5% (Propeller is cavitating badly).

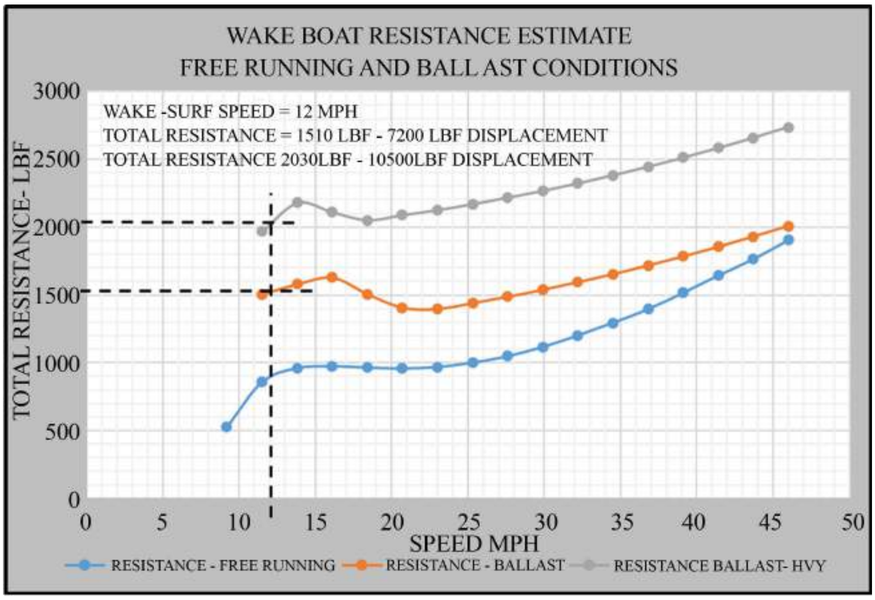

Figure 3. Wake boat resistance estimate.

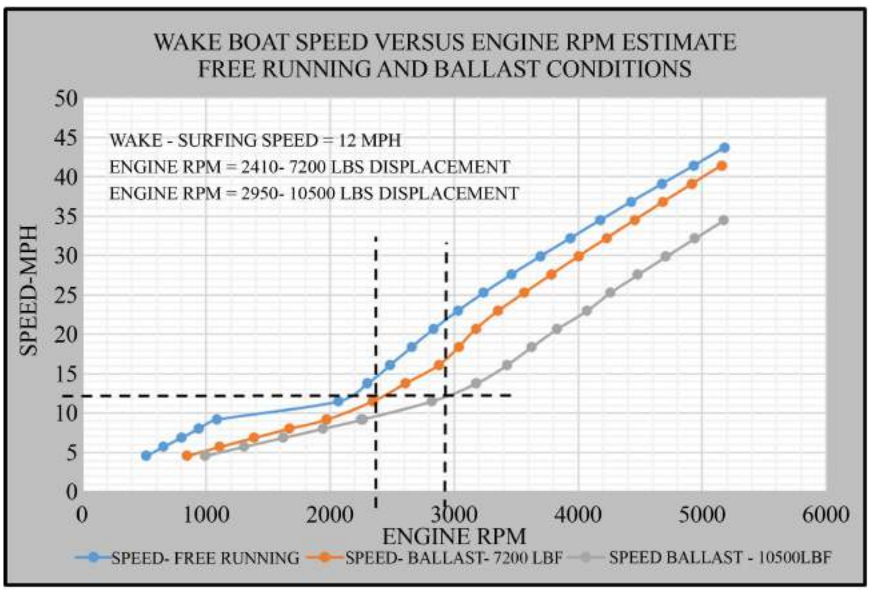

Figure 4. Speed versus engine RPM for a typical wake boat.

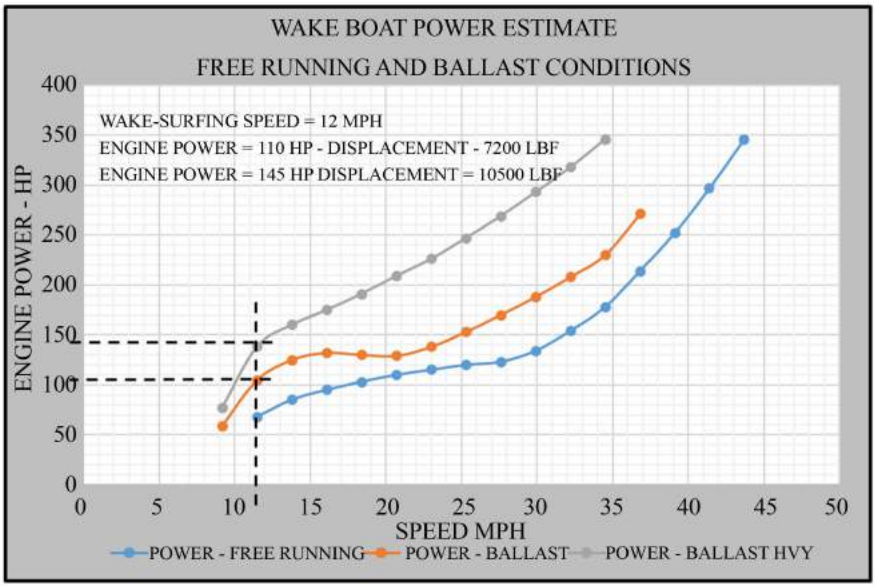

Figure 5. Engine power versus speed for a typical wake boat.

The wave making resistance can be estimated by subtracting the frictional resistance, appendage and wind drag. We can assume the wind drag is small compared to the overall resistance at 12 mph. The appendage drag is approximately 32 lbf and the frictional resistance is estimated at 87 lbf (387 N) with a frictional coefficient of 2.625 (10^{-3}) and a wetted surface of 117 ft^2. The result is the wave making resistance is 1391 lbf at 12 mph (17.62 ft/sec).

If we take the wave making resistance figure and multiply it by the speed of the boat we can estimate the power input into the development of the wave field following the boat. The wave making power in ft-lbf/sec is 24,509 ft-lbf/sec or 33.23 kW or 33,230 joules per second.

The results of the ballasted condition at 10,500 lbs (4760 kg) are:

Vessel Speed 12 mph (17.3 kph);

Total Resistance 2030 lbf (9.03 kN);

Engine RPM 2950;

Engine Power 145 horsepower (108.2 kW);

Overall Efficiency 44.8% (Propeller is cavitating badly).

The wave train behind the boat is travelling at the same speed of boat assuming the boat is travelling at a constant speed. The wave train shows an angle behind the boat with a series of waves traveling at an angle of 19 degrees off the line of travel or an included angle of 38 degrees. The waves are a group of disturbances from the boat's disruption of the free surface. The waves are dispersive meaning they are a series of waves emanating from a single source.

Figure 2 shows an aerial view of a planning craft running at approximately 30 mph. The waves are described as gravity waves and are travelling at the speed of the boat appearing that they are attached to the boat. The appearance of the "V" shaped angle is related to the phase velocity and group velocity of the waves following behind the boat. The "V" shape is described as the Kelvin Wedge. The angled layout is shown in **Figure 6** with each leg at an angle of 19.4 degrees [2]. The wave crests are set at an angle to the line of travel at approximately 35.3 degrees. The velocity of the crest relative to the speed of the boat is:

$$c = V\cos(\theta) \quad (3)$$

where

c: phase velocity, ft/sec;

θ: Wave angle to line of travel = 35.3 degrees;

V: Boat speed, ft/sec.

The phase velocity is the speed of the crest of the largest or dominant wavelength and has a speed of 0.816 (V). The wavelength of the gravity wave in deep water is:

$$L_c = 2\pi \frac{c^2}{g} \quad (4)$$

where

L_c: wavelength of the dominant wave on the crest of the wake;

g: acceleration due to gravity.

The group velocity of the wave train because of the range of wavelengths that make up the full wake is equal to one-half of the phase velocity in deep water. The definition of deep water is dependent on speed. As the depth is reduced for a particular wave train the group velocity will approach the phase velocity causing the waves to bunch up and begin to break at the crest.

The estimate for the group velocity in shallow water can be estimated based on the following equation. The factor (n) will be used to determine the energy in a wave train for comparison to typical waves generated by wind.

$$n = \frac{U}{c} = \frac{1}{2}\left[1 + \frac{4\pi \frac{h}{L_c}}{\sinh\left(\frac{4\pi h}{L_c}\right)}\right] \quad (5)$$

where

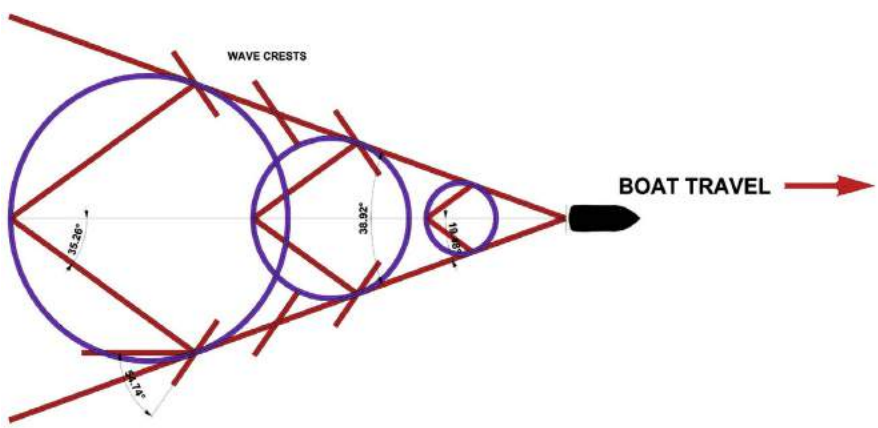

Figure 6. Kelvin wave boundary.

n: Velocity factor;

U: Wave group velocity, ft/sec;

c: Phase velocity, ft/sec;

h: Water depth, feet.

The velocity factor with the boat wake surfing at 12 mph at different depths is shown in **Table 1**. The phase velocity is the cosine of 35.4 degrees times the vessel speed of 17.62 feet per second or 14.37 ft/sec. The group velocity in deep water is 7.19 feet per second.

The wave energy can be estimated by the following equation for a standard gravity wave.

$$E = E_k + E_p \tag{6}$$

where:

E_k: waves kinetic energy;

E_p: wave potential energy.

The wave running in deep water ignoring dissipation of the wave energy the kinetic energy equals the potential energy and can be estimated based on the following equation.

$$E = E_k + E_p = \frac{\rho g H^2 L_c}{16} + \frac{\rho g H^2 L_c}{16} = \frac{\rho g H^2 L_c}{8} \tag{7}$$

where:

H: mean wave height in the wave train in the Kelvin wedge.

The power in the wave can be derived from the following equation based on the total energy represented by the dominant wave in the wave train.

$$P = Enc \tag{8}$$

where:

P: Wave power in ft-lbf per second;

n: Group velocity factor;

E: Wave energy in lbf;

c: Phase Velocity.

If we equate the wave making effective power to the wave power, we can estimate the height of the wave that would be recognized by the observer on the shore. The total effective power for wave making is estimated at 1391 lbf times

Table 1. Group velocity at different depths at 12 mph (17.61 ft/sec).

Depth, h feet	n	Group Velocity, U
5	0.845	12.14
10	0.640	9.20
15	0.545	7.83
20	0.512	7.36
25	0.503	7.23

the speed of 17.61 feet per second or 24,509 ft-lbf per second. The wave train is split in two with a wave train running on either side of the vessel. The single dominant wave is assumed to be close to root mean square of the group of waves travelling with the boat. The effective power for the dominant wave is the RMS value of one half the total power or 8664 ft-lbf/sec. So the estimated wave height can be determined from the effective wave making power of the power and the above equation for the effective power of the wave.

$$8664 = Enc = \frac{\rho g H^2 L_c}{8} nc \quad (9)$$

$$11900 = Enc = \frac{\rho g H^2 L_c}{8} nc \quad (10)$$

The estimated wave height of the dominant wave is 1.85 feet (0.564 m) at 7200 lbs displacement and 2.26 feet (0.688 m) at 10,500 lbs displacement.

