UNIT - 2

# 2 MATHEMATICAL EXPECTATION



## 1. Introduction :

We know that statistics is one of the scientific technique by using which we can collect the data, analyse the data and on the basis of it we can make prediction regarding the population characteristic under study. For collecting data we use sampling methods and then by using classification and tabulation we can arrange the data in order. Now we have to analyse this data and for that we determine the measures of average, dispersion, skewness and kurtosis. These measures we can easily find when we deal with frequency distribution, but when we have to deal with probability data and we suppose to find the above four measures then the concept of mathematical expectation is used. For understanding the term mathematical expectation first of all we suppose to understand the meaning of random variable and the probability distribution.

If two coins are tossed simultaneously the following sample space is generated :

$$S = \{HH, HT, TH, TT\}$$

In the experiment of tossing of two coins, suppose we are interested in number of heads obtained. If $x$ denotes the number of heads the values of $x$ can be 0, 1, 2. Here $x$ is said to be a discrete random variable. Thus a variable whose values can be obtained from the results of a random experiment is called a random variable. A random variable is a function associated with a sample space of a random experiment, and it takes different values with different probabilities. A random variable is usually denoted by $x$. A random variable can either be discrete or continuous. If a random variable can take finite values or countable infinite values it is known as a discrete variable. The number of heads obtained in the above illustration is a discrete variable. Similarly the number of children in a family, number of accidents are examples of discrete variables.



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If the number of defective screws manufactured by a company is denoted by $x$ then the values of $x$ can be 0, 1, 2, 3, 4, 5, 6..... etc. The value of discrete variable in this case are countable infinite values. If a random variable $x$ can take all values within an interval, it is known as a continuous variable. The age of persons, weight, height etc. are the illustrations of continuous variables.

Now we understand the concept of probability distribution.

If two coins are tossed and if $x$ denotes the number of heads obtained then, $x$ can take values 0, 1 and 2 with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$. The different values of $x$ together with their respective probabilities can be represented as follows :

| Variable $x_i$ | Probability $p(x_i)$ |
|---|---|
| 0 | $\frac{1}{4}$ |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |
| Total | 1 |

The table showing different values of a variable $x$ and their respective probabilities, is said to be a probability distribution. Here the total probability 1 is distributed among different values of a variable.

**Definition of probability mass function :**

If $x_1, x_2, x_3, \ldots.. x_n$ are different values of a discrete random variable and $p(x_1), p(x_2), \ldots\ldots.. p(x_n)$ are their respective probabilities and if

(i) $p(x_i) \geq 0$ i.e. all probability values are non-negative

(ii) $\sum p(x_i) = 1$. i.e. total probability is one.

then the function p is known as the probability mass function of random variable $x$ and the values of $p(x_i)$ are known as probability distribution of a random variable $x$. Thus in a probability distribution the values of $p(x_i)$ should be non-negative and the sum of probabilities of all values of $x$ should be equal to 1. Let us take one more illustration of a probability distribution.

If an unbiased die is thrown, any one number from 1, 2, 3, 4, 5, 6 can be obtained on the upper face. The table shows the different values of $x$ and their probabilities.

| No.on the die $x_i$ | Probability $p(x_i)$ |
|---|---|
| 1 | $\frac{1}{6}$ |
| 2 | $\frac{1}{6}$ |
| 3 | $\frac{1}{6}$ |
| 4 | $\frac{1}{6}$ |
| 5 | $\frac{1}{6}$ |
| 6 | $\frac{1}{6}$ |
| Total | 1 |



It is clear that $p(x_i) \geq 0$ and $\sum p(x_i) = 1$

## 2. Mathematical expectation of a discrete variable :

The expected value of a discrete random variable is the average value of the random variable. If $x_1, x_2, x_3, \ldots.. x_n$ are different values of a random variable $x$ with their respective probabilities $p(x_1), p(x_2), p(x_3), \ldots\ldots.. p(x_n)$ then the expected value of the random variable $x$ is defined as follows :

$$E(x) = x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \cdot p(x_3) + \ldots\ldots.. x_n\, p(x_n)$$
$$= \sum x_i\, p(x_i)$$

The expected value of $x$ is the mean of $x$ and is denoted by $\mu$.

$\therefore \quad E(x) = \mu$

**Some properties of expected value :**

(i) The expected value of a constant is a constant itself i.e. $E(k) = k$.