Figure 7 shows the wake height of the boat at 7200 lbs displacement at 12 mph in 15 feet of water. The second wave crest behind the boat is the dominant wavelength running at the group velocity. The height of the wave from trough to crest is approximately 0.50 m. It is also clear in the figure that the wave height dissipates quickly as it expands outward and away from the boat. The height of the dominant wave may not carry the RMS value of the energy in the wave train and the figure may be closer to half than the RMS value of 70.7 percent.

Figure 8 shows a second view of the wave train behind the wake boat running at 12 mph. The peak wave at the tip of the train has a height of 0.50 meters (1.64 feet). The center peak is the vertex of the surface disturbance. The figure shows how the wave train is a series of waves with diverging and transverse waves.

For comparison, the wave power at a free running speed of 30 mph at a displacement of 5500 lbf is estimated. The wave making resistance is estimated at 463 lbf. The speed of 30 mph equals 44 feet per second. The effective wave making power is the speed times the force applied to the water surface or 20,372 ft-lbf per second. The wave making power is higher at 30 mph at a lighter displacement. The data is presented below.

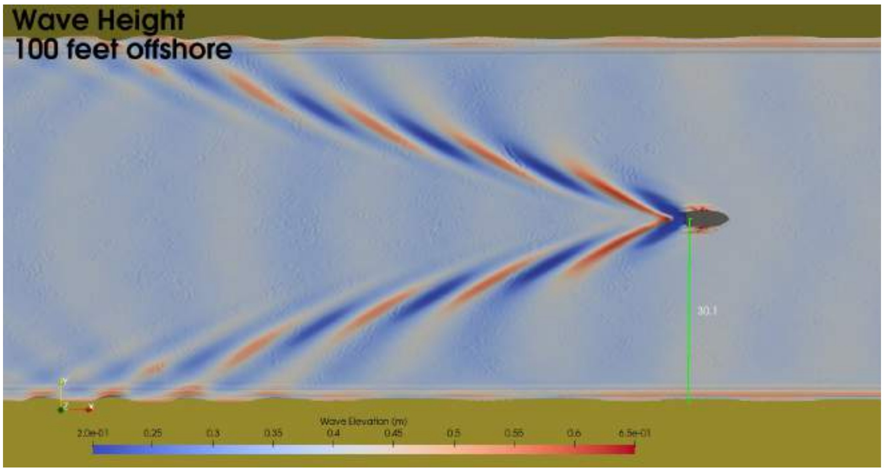

Figure 7. Wake Height of Wake boat 100 feet (30.5 m) from shore in 15 feet of water.

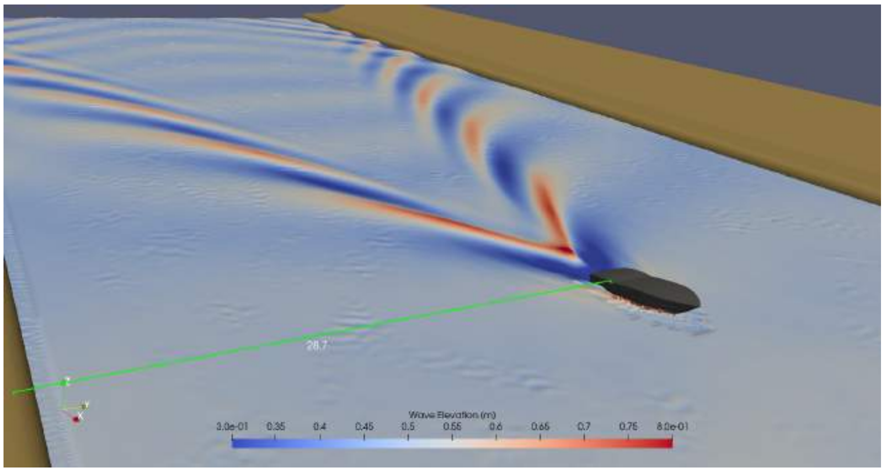

Figure 8. Wake boat at 12 mph in 15 feet of water approximately 100 feet from shore.

Speed: 30 mph = 44 feet per second;

Displacement: 5500 lbf;

Wave Making Resistance: 463 lbf;

Effective Power $R_r V$: 20,372 ft-lbf/sec;

Wave Phase Velocity—c: 35.9 ft second;

Group Wavelength L_c: 251 feet;

Group Velocity Factor—n: 0.956 (15 foot water depth);

Group Wave Energy: 7203 ft-lbf/sec;

Estimated Wave Height: 0.327 feet (0.1 m).

Although the wave height is significantly lower, the wave energy is only seventeen percent lower than the wake surfing condition. In addition, the longer wavelength will allow the high-speed wave to travel further with less dispersion. The higher energy is primarily a function of the higher velocity of the wake. The wave loses a certain amount of energy during each period. The longer wavelengths are travelling faster and have a longer period. The period of the wake surfing wave group is 2.81 seconds compared to 7.0 seconds for the wave group in the free running condition at 30 mph. The higher speed craft running at the same distance from shore will generate similar wave energy striking the shoreline than the wake boat at a heavier displacement running at 12 mph. In each case, the recommendation is to operate the boat 200 feet from shore and in water depths greater than 10 feet.

3. Computational Fluid Dynamic Analysis of Wake Surf Boat

The purpose of this section is to evaluate the waves generated by a wake surf type of boat. The initial development was untaken in Sections 1 and 2 to prepare the reader for the application of CFD in predicting the height, period and distribution of the wave train generated by a passing wake surf boat. The difficulty of developing a model is that only a single condition can be run at one time. The wake is influenced by obvious factors including the vessel weight, running trim and center of gravity. As discussed in Section 2, the running trim and displacement are directly related to the resistance of the vessel. Other factors include the water depth and distance from shore. To a lesser extent, the type of shoreline also may have an influence on reflected waves.

Based on the speed of the wake surfing boats, a depth of ten feet or higher will have little effect on shape, height, and period of the wave train [3]. As the wave train approaches a sloped beach the waves will tend to pile up. The range of CFD analysis includes two operating ballast conditions. The ballast conditions are outlined in Section 2 at displacements of 7200 lbf and 10,500 lbf. The heavier load is an extreme case to define the heavier range of wake surfing boats. The waterline length of the test boat is 19.63 feet at a displacement of 10,500 lbf is at the limit for the boat. The wake surf boat market includes heavier boats, but they have longer waterlines, and the expected running trim would be lower.

The boats were run at two basic speeds to represent wake surfing at 10 - 12 mph and 20 - 22 mph to represent wake boarding. The CFD domains are expanded to provide the wave train well beyond and behind the boat to show the expected dissipation of the wave train over time. The time range of the domain based on the speed is about 40 seconds in the wake surfing mode and 25 seconds in the wake boarding condition. The size of the domain for the study in shallow water is 47 million cells and 67 million cells for the deep-water domain. A third domain modeling a channel with a shoreline on each side was developed to show the wave impact with the shoreline and reflected wave shapes. The size of the channel domain is 45 million cells. The channel model shows the boat approximately 100 feet from shore while the deep-water model shows the boat 200 feet from shore. The shallow water model shows the boat 150 feet from shore. The depth of the channel model is 16.4 feet (5 m), the depth of the deep-water model is 33 feet (10 m) and the shallow water model is 10 feet (3 m). All the models were run at the Super Computing Center at Ohio State University (https://www.awesim.org/).

3.1. CFD Model Domains

In general, for the CFD modeling of hulls, the domain is minimized to reduce the computational time required to solve the equations. The purpose is normally to estimate the resistance of the hull at a given speed, operating displacement, and center of gravity. In the present case, the shape, extent and size of the wake is the primary goal. To model the wave train, the domain needed to be refined well beyond the normal resistance estimate.

The domain was extended side to side to capture the wave train as it developed behind the boat. The depth was set to determine impact of the water depth on the shape, height and extent of the wave train. The largest impact on the model was the refinement of the mesh that made up the body of water. The mesh at the surface was critical to accurately define the wave surface and the mesh below the surface was critical to determine the impact of the water depth.

The canal model was developed to show the effects of the beach on either side and the impact of reflecting waves. The shallow water model depth was constant to starboard and sloped to nothing on the port side to show the effects of waves approaching a shoreline.

To develop a full wave profile over the whole domain the models were all run for 2000 seconds with the boat fixed on the surface. The forces and moments were evaluated, and the position of the boat was balanced based on the vessels weight and center of gravity for each speed point.

The domain of the canal model is shown in **Figure 9**. The figure shows the water mesh from the inlet to the domain with the boat 80 meters into the model. The green boundary is the beach with a twenty-degree slope on each side. The mesh around the boat is more refined by a factor of eight and a layered mesh of three elements is added to the boat surface. An additional layer of two elements is added to the beach surface to model viscous effects.

Figure 10 shows the domain of the shallow water model. The width of the model has been extended to 45 meters (150 ft) to look at the increased distance from shore. The water depth reduces to 0.45 m to port and is a constant 3 m (10 feet) to starboard. The boat and bottom have a layer of cells added to model the viscous effects.

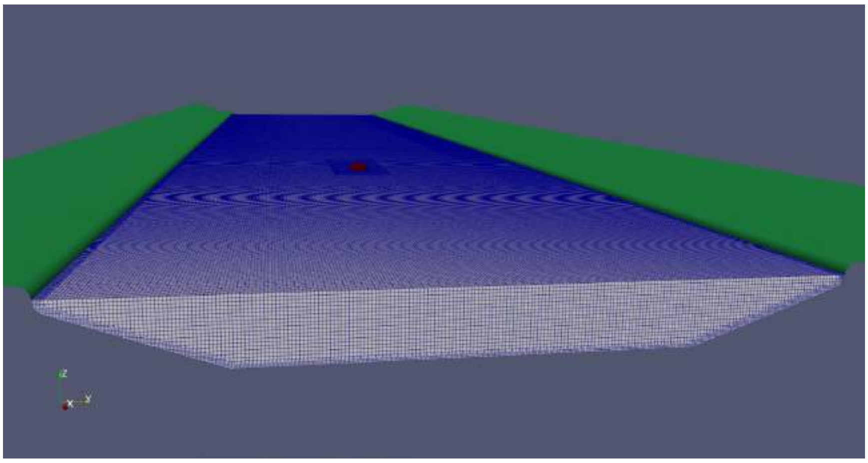

Figure 9. Channel model showing the water mesh.

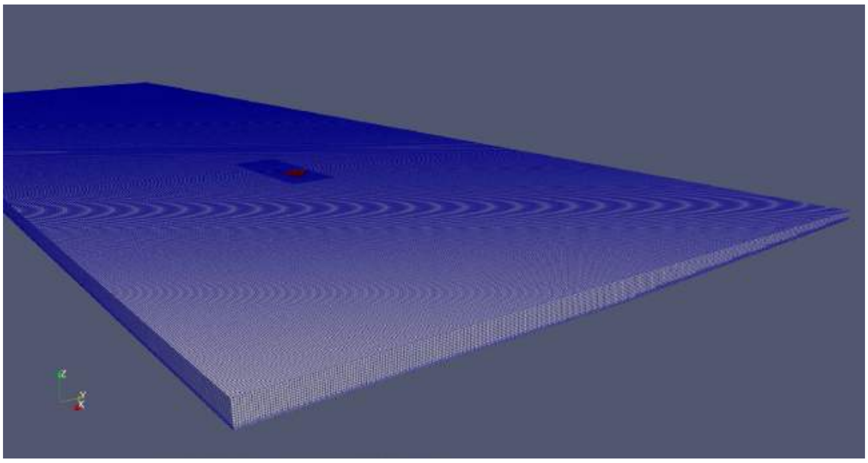

Figure 10. Shallow water model showing water mesh.

Figure 11 shows the refined mesh around the boat with the added mesh layers on the boat hull. The red in the model is the water while the blue is the air with the boat running at 11 mph. The white is the transition between the two liquids. The fluids are immiscible and are split by cells.

Figure 12 shows the water mesh for the deep-water model. The water depth goes to zero on the port side and remains at 10 m (30 feet) to starboard.

3.2. Computational Results

The iterative process of the position of the boat on the surface was made easier using the simulation in Section 2 as a starting point. Figure 13 shows the convergence of the forces and moments on the hull in shallow water model at 12 mph.

The different effects that were investigated include the distance from shore, the depth of water and the type of beach. The Canal model shows the distance from shore of approximately 100 feet with a 20-degree sloping beach.

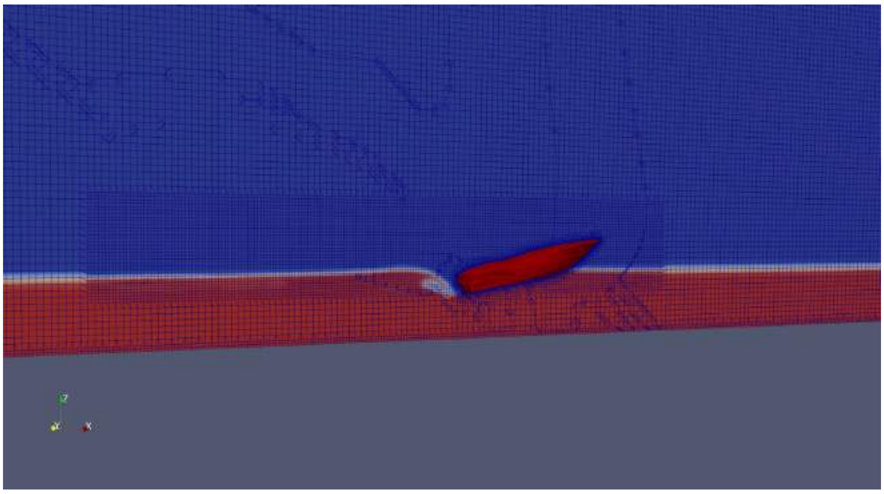

Figure 11. Shallow water model showing the refined mesh.

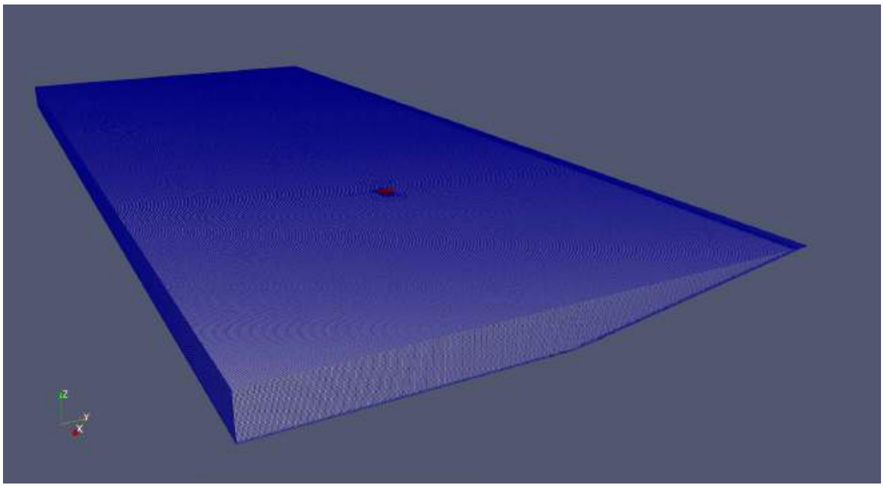

Figure 12. Deep water model water mesh.

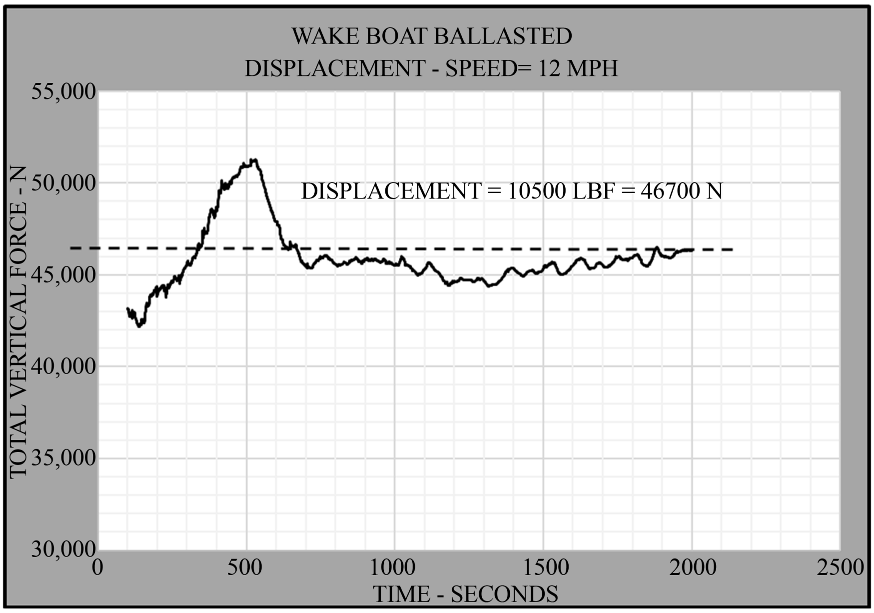

Figure 13. Convergence of vertical force.

The canal model is a good model to show the change in the wave train due to boat speed. The runs were done at the same displacement of 7200 lbs at 11, 17 and 23 mph. Figures 14-16 show the change in the wave train shape based on vessel speed in 5 m (16 feet) of water.

The figures show the wake stretching further behind the boat as the divergent waves tend to turn towards shore as the speed increases. The shallow water begins to have some impact at 23 mph with a larger series of reflected waves.

Figure 17 shows the canal model in perspective with the shore running behind the boat. The distance of the beach is 28 meters (92 feet). Figure 18 shows the shallow model at 45 meters (148 feet) off the beach and Figure 19 shows the deep model that is 60 meters (197 feet) from the edge of the domain. The boat is traveling at 11 mph in each example at a displacement of 10,500 lbf ballast for wake surfing.

In each figure the number of waves striking the shore increases as the distance increases. The number of waves striking the shore 28 meters in Figure 17 is five. The number in Figure 18 at 45 meters is eight and the number at 60 meters in Figure 19 is eleven. The group of waves tends to separate out into its components centered around the group velocity. The wave train shape height and amplitude will be discussed further in Section 3.3.

Figure 20 and Figure 21 show the change in depth at the heavy displacement. Figure 20 is run in deep water at 30 feet deep and Figure 21 is run in shallow water at 10 feet (3 m) deep.

The shallow water runs in Figure 21 shows more wave peaks in view meaning that the group of waves is separated quicker in shallow water. The wave train begins as a single large wave at centerline and the wave group disperses outward from the center at an included angle of 38.4 degrees.

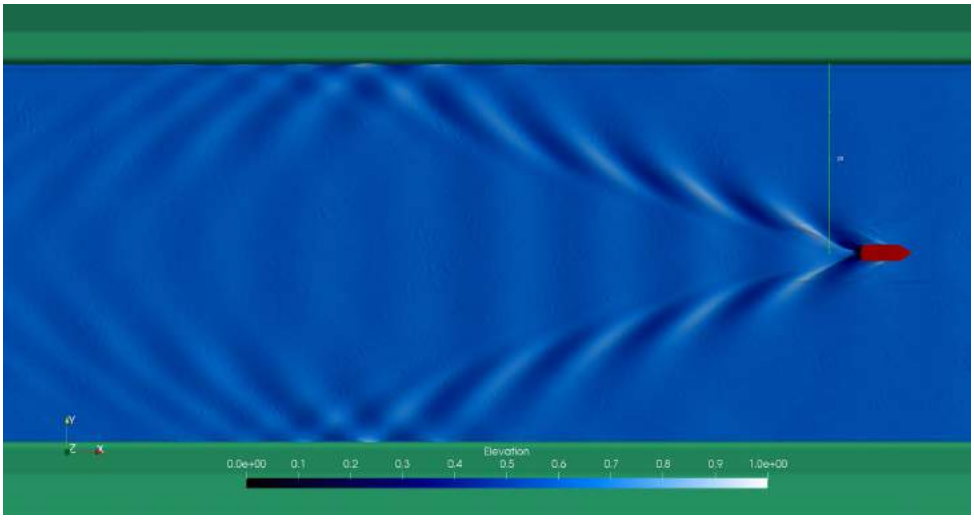

Figure 14. Canal model at 11 mph.

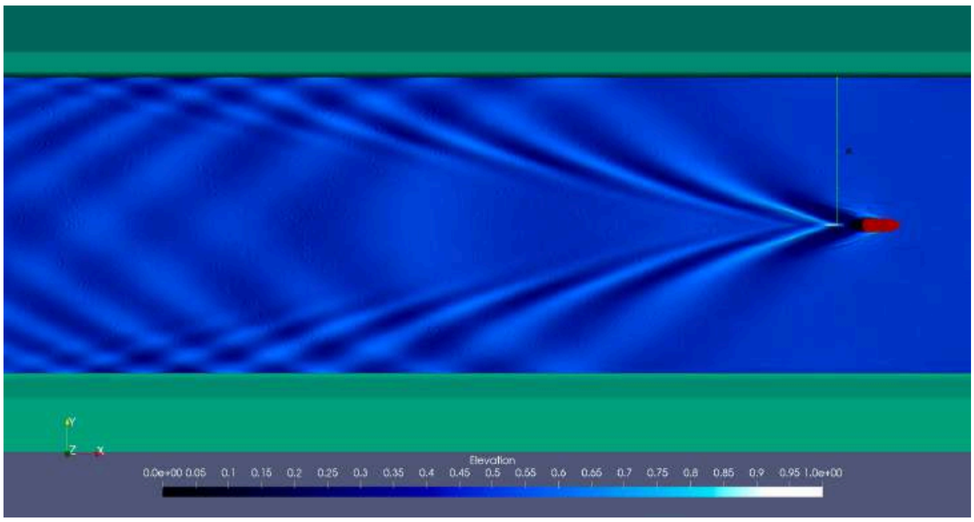

Figure 15. Canal model at 17 mph.

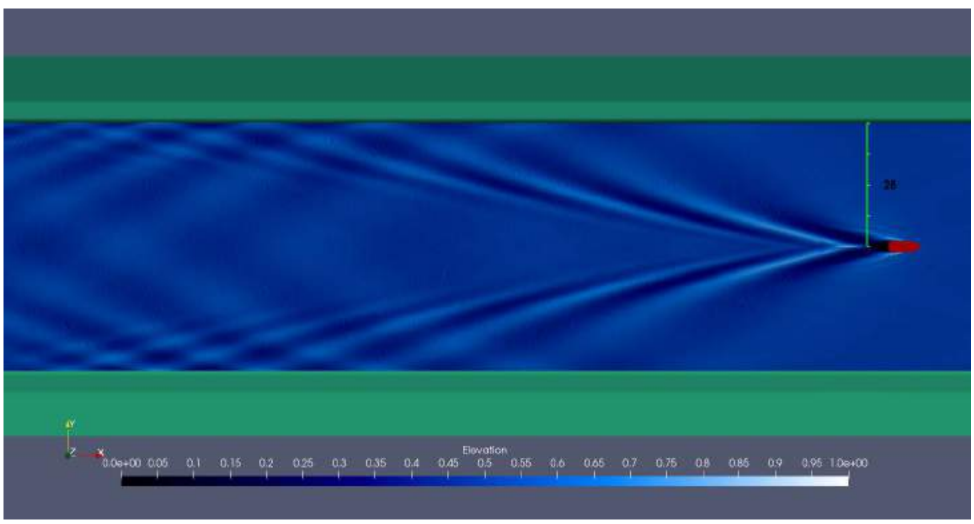

Figure 16. Canal model at 23 mph.