(ii) If $k$ is a constant, $E(kx) = k \cdot E(x)$.

(iii) $E(ax + b) = aE(x) + b$.

i.e. Expectation is not independent of change of origin and scale.

(iv) If $x$ and $y$ are two random variables,

$E(x + y) = E(x) + E(y)$.

i.e. Epectation of sum of two random variables, is the sum of their expectations.

(v) If $x$ and $y$ are two independent random variables,

$E(xy) = E(x) \cdot E(y)$

i.e. Expectation of the product of two independent random variable is product of their expectations.

(vi) $E(x - \mu) = E(x) - E(\mu)$
$= \mu - \mu$
$= 0$

i.e. Expectation of the deviations taken from mean is zero.

(vii) If $g(x)$ is a function of $x$,

$$E[g(x)] = \sum g(x) \cdot p(x)$$

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**Illustration 1: An unbiased die is thrown. Find the expected value of the number on the die.**

**Ans :**

| Number on the die $x_i$ | probability $p(x_i)$ | $x_i p(x_i)$ |
|---|---|---|
| 1 | $\frac{1}{6}$ | $\frac{1}{6}$ |
| 2 | $\frac{1}{6}$ | $\frac{2}{6}$ |
| 3 | $\frac{1}{6}$ | $\frac{3}{6}$ |
| 4 | $\frac{1}{6}$ | $\frac{4}{6}$ |
| 5 | $\frac{1}{6}$ | $\frac{5}{6}$ |
| 6 | $\frac{1}{6}$ | $\frac{6}{6}$ |
| Total | 1 | $\frac{21}{6}$ |

$$E(x) = \sum x_i p(x_i)$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6}$$

$$= \frac{21}{6}$$

$$= \frac{7}{2}$$

$$= 3.5$$



**Illustration 2 : Two dice are thrown. Find the expected value of the total on the dice.**

**And. :**

| Total on the dice $x_i$ | Probability $p(x_i)$ | $x_i p(x_i)$ |
|---|---|---|
| 2 | $\frac{1}{36}$ | $\frac{2}{36}$ |
| 3 | $\frac{2}{36}$ | $\frac{6}{36}$ |
| 4 | $\frac{3}{36}$ | $\frac{12}{36}$ |
| 5 | $\frac{4}{36}$ | $\frac{20}{36}$ |
| 6 | $\frac{5}{36}$ | $\frac{30}{36}$ |
| 7 | $\frac{6}{36}$ | $\frac{42}{36}$ |
| 8 | $\frac{5}{36}$ | $\frac{40}{36}$ |
| 9 | $\frac{4}{36}$ | $\frac{36}{36}$ |
| 10 | $\frac{3}{36}$ | $\frac{30}{36}$ |
| 11 | $\frac{2}{36}$ | $\frac{22}{36}$ |
| 12 | $\frac{1}{36}$ | $\frac{12}{36}$ |
| Total | 1 | $\frac{252}{36}$ |

$$E x = \sum x_i \cdot p(x_i) = \frac{252}{36} = 7$$

## 3. Variance of a random variable :

The average of the squares of the deviations from the mean of a random variable $x$ is said to be its variance.

If the mean of a random variable $x$ is $E(x) = \mu$ the variance of $x$ can be given as follows :

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$$V(x) = E(x-\mu)^2$$
$$= E(x^2 - 2x\mu + \mu^2)$$
$$= E(x^2) - 2\mu E(x) + E(\mu^2)$$
$$= E(x^2) - 2\mu \cdot \mu + \mu^2$$
$$= E(x^2) - 2\mu^2 + \mu^2$$
$$= E(x^2) - \mu^2$$
$$= E(x^2) - [E(x)]^2$$
$$V(x) = E(x^2) - [E(x)]^2$$

**Mean Variance and Covariance**

We have seen that

Mean $= E(x) = \bar{x}$

Variance $= E(x - \bar{x})^2$

$= E(x^2) - [E(x)]^2$

$\therefore \; V(x) = E(x^2) - \bar{x}^2$

The variance of $x$ measures the variations in the values of $x$ and the variance of $y$ measures the variations in $y$. The simultaneous variations in the values of $x$ and $y$ can be measured by covariance. The co-variation between two variables $x$ and $y$, denoted by cov $(x, y)$ can be defined in the following way.