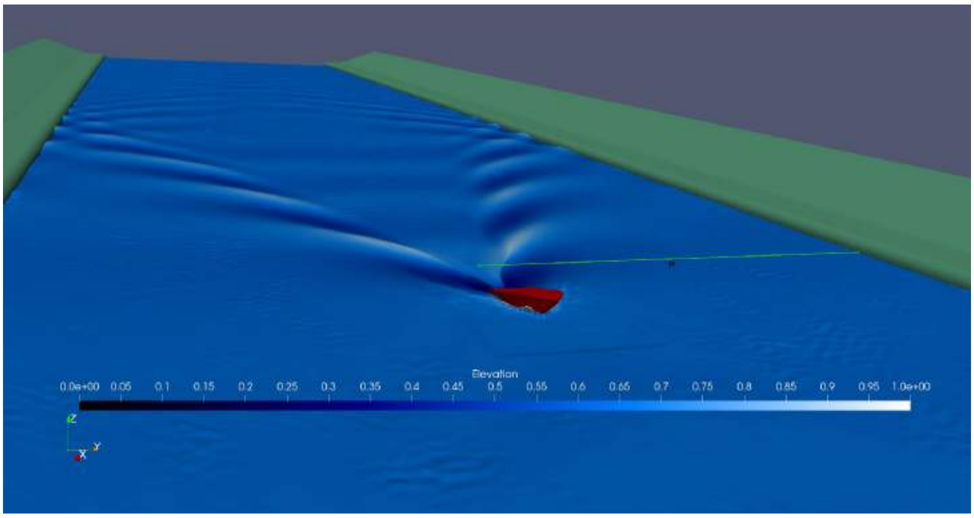

Figure 17. Canal model at 11 mph.

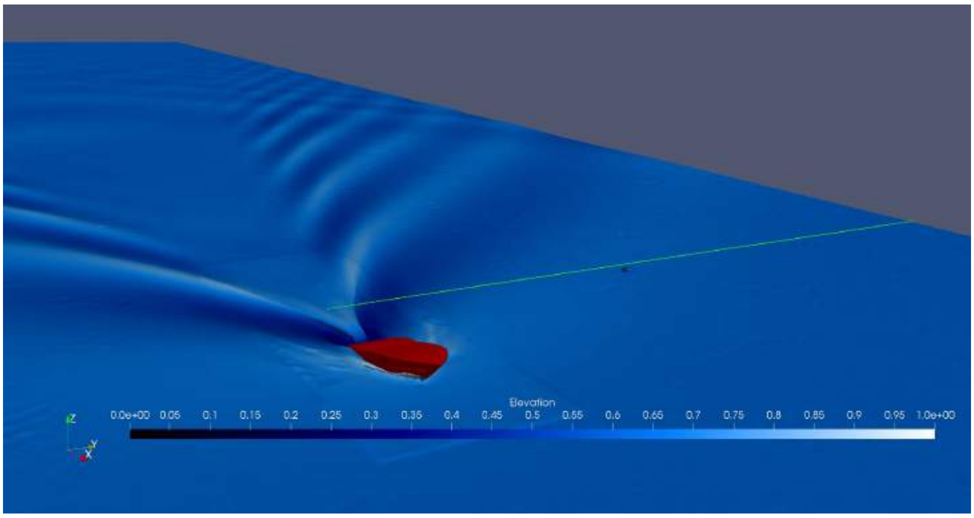

Figure 18. Shallow water model at 11 mph.

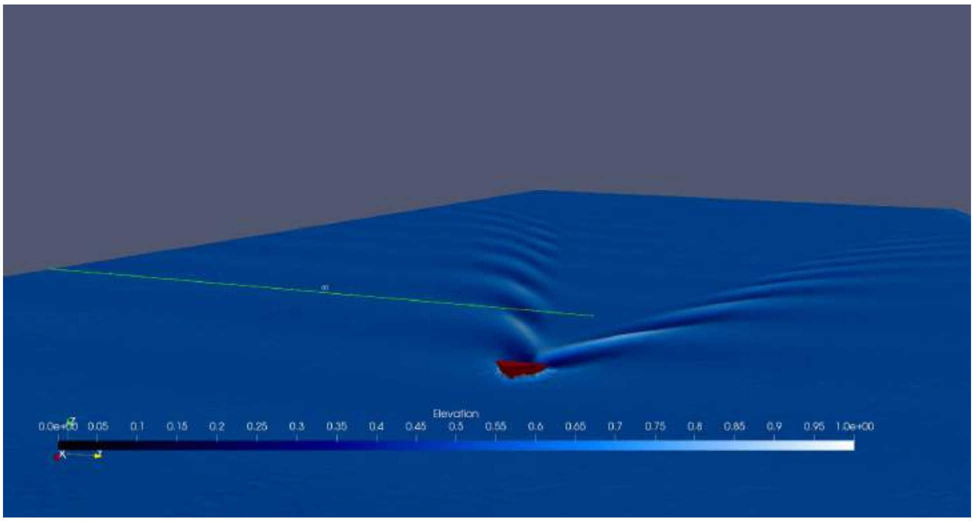

Figure 19. Deep water model at 11 mph.

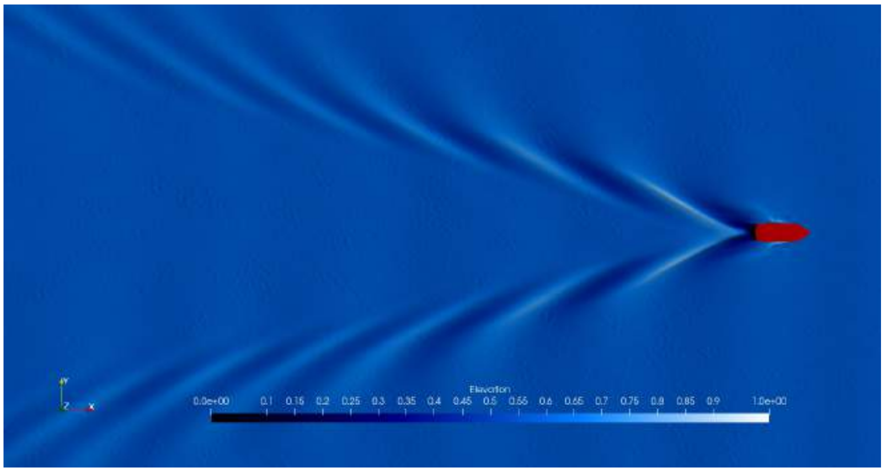

Figure 20. Deep water runs at 12 mph at 10,500 lbs displacement.

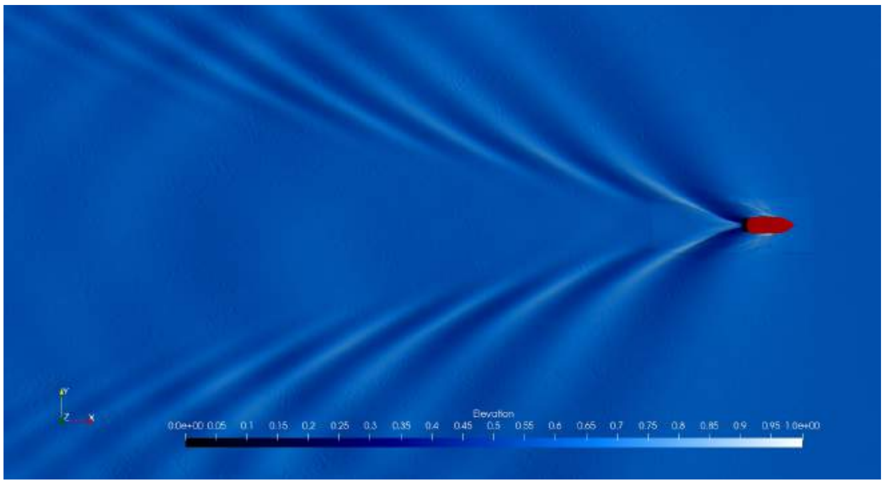

Figure 21. Shallow water runs at 12 mph at 10,500 lbs displacement.

3.3. Wave Shape and Group Velocity

If we take cuts at different distances off centerline, the wave profile will begin as a single major wave where the surfer is on the wave. Moving outboard from centerline, the wave train breaks into smaller waves with periods and wavelengths around the group wavelength based on the wave celerity. The wavelength is the longest in front or in the lead as the faster waves outpace the slower ones until the group separates out into a group of around seven peaks.

The group of waves satisfies the simple dispersion equation [2] as follows.

$$c = \left(\frac{g\lambda}{2\pi}\right)^{1/2} \tag{11}$$

where:

c: wave speed (feet per second);

λ: wavelength (feet).

If we simplify and rearrange the variables, we get the Froude number showing the wave speed and wavelength are proportional to boat speed.

$$F_n = \frac{V}{\sqrt{gl}} = \frac{2\pi c^2}{g\lambda} = \frac{c}{\sqrt{g\lambda}} \tag{12}$$

The wave trains from the deep-water model were cut at 5 m (16 feet), 10 m (33 feet) and 20 m (65 feet) to show the shape of the wave train as it traveled away from the boat. Figure 22 shows the section through the surface at 5 meters (16 feet) off centerline. The wave is primarily a single large wave. Figure 23 shows a section at 10 m (33 feet) and Figure 24 shows the section at 20 m (65 feet). The sections are taken at 12 mph in the deep-water model.

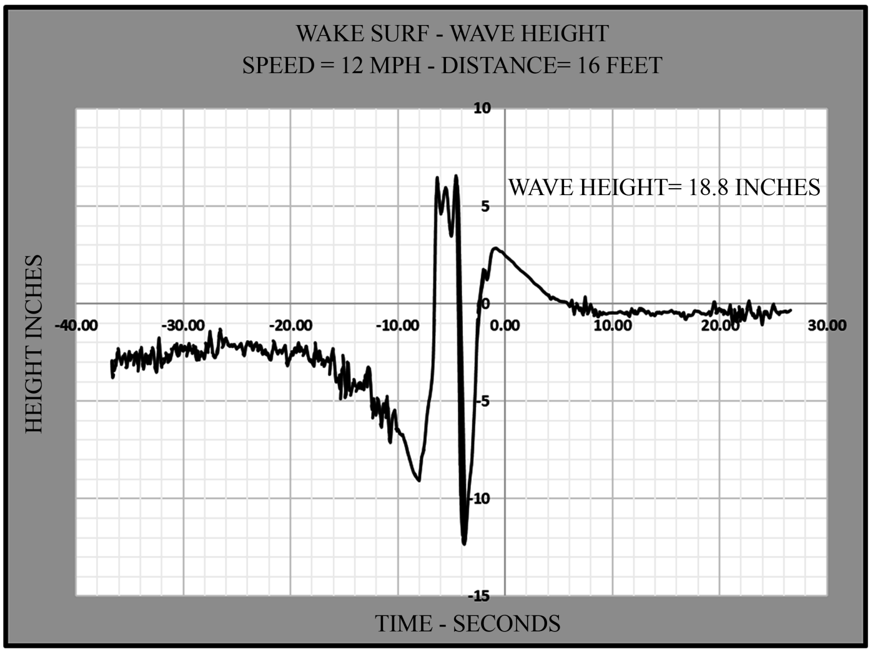

Figure 22. Surface section at 5 m off centerline in deep water at 12 mph.

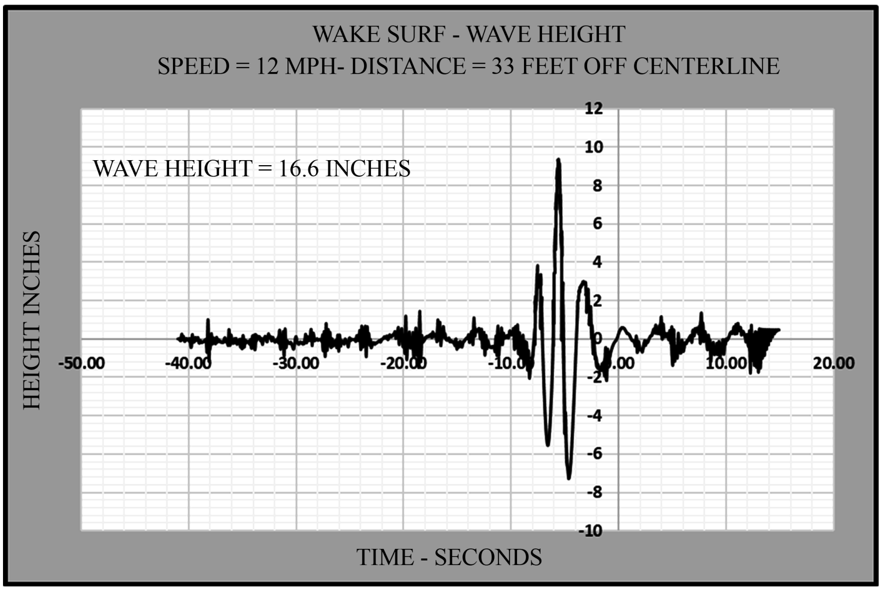

Figure 23. Surface section at 10 m off centerline in deep water at 12 mph.

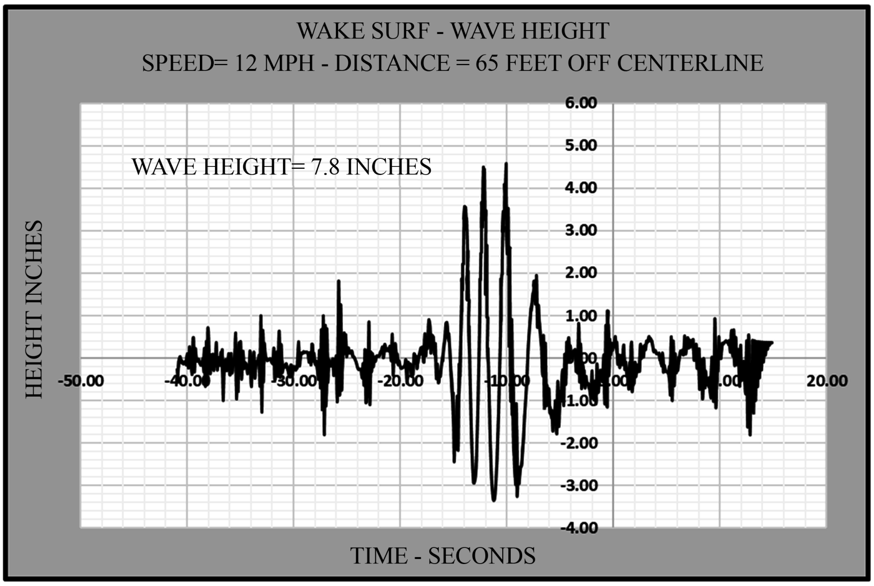

Figure 24. Surface section at 20 m off centerline in deep water at 12 mph.

The deep-water model is not affected by the depth and the group velocity will be half of the wave velocity allowing the wavelength to disperse. The shape of the wave profile in **Figure 22** is most likely missing a bit of the top of the wave because the wave is breaking. The wake clearly shows a single disturbance. In **Figure 23**, the wave train has split in two at one period (2.8 seconds) behind the first section at 5 meters. **Figure 24** is about four wave cycles behind the boat and the section clearly shows five wave peaks. The wave train has separated into a group of waves as it travels away from the boat. The longer wavelengths lead the group with the peak amplitude wave in the middle. The wave period at the front of the group in **Figure 24** is 3.73 seconds. The last wave has a period of 1.86 seconds. The peak wave or dominant wave has a period of 2.81 seconds.

The time plot is the longitudinal position from the transom divided by the speed of the boat. The time zero is the boat passing at the transom and the negative time is the distance behind the boat. Positive time begins forward of the transom and negative time behind the transom corresponds to the domain dimensions with the origin at the transom.

4. Experimental Field Data

There are numerous ways to estimate the height of waves on the water. In deep water the wave disturbance from boats has been measured with submerged pressure sensors. The pressure sensors needed to be calibrated and the distance below the surface can be affected by current and the orbital velocities in the wave. A common approach in the study of offshore waves is to measure the maximum velocity at the surface using a high-speed GPS recorder.

Another approach has been to measure the wave height directly with sensors that are in the water column and measure the running surface height. In general, these units are floating and can be affected by the orbital velocity, background waves and fundamental heave frequencies as the wave passes. The measurement devices are generally floating and anchored to the bottom.

Reference [4] took a series of wake measurements in shallow and deep water at various distances from shore. The boat used in the experiment was a wake surf boat at a displacement of approximately 10,500 lbs. The wave heights were measured perpendicular to the shoreline, but the boat ran at an angle to the shoreline to produce a wave train that travelled perpendicular to the shoreline. The angle is assumed to be approximately 19 degrees based on a Kelvin wave. The data used for comparison were taken using pressure sensors located below the surface. The sensitivity and calibration of the sensors are difficult when trying to measure the wave heights in shallow water. The limited depth reflects a pressure field off the boat as it approaches and then quickly dissipates as the boat passes. The plots shown are taken from data in relatively deep water with approximately ten feet of water below the sensor. The shallow water wave height data was not consistent and ignored for comparison.

The CFD analysis shows the initial wake is a disturbance that breaks up into its component waves as it moves away from the boat. The wake sections taken off centerline represent the travel time of the wave train away from the boat. The plotted wave profiles from [5] show the wave profile of a boat in wake surfing mode at different distances from the path of the boat. Each sensor represents a distance and time from the path of the boat after it has passed. The time represents wave cycles as they travel over the surface of the water. **Figure 25** shows the field test wave train over time while it travels away from the boat. The time is difficult to compare since the sensors are perpendicular to the shore and at an unknown angle to the line of travel of the boat. The field data time plot is roughly from the time the boat passed the line of sensors. The CFD data follows

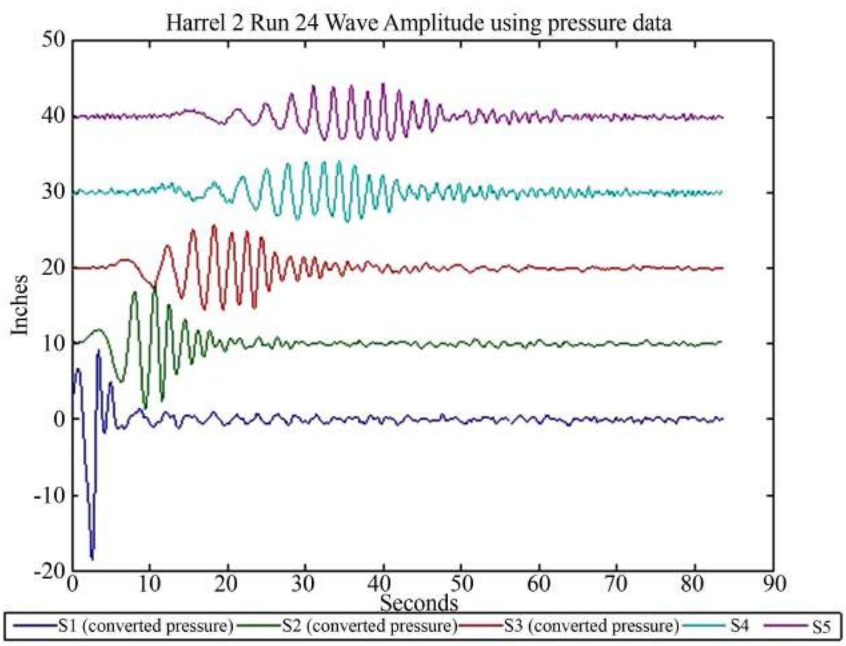

Figure 25. Data provided by [4] at 11 mph at a displacement of 10,500 lbf.

the sign of the position of the boat from the transom. The CFD model had 67 million cells shown in Figure 12. The resolution near the boat is good but deteriorates at more than three boat lengths away from the boat making it impossible to plot the expected wave height more than 100 feet from the track of the boat.

The CFD analysis data is taken at a point in time or picture of the wave train. By taking cross sections through the CFD domain at positions off centerline parallel to the path of the boat, the shape of the waves can be estimated. The time on the plot utilizes the speed of the domain and distance to estimate time. Figure 26 shows the CFD wave profile at 5 m (16 ft) off centerline. The disturbance is very similar to the provided shown at the bottom of Figure 25 where the sensor was close to the path of the boat. Figure 27 shows the CFD profile at 20 m (66

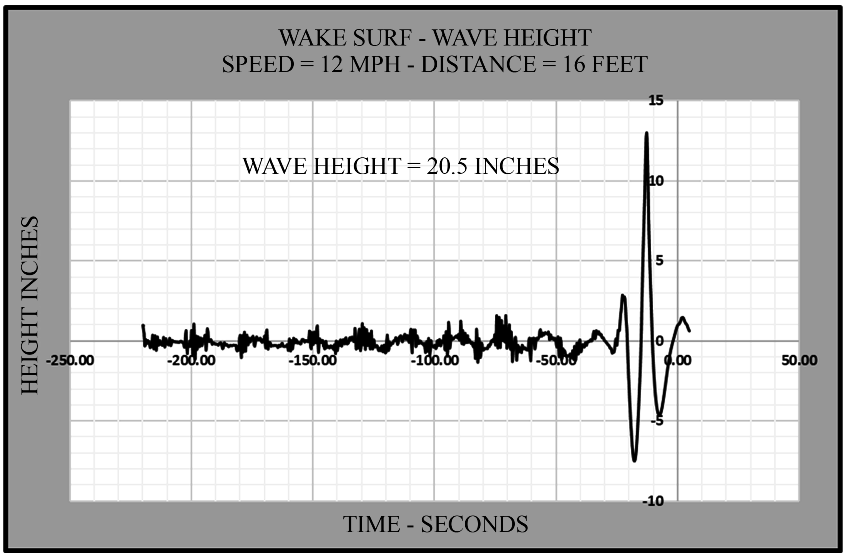

Figure 26. Wake profile 5 m off centerline at 11 mph at a displacement of 10,500 lbf.

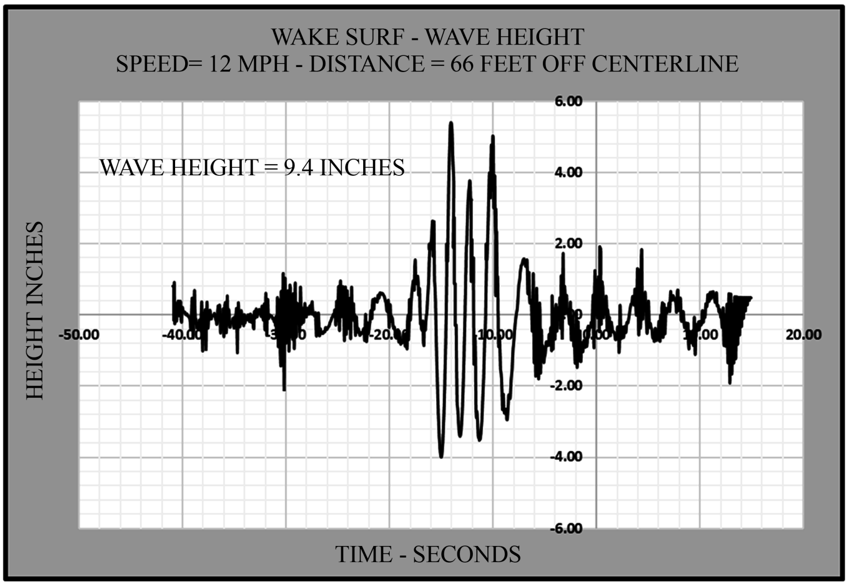

Figure 27. Wave profile at 20 m off centerline.

feet) off centerline. The figure represents the third profile from the bottom in Figure 25. The CFD analysis captures the shape, height and distributions of the wave train generated by the boat in the field test. Although the two boats are different their basic configuration and weight are similar.