If the means two variables $x$ and $y$ are $\bar{x}$ and $\bar{y}$ then

$\text{Cov}(x, y) = E(x - \bar{x})(y - \bar{y})$

This formula can also be expressed as

$\text{Cov}(x, y) = E(x - \bar{x})(y - \bar{y})$

$= E(xy - \bar{x}y - \bar{y}x + \bar{x}\bar{y})$

$= E(xy) - \bar{x}E(y) - \bar{y}E(x) + E(\bar{x}\bar{y})$

$= E(xy) - \bar{x}\bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y}$

$= E(xy) - \bar{x}\bar{y}$

$= E(xy) - E(x).\, E(y).$

**If $x$ andy are independent variables Cov $(x, y) = 0$.**

$\text{Cov}(x, y) = E(xy) - E(x)\,.\,E(y)$

For two independent variables $x$ and $y$

$E(xy) = E(x)\,.\,E(y)$

$\therefore \text{Cov}(x, y) = E(x)\,.\,E(y) - E(x)\,.\,E(y)$

$= 0.$

Thus for two independent variables Cov $(x, y)$ is 0.



**The following are some useful results about variance and covariance of two variables.**

(1) $V(x + a) = V(x)$

(2) $V(ax + b) = a^2 V(x)$

(3) $\text{Cov}(ax, by) = ab\,\text{Cov}(x, y)$

(4) $\text{Cov}(x + a, y + b) = \text{Cov}(x, y)$

(5) It $x$ and $y$ are independent $\text{Cov}(x, y) = 0$

## 5. Illustrations :

**Illustration 3 : A person throws an unbiased die and he gets the amount in rupees equal to the square of the face value obtained. Find the mathematical expectation of his amount.**

**Ans. :**

| Face value | The amount gained $x_i$ | probability $p(x_i)$ | $x_i p(x_i)$ |
|---|---|---|---|
| 1 | 1 | $\frac{1}{6}$ | $\frac{1}{6}$ |
| 2 | 4 | $\frac{1}{6}$ | $\frac{4}{6}$ |
| 3 | 9 | $\frac{1}{6}$ | $\frac{9}{6}$ |
| 4 | 16 | $\frac{1}{6}$ | $\frac{16}{6}$ |
| 5 | 25 | $\frac{1}{6}$ | $\frac{25}{6}$ |
| 6 | 36 | $\frac{1}{6}$ | $\frac{36}{6}$ |
| Total | | 1 | $\frac{91}{6}$ |

$$E(x) = \sum x_i \cdot p(x_i)$$
$$= \frac{91}{6} \quad = 15.17$$

**Illustration 4 : The probability distribution of a random variable $x$ is as follows. Find (i) The value of $p$ (ii) $E(x)$.**

| $x_i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $p(x_i)$ | $\frac{1}{16}$ | $p$ | $\frac{3}{8}$ | $p$ | $\frac{1}{16}$ |

**Ans. :** (i) For a probability distribution

$$\sum p(x_i) = 1$$
$$\therefore \quad \frac{1}{16} + p + \frac{3}{8} + p + \frac{1}{16} = 1$$

$1 + 16p + 6 + 16p + 1 = 16$

$\therefore \quad 32p = 8$

$\therefore \quad p = \frac{8}{32}$

$\therefore \quad p = \frac{1}{4}$

$\therefore$

Now (ii) $E(x) = \sum x_i \, p(x_i)$

$= 0\left(\frac{1}{16}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{16}\right)$

$= 0 + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{4} = 2$

**Illustration 5 : The probability distribution of a random variabl $x$ is as follows. (i) Find the value of $k$ (ii) Obtain the probabili distribution of $x$.**

| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p(x_i)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

**Ans. :** For a probability distribution the total probability $\sum p(x_i) = 1$

$\therefore \; 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\therefore \quad 10k^2 + 9k = 1$