The wave generated by any planning craft will produce a disturbance on the water surface. As the boat passes the wake breaks into a series of smaller waves. The wake extends outward and breaks into its component parts. The measured data and CFD data results show the wave disturbance at the transom and the group of waves trailing behind the vessel. The wave heights, period, and shape of the wave train match well. The wave height drops in half after about 15 cycles or 30 seconds. The field test data and CFD analysis show the same reduction in wave height as the wave train moves away from the path of the boat.

A similar review of the wave profile of the boat at 20 mph in a wake boarding condition at 10,500 lbf between the CFD analysis and the field data shows similar results. The wave profiles show the same initial large disturbance, and the group components separate into a range of individual waves of different wavelengths.

Figure 28 shows the wave height plot at the different sensors running in deep water. The time starts as the boat passes the line of sensors. The boat passed at a reported ten feet from the first sensor or about one wavelength based on the plot.

Figure 29 and Figure 30 show the wave profile at 20 mph during wake boarding. The wave shape, height and period are all consistent between the field test data and the CFD analysis. Figure 29 shows the wake near the transom at 5 m off centerline and Figure 30 shows the wake profile 20 m off centerline. The higher speed of the vessel puts the CFD plots at the first two plots from the bottom in Figure 28 showing the wave profiles of the field test.

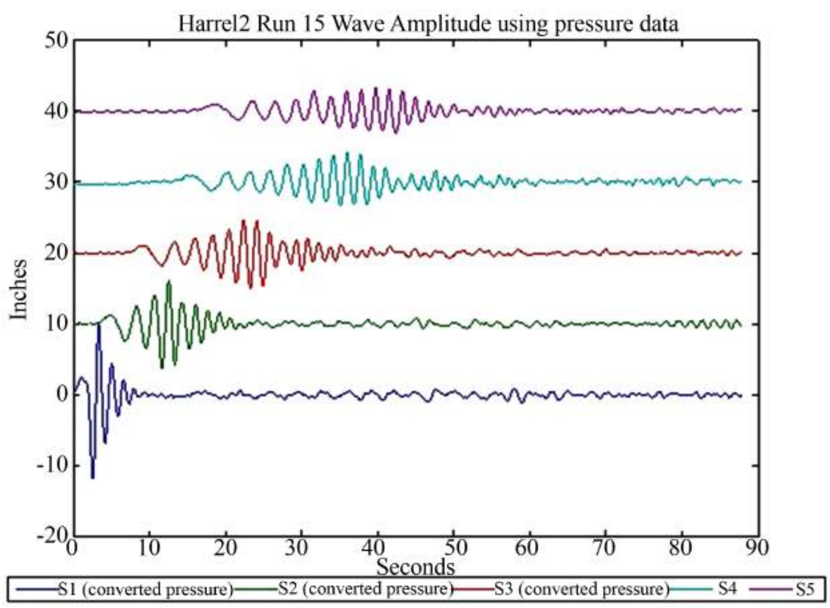

Figure 28. Wake boarding wave train profile test data 20 mph at a displacement of 10,500 lbf.

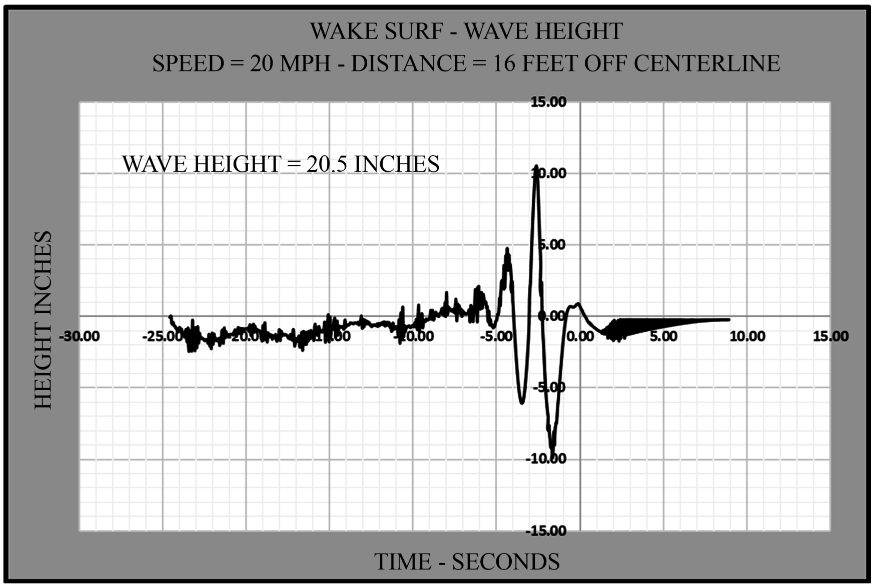

Figure 29. CFD results at 20 mph at 5 m off centerline at a displacement of 10,500 lbf.

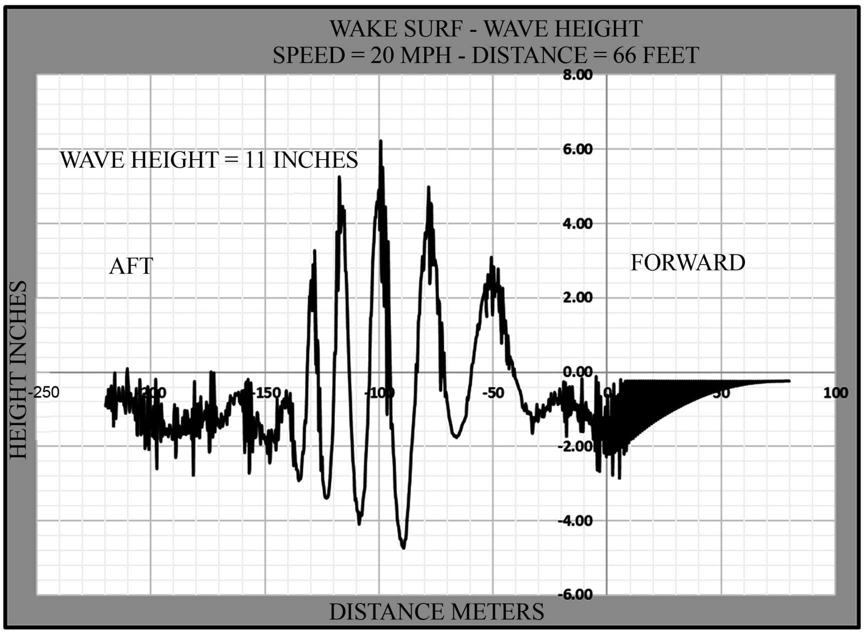

Figure 30. CFD results at 20 mph at 20 m off Centerline at a displacement of 10,500 lbf.

5. Wave Energy Attenuation

The wave train behind the boat is a group of waves following the boat with a range of periods and wavelengths centered around the group velocity. The wave train expands into the individual waves as it travels away from the boat and its energy will dissipate as it travels.

The primary loss in energy as the wave train travels in shallow water generally found in confined bodies of water is through bottom friction [2]. The bottom friction is a result of the orbital motion of the wave interacting with the bottom as the wave travels. The amount of energy lost per cycle can be significant depending on the depth of the water and the average wavelength.

The average wavelength is proportional to the vessels' speed. For example, the average wavelength from a wake surfing boat running at 12 miles per hour is approximately 40 feet while the average wavelength from the same boat at a wake boarding speed of 20 miles per hour is 112 feet.

The loss of energy is a function of the viscosity, frequency, and depth of the water. The following formula is provided [2]. The square brackets are the viscous components, and the curling brackets provide a factor for the wave number and water depth.

$$\text{Loss per Cycle} = \left[\frac{2\pi\left(\frac{\upsilon}{2\omega}\right)^{\frac{1}{2}}}{h}\right]\left\{\frac{2kh}{\sinh(2kh)}\right\} \tag{13}$$

where υ: kinematic viscosity (ft^2/sec^2)

ω: wave frequency (radians per second);

h: water depth – feet;

k: wave number: $k = \frac{4\pi^2}{gT^2}$;

T: wave period—seconds.

The second term provides the factor based on wavelength and depth. Table 2 and Table 3 show the wavelength, L_w and water depth, h ratio comparing the wake surfing and wake boarding conditions.

Figure 31 shows a plot of the data in Table 2 and Table 3. A cycle is equivalent to one period or one wavelength in distance. The longer wavelength of the

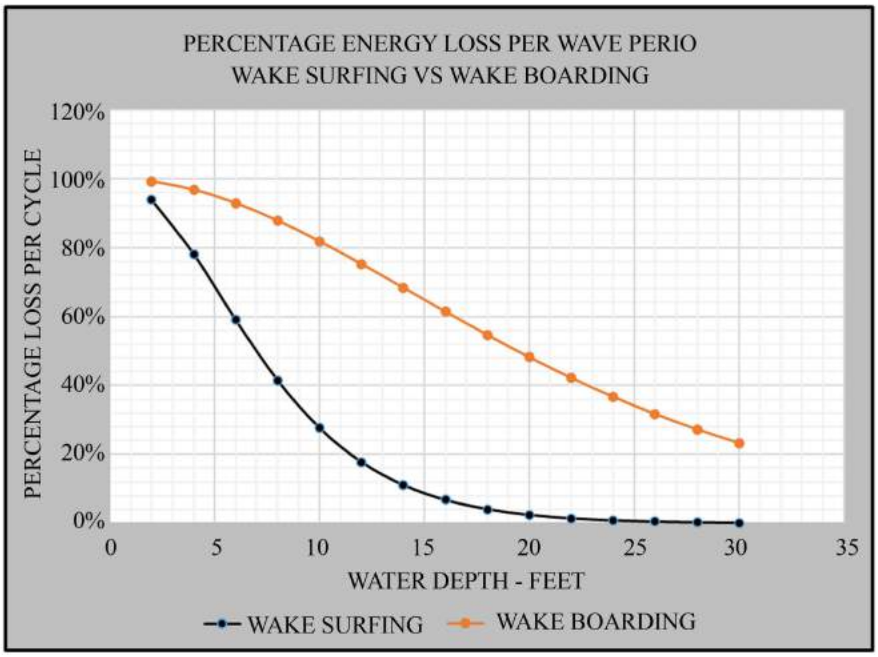

Figure 31. Percentage energy loss per wave period.

Table 2. Wave surfing.

VESSEL SPEED = 12 MPH (19.3 KPH)	WAVE, C = 14.38 FT/SEC (4.38 M/S)
WAVELENGTH = 40.33 FT (12.3 M)	WAVE PERIOD = 2.81 SECONDS
WAVE NUMBER, K = 0.156	CYCLE TIME = 2.81 SECONDS

DEPTH	RATIO	ENERGY
h	L_w/h	LOSS
2	20.17	93.81%
4	10.08	78.14%
6	6.72	59.06%
8	5.04	41.51%
10	4.03	27.69%
12	3.36	17.79%
14	2.88	11.13%
16	2.52	6.82%
18	2.24	4.11%
20	2.02	2.45%
22	1.83	1.45%
24	1.68	0.85%
26	1.55	0.49%
28	1.44	0.28%
30	1.34	0.16%

Table 3. Wake boarding.

VESSEL SPEED = 20 MPH (32.2 KPH)	WAVE, C = 23.96 FT/SEC (7.30 M/S)
WAVELENGTH = 112 FT (34.1 M)	WAVE PERIOD = 4.68 SECONDS
WAVE NUMBER, K = 0.0561	CYCLE TIME = 4.68 SECONDS

DEPTH	RATIO	ENERGY
h	L_w/h	LOSS
2	56.02	99.17%
4	28.01	96.72%
6	18.67	92.83%
8	14.00	87.74%
10	11.20	81.75%
12	9.34	75.16%

Continued

14	8.00	68.27%
16	7.00	61.34%
18	6.22	54.58%
20	5.60	48.15%
22	5.09.	42.15%
24	4.67	36.64%
26	4.31	31.67%
28	4.00	27.22%
30	3.73	23.29%

wake boarding wave will interact with the bottom in deeper water than the wake surfing waves. The observer would see the larger waves impacting the shore from the wake surfing boat due to the shorter wavelengths and lack of interaction with the bottom.

In terms of the shape of wave for wake surfing, the depth Froude Number and its impact on the shape of the wave train are shown in Equation (14). The critical depth Froude number is defined as follows [6].

$$Fr_h = \frac{U}{\sqrt{gh}} \tag{14}$$

where:

Fr_h: Depth Froude Number;

U: Vessel Speed in meters per sec;

g: Acceleration due to gravity 9.81 m/s^2;

h: Water Depth in meters.

The critical depth at 11 mph is 8.1 feet and the depth to minimize bottom effects is at a Fr_h of 0.75 resulting in a depth of 14.4 feet. Anecdotally, the depth of 16 feet has been noted by wake surf enthusiast as the minimum depth for the best wave. In the case of wake boarding, the critical depth is 26.8 feet. Assuming a speed of 20 mph, the Froude depth number is 1.33 at a depth of 15 feet. Reference [6] discusses Fr_h numbers above one as super critical where the wave train produces no transverse waves only divergent long crested waves. The shape of the divergent waves in the absence of the transverse waves would provide an optimum experience for the wake boarder providing clean water between the divergent waves.

6. Turbidity

The power boat is driven through the water by the thrust from the propeller. The propeller generates the thrust required to overcome the hull resistance that includes the power to generate the wave train travelling on the boat. The thrust is generated by a change in momentum of water running through the propeller disk. The added momentum generates a high velocity column of water travelling through the propeller and behind the boat. The change in momentum generates the thrust needed to propel the boat. The following equation idealizes the estimate of the thrust.

$$T = \rho A V_p \left(V_p - V_0 \right) \tag{15}$$

where:

T: Thrust in lbf;

A: Area of the Propeller Disc (ft^2);

ρ: water density;

V_p: Water Velocity in propeller stream (nP/12) (ft/sec);

n = propeller speed in revs/sec;

P = propeller pitch in inches;

V_0: Boat Speed (ft/sec).

In Section 2 the performance at 12 mph was estimated, providing an engine RPM and required thrust. The resistance of the hull was estimated at 1056 lbf (4697 N). A thrust deduction is added based on the interaction of the propeller induced velocities on the hull. The estimated total thrust required is 1290 lbf (5737 N) based on a thrust deduction of about 20 percent based on the shaft angle and rudder position. The propeller RPM is 1680 with a reduction ratio of 1.57:1 based on the engine RPM of 2636. The required power is estimated at 91 horsepower (67.9 kW).

The propeller pitch is 16 inches and a shaft RPM of 1680 V_p is 37.3 feet per second (25 mph) and V_0 is 17.61 feet per second (12 mph). The stern gear arrangement is shown in **Figure 32** with a shaft angle of 18 degrees. **Figure 33** shows the CFD model with a thrust disc to represent the propeller with the boat operating at 12 mph. The resulting ideal thrust based on the thrust equation is 1746 lbf (7775 N). The actual thrust based on the propeller performance integrated with the shaft line and rudder is 1290 lbf (5740 N).

Seconds. There will be some continued mixing near the surface generated by the wake and the turbulence in the boundary layer of the boat. **Figure 34** provides some insight into a rotating propeller and the vertical movement of water from the surface to the propeller under the boat. The domain has a depth of 5 m (16 ft) at a displacement of 7200 lbs.

The flow is shown in the streamlines through the propeller with the boat hull over the top. The oxygenation of the water through power boat activity has been monitored by the Environmental Protection Agency [7] to show that the body of water sees a general increase in oxygen content during boating activity.

The illustrations of the flow of the propeller in **Figures 33-35** show that the wash does not travel toward the bottom with the movement of the boat through the water. In **Figure 35** the wash reaches approximately seven and a half feet below the surface with the propeller at approximately three feet below the surface.

This is further seen in **Figure 36** which depicts velocity on planes located at various depths below the water surface. Note that this simulation was performed with infinite depth (no bottom).

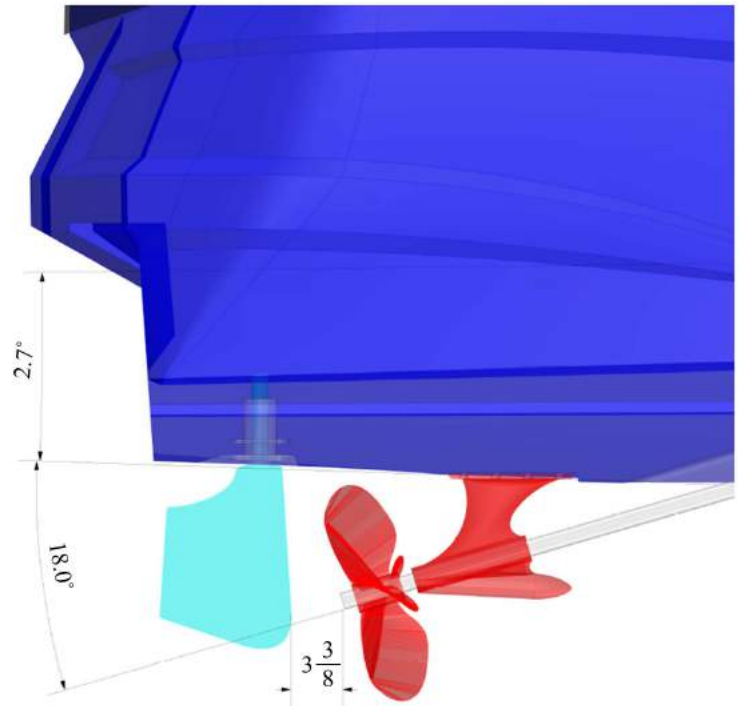

Figure 32. Propeller shaft and rudder arrangement.

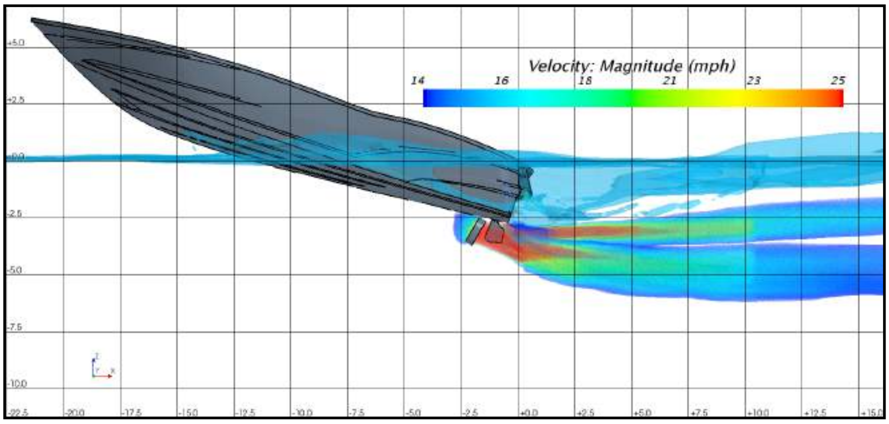

Figure 33. Propeller wash velocity (25 mph = 36.7 ft/sec).

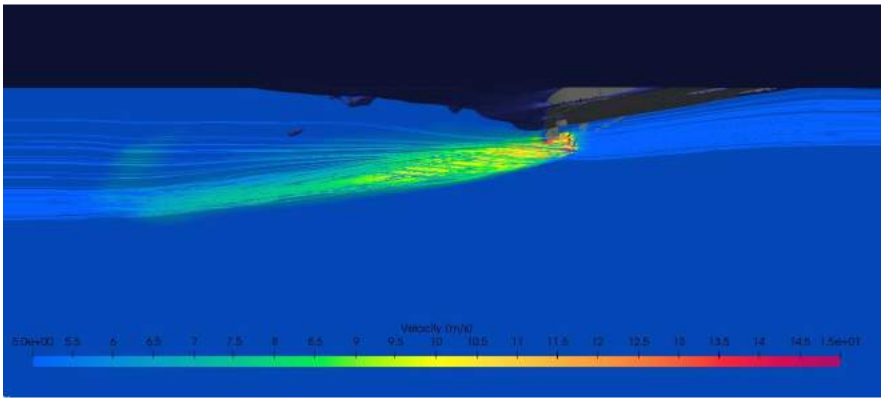

Figure 34. Propeller streamlines showing vertical mixing.

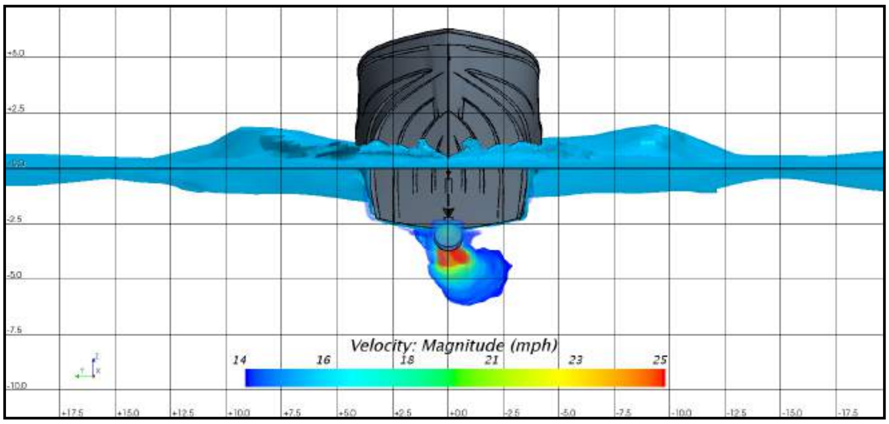

Figure 35. Water velocity below the surface.

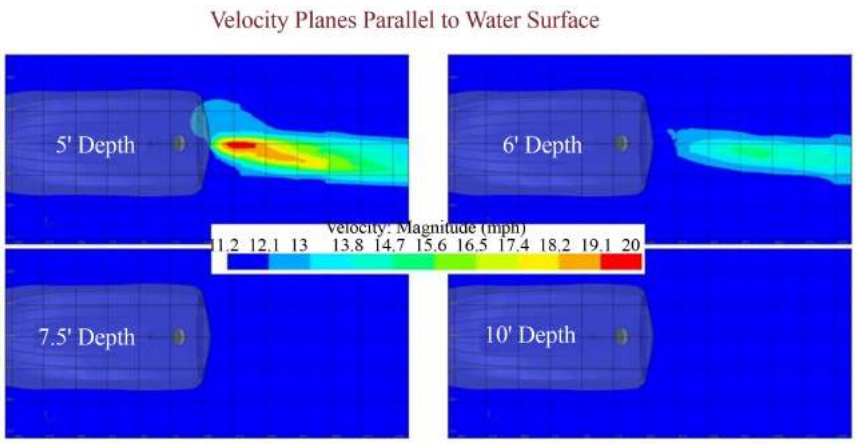

Figure 36. Velocity profiles at different depths.

The inclusion of a bottom could serve to increase propwash penetration depth by as much as 0.5 ft, depending on the proximity of the propeller to the bottom. Additionally, a vessel with larger displacement such as 10,500 lbs from Section 3 could place the propeller slightly lower than 3 ft (1 m) below the surface and require higher thrust to overcome increased drag. For these reasons, the recommended depth for wake surf operation is conservatively set at 10 ft (3 m).

7. Wind and Waves

The wind level around the lake depends on the fetch and buffer zones around the lake. Most lakes experience a significant number of days per year with a steady wave train breaking on the shores generated by seasonal prevailing winds and weather systems. The wind generated waves help mix the water column to distribute nutrients and oxygen rich water. The wind generated waves disturb the shoreline causing widespread turbidity in combination with well oxygenated water. The wind generated waves can cause minor erosion early in the season where the shoreline has been impacted by ice pushing or loading.

Wind generated waves on lakes and small bodies of water are unique that they are generally small but develop very quickly. For a wind speed of 25 miles per hour, the wave train is fully developed in approximately twelve minutes over a fetch of 800 yards. The significant wave height is ten inches with a wave period of 1.6 seconds and a wavelength of 8.4 feet.