$\therefore \quad 10k^2 + 9k - 1 = 0$

$\therefore \quad 10k^2 + 10k - k - 1 = 0$

$\therefore \quad 10k(k + 1) - 1(k + 1) = 0$

$\therefore \quad (k + 1)(10k - 1) = 0$

$\therefore \quad k + 1 = 0$ or $10k - 1 = 0$

$\therefore \quad k = -1$ or $k = \frac{1}{10}$

$k = -1$ is not possible

$\therefore \quad k = \frac{1}{10}$

(ii) The probability distribution of $x$ will be as follows :

| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|---|
| $p(x_i)$ | 0 | $\frac{1}{10}$ | $\frac{2}{10}$ | $\frac{2}{10}$ | $\frac{3}{10}$ | $\frac{1}{100}$ | $\frac{2}{100}$ | $\frac{17}{100}$ | 1 |



**Illustration 6 : The probability distribution of a random variable $x$ is as follows :**

| $x_i$ | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| Probability | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

**Find mean and variance of $x$.**

**Ans. :**

| $x_i$ | Probability $p(x_i)$ | $x_i . p(x_i)$ | $x_i^2 \cdot p(x_i)$ |
|---|---|---|---|
| −1 | $\frac{1}{8}$ | $-\frac{1}{8}$ | $\frac{1}{8}$ |
| 0 | $\frac{1}{8}$ | 0 | 0 |
| 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| 2 | $\frac{1}{4}$ | $\frac{2}{4}$ | 1 |
| 3 | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{9}{8}$ |
| 4 | $\frac{1}{8}$ | $\frac{4}{8}$ | $\frac{16}{8}$ |
| Total | 1 | $\frac{12}{8}$ | $\frac{36}{8}$ |

Mean $= E(x) = \sum x_i \cdot p(x_i)$

$= \frac{12}{8}$

$= 1.5$

Now $E(x^2) = \sum x_i^2 \cdot p(x_i)$

$= \frac{36}{8}$

$= 4.5$

Variance $\sigma^2 = E(x^2) - [E(x)]^2$

$= 4.5 - (1.5)^2$

$= 4.5 - 2.25$

$= 2.25$

**Illustration 7 : The probability distribution of a random variable $x$ is as follows :**

| $x_i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Probability | $\frac{1}{10}$ | $p$ | $\frac{3}{10}$ | $p$ | $\frac{1}{10}$ |

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(i) Find the value of $p$.

(ii) Find $E(x+1)$

**Ans. :** For a probability distribution, total probability $= \sum p(x_i) = 1$.

$$\frac{1}{10} + p + \frac{3}{10} + p + \frac{1}{10} = 1$$

$\therefore$ $$\frac{1+10p+3+10p+1}{10} = 1$$

$\therefore$ $$20p + 5 = 10$$

$\therefore$ $$20p = 5$$

$\therefore$ $$p = \frac{5}{20}$$

$\therefore$ $$p = \frac{1}{4}$$

Now $E(x+1) = E(x) + E(1)$

$= E(x) + 1$

$E(x) = \sum x_i \cdot p(x_i)$

$$= 0\left(\frac{1}{10}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{3}{10}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{10}\right)$$

$$= 0 + \frac{1}{4} + \frac{6}{10} + \frac{3}{4} + \frac{4}{10}$$

$$= \frac{0+5+12+15+8}{20}$$

$$= \frac{40}{20}$$

$$= 2$$

$\therefore$ $E(x+1) = E(x) + E(1) = 2 + 1 = 3.$

**Illustration 8 : The probability distribution of demand of a commodity is given below :**

| Demand $x$ | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| Probability p(x) | 0.05 | 0.1 | 0.3 | 0.4 | 0.1 | 0.05 |

**Find the expected demand and its variance.**



**Ans. :**

| Demand $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ | $x_i^2 \cdot p(x_i)$ |
|---|---|---|---|
| 5 | 0.05 | 0.25 | 1.25 |
| 6 | 0.1 | 0.60 | 3.60 |
| 7 | 0.3 | 2.10 | 14.70 |
| 8 | 0.4 | 3.20 | 25.60 |
| 9 | 0.1 | 0.90 | 8.10 |
| 10 | 0.05 | 0.50 | 5.00 |
| Total | 1 | 7.55 | 58.25 |

Expected Demand $= E(x) = \sum x_i p(x_i) = 7.55$

$E(x^2) = \sum x_i^2 p(x_i) = 58.25$

$\therefore$ Variance $\sigma^2 = E(x^2) - [E(x)]^2$

$= 58.25 - (7.55)^2$

$= 58.25 - 57$

$= 1.25.$

**Illustration 9 : There are 3 black and 2 white balls in a box. Two balls are taken at random from it, find the expected number of white balls.**