The wind and wave data were developed from Army Corps, Coastal Engineering Manual (2015). The equations were extrapolated to accept shorter fetches and were checked based on observation and computational fluid dynamics. The equations are split into a wave height equation and a wave period equation and include a function for water depth.

The wave height equation:

$$\frac{gH}{U^2} = 0.283\tanh\left[0.578\left(\frac{gd}{U^2}\right)^{0.75}\right]\times\tanh\left[\frac{0.0125\left(\frac{gF}{U^2}\right)^{0.42}}{\tanh\left[0.578\left(\frac{gd}{U^2}\right)^{0.75}\right]}\right] \quad (16)$$

The wave period equation:

$$\frac{gT}{2\pi U} = 1.20\tanh\left[0.520\left(\frac{gd}{U^2}\right)^{0.375}\right]\times\tanh\left[\frac{0.077\left(\frac{gF}{U^2}\right)^{0.25}}{\tanh\left[0.520\left(\frac{gd}{U^2}\right)^{0.375}\right]}\right] \quad (17)$$

where

H: Wave height—feet;

U: Wind speed—feet per second;

g: Acceleration due to gravity 32.2 ft/sec^2;

F: Fetch distance in feet;

d: Water depth in feet;

T: Wave period in seconds.

The significant wavelength, L_w:

$$L_w = \frac{gT^2}{2\pi}\left[\tanh\left(\frac{4\pi^2 d}{T^2 g}\right)\right]^{\frac{1}{2}} \quad (18)$$

The equation has been analyzed to include smaller fetches and checked using computational fluid dynamics (CFD). Table 4 shows the results of the equation at a wind speed of 35 mph (15 m/s) and at a depth of 16 feet (5 m). The line in the table is highlighted (fetch of 800 yards) that shows the details of the CFD model.

The table takes the fetch, wind speed and water depth and provides the wave height, wave period and time in minutes for the wave state to become fully developed. In the case of a fetch of 800 yards, the waves will become fully developed after a period of ten minutes. The wave height is 1.11 feet (0.338 m), the wavelength is 9.5 feet (2.9 m) and the wave period is 1.86 seconds. The impact of the wind event would have a 1.16 foot (0.354 meter) high wave impacted the shoreline every 1.86 seconds or approximately 1940 times per hour. Another interesting observation factor is that the sea state only takes ten minutes to become fully developed at a wind speed of 35 miles per hour. Figure 37 shows a photograph of the wave spectra on a lake with a fetch of about one mile in 15 - 20 mph (24 - 32 kph) of wind.

Figure 38 shows the results of the CFD analysis of the wave train over the 800-foot fetch. The CFD model uses 32 million cells and a long eddy simulation to model the wind to water interface. The elevation shows a wave height of 0.32 meters which corelates to the wind-wave model shown above.

Figure 37. Image of wind driven waves at 25 - 35 mph.

Table 4. Wave height estimate.

FETCH YARDS	FETCH FEET	WIND FT/SEC	DEPTH FEET	WAVE HEIGHT	WAVE PERIOD	WAVELENGTH FEET	TIME MINUTES	WAVESPEED FT/SEC
100	300	51.4	16	0.267	0.941	4.82	2.71	2.13
200	600	30	16	0.357	1.101	5.64	4.59	2.49
300	900	30	16	0.422	1.205	6.17	6.23	2.73
400	1200	30	16	0.475	1.282	6.57	7.75	2.90
500	1500	30	16	0.521	1.344	6.89	9.18	3.04
600	1800	30	16	0.561	1.396	7.15	10.54	3.16
700	2100	30	16	0.598	1.440	7.38	11.85	3.26
800	2400	30	16	0.631	1.480	7.58	13.12	3.35
900	2700	30	16	0.662	1.515	7.76	14.34	3.43
1000	3000	30	16	0.691	1.547	7.93	15.54	3.50
1200	3600	30	16	0.743	1.602	8.21	17.85	3.63
1400	4200	30	16	0.791	1.649	8.45	20.06	3.73
1600	4800	30	16	0.834	1.691	8.67	22.21	3.83
1800	5400	30	16	0.873	1.728	8.85	24.29	3.91
2000	6000	30	16	0.910	1.761	9.02	26.31	3.99

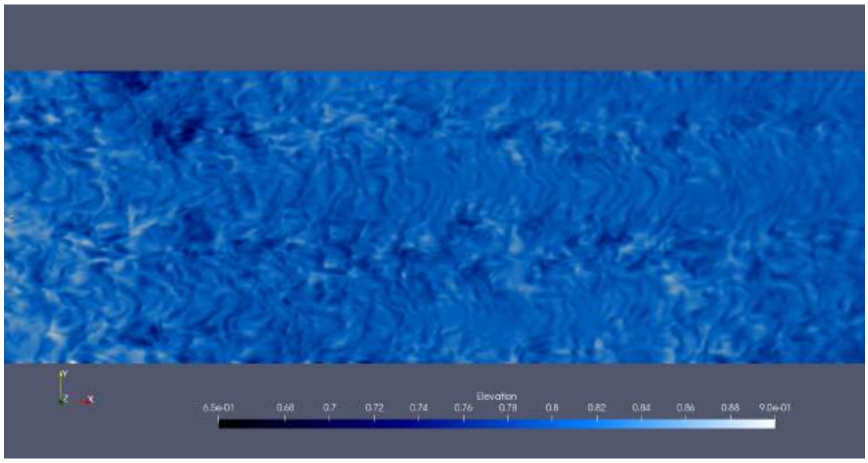

Figure 38. Wave height elevation at 35 mph over a fetch of 800 yards.

Figure 39 shows the wind velocity profile over the water surface. The wind speed is set at the top of the domain at an elevation of 30 meters which is defined as the standard for the measurement of wind velocity overland. The level of turbulence over the water is an indication of the wind and water interaction as the sea state develops.

8. Conclusions

The report has shown that the operation of wake boats on a lake has a minor impact on the environmental health of the body of water.

In an Australian study [8], the goal was to develop a decision support tool (DST) to objectively access the vulnerability of a particular shoreline to erosion. The study references a range of papers that describe the wave energy threshold for erosion. The range of wave heights noted by the author does not include any reference to wind waves and the author states, "Importantly, the previously proposed wave management criteria do not take into account the natural background wave energy, nor the condition of the bank." The quote from the author is true, but the studies cited were all done for a specific body of water. The wave heights noted generally agree on a maximum wave height of 28 cm (11 inches) as it approaches the shore. A broader definition [9] uses the following equation to define a maximum wave height.

$$H_h \le 0.5\sqrt{\frac{4.5}{T_h}} \tag{19}$$

where:

H_h: Maximum wave height (meters);

T_h: Mean Wave period (seconds).

The higher speed wake at 20 mph will cause turbidity through bottom friction while producing a smaller series of waves at the shoreline. The impact of rain events and modest wind events also tend to raise the level of turbidity and are the primary cause of erosion on shorelines and the introduction of sediment into the lake.

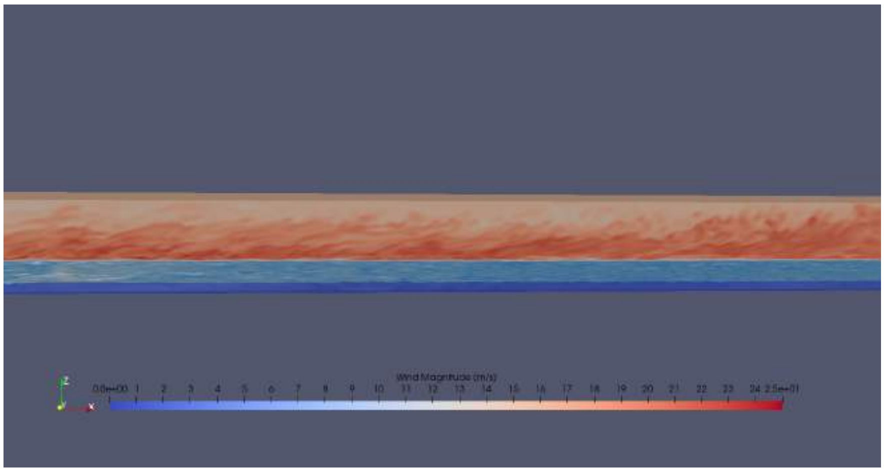

Figure 39. Wind profile over the water surface – 15 m/s at 30 m above the water surface.

In **Section 2** the predicted wave heights based on the wave energy equation assume the wake is a single wave. The equation is accurate close to the boat where the initial disturbance generates the wake. The predicted height of the wave by calculation matches closely with the measured wave height in **Figure 25** and the CFD results shown in **Figure 26**. As shown in the CFD analysis and in field test data, the initial disturbance of the water surface breaks into a group of waves as the wave train moves away from the boat and cycles over the surface of the water. The total energy of the wave train remains relatively constant while dissipating with every cycle, but the initial large wave breaks apart into smaller and smaller waves as the group travels away from the boat. The wave train energy that reaches the shore is reduced as the initial large surfing wave breaks into several smaller waves that will have little or no impact on the shoreline.

Based on both the field data and CFD data, the key to reducing the impact of wake surfing is to operate the boat far enough offshore to allow the wake neat the boat to dissipate into its component parts where the individual wave heights of the group are reduced to a height less than 28 cm (11 inches). The field test data [4] found 200 feet to be adequate to reduce the wave heights to under 28 cm (11 inches). In comparison to wind generated waves, the wave height of 28 cm is common in a modest wind event on lakes with a fetch of a half mile (0.8 km) at a wind speed of 20 mph (9.0 m/s). The full wave spectrum would be fully developed in less than 20 minutes and the average wave period would be 1.5 seconds.

The turbidity question is answered in the CFD analysis where motor craft should not operate at planning speeds in water depths under ten feet. At this depth, the turbidity levels would remain well above the bottom and the wash from the propeller(s) would not endanger any native water plants or disturb small fry. In lakes that are relatively shallow and have large ranges of shallow water, further restrictions may be necessary to reduce the bottom friction generated by turbidity caused by wakes of passing motor craft.

In a study, it has been observed that a wake-surf boat wake will dissipate completely in 300 meters from the boat path while operating in deep water [8]. Operating a boat that far off a shoreline may not be possible due to the size of the lake. The testing [4] [8] suggests a distance of 200 feet allows the wave train to dissipate enough to cause little or no impact on the shoreline. The commonsense approach includes a few operating guidelines for wake surfing. Always operate the boat at least 200 feet from shore and in a water depth greater than ten feet. If possible, run parallel to shore and make only lateral runs without turning at speed to reduce the large wake produced during a turn. If the lake is large enough, relocate within the lake to reduce the time in a particular area.

9. Epilogue

While operating any motorboat on a small body of water, the depth of water and the proximity to shore should be considered for the people on shore as well as the health of the lake. On large lakes in Ontario, a speed limit is imposed within 100 m (330 ft) from shore of 10 kph (6.3 mph) and 70 kph (44 mph) over the remainder of the lake. In New Hampshire there is a no-wake zone within 150 feet of the shore. Many states' focuses are on enforcement of existing laws on the books which state that the boat operator is responsible for their wake and any damage it may cause. The price of a ticket for a wake that causes damage or injury can be as high as $720. The law in Oregon reads if a skipper operates a boat in a way that damages or is likely to damage private property or cause injury, ORS 830.305 clearly states it as a citable offense. At this time many states are opting for the Play Away approach that everyone has a right to be on the water, but anyone that endangers others will be cited.

In some states, they are looking at imposing restrictions on lakes with an average water depth under fifteen feet. Wake surf boats should operate 200 feet offshore to minimize the wave impact on shore to allow the wave to break into their group components to an average height lower than the suggested limit of 11 inches in height. The rules going forward will include all power boats, but the wake surfing community needs to embrace their responsibility as operators to minimize the confrontations with other boats and people on shore. The conspicuous nature of wake surfing by generating a larger wake at a slower speed and staying in the area tends to draw attention to the activity. Sometimes the effected shoreline needs a break from the action, and they could move to a new location. The wake-surfers need to be sensitive to people on shore as everyone has a right to enjoy the water.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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