**Ans. :**

| Number of white balls $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|
| 0 | $\frac{^2C_0 \times {^3C_2}}{^5C_2} = \frac{3}{10}$ | 0 |
| 1 | $\frac{^2C_1 \times {^3C_1}}{^5C_2} = \frac{6}{10}$ | $\frac{6}{10}$ |
| 2 | $\frac{^2C_2 \times {^3C_0}}{^5C_2} = \frac{1}{10}$ | $\frac{2}{10}$ |
| Total | 1 | $\frac{8}{10}$ |

$E(x) = \sum x_i \cdot p(x_i)$

$$= \frac{8}{10} = \frac{4}{5}$$



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**Illustration 10 : There are 2 white and 4 black balls in a box. A person takes 3 balls at random from the box. If he receives Rs. 10 for each white ball and receives Rs. 5 for each black ball, find the expected value of the amount received by him.**

**Ans. :**

| Number of white balls | Amount received in Rs. $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|---|
| 0 | 15 | $\frac{{}^2C_0 \times {}^4C_3}{{}^6C_3} = \frac{4}{20}$ | $\frac{60}{20}$ |
| 1 | 20 | $\frac{{}^2C_1 \times {}^4C_2}{{}^6C_3} = \frac{12}{20}$ | $\frac{240}{20}$ |
| 2 | 25 | $\frac{{}^2C_2 \times {}^4C_1}{{}^6C_3} = \frac{4}{20}$ | $\frac{100}{20}$ |
| Total | | 1 | $\frac{400}{20}$ |

$$E(x) = \sum x_i \cdot p(x_i)$$

$$= \frac{400}{20}$$

$$= \text{Rs. } 20$$

**Illustration 11 : There are two coins. On one face of each coin 1 is written and on the other face 2 is written. The coins are tossed simultaneously. Find the expected value of the total on the coins.**

**Ans. :** Suppose 1 is written on the face showing Head and 2 is written on the face showing Tail.

| Number of heads | Total obtained on the coins $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|---|
| 0 | 4 | $\frac{1}{4}$ | $\frac{4}{4}$ |
| 1 | 3 | $\frac{1}{2}$ | $\frac{3}{2}$ |
| 2 | 2 | $\frac{1}{4}$ | $\frac{2}{4}$ |
| Total | | 1 | $\frac{12}{4}$ |



The expected value of the total on the coins

$$= E(x)$$

$$= \sum x_i\, p(x_i)$$

$$= \frac{12}{4}$$

$$= 3.$$

**Illustration 12 : There are 6 slips in a box and numbers 1, 1, 2, 2, 3, 3 are written on these slips. Two slips are taken at random from the box, find the expected value of the sum of the numbers on the two slips.**

**Ans. :**

| Numbers on slips | Sum of the numbers $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|---|
| (1, 1) | 2 | $\frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$ | $\frac{2}{15}$ |
| (1, 2) | 3 | $\frac{{}^2C_1 \times {}^2C_1}{{}^6C_2} = \frac{4}{15}$ | $\frac{12}{15}$ |
| (1, 3) | 4 | $\frac{{}^2C_1 \times {}^2C_1}{{}^6C_2} = \frac{4}{15}$ | $\frac{16}{15}$ |
| (2, 2) | 4 | $\frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$ | $\frac{4}{15}$ |
| (2, 3) | 5 | $\frac{{}^2C_1 \times {}^2C_1}{{}^6C_2} = \frac{4}{15}$ | $\frac{20}{15}$ |
| (3, 3) | 6 | $\frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$ | $\frac{6}{15}$ |
| | | 1 | $\frac{60}{15}$ |

Expected value of the sum of the numbers on the two slips

$$= \sum x_i \cdot p(x_i)$$

$$= \frac{60}{15}$$

$$= 4.$$

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**Illustration 13 : 10,000 tickets each of Re. 1 are sold in a lottery. There is only one ticket in the lottery bearing a prize of Rs. 8000. A person is having one ticket of the lottery. Find his expectation.**

**Ans.**

| The amount gained $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|
| $-1$ | $\frac{9999}{10000}$ | $-\frac{9999}{10000}$ |
| $7999$ | $\frac{1}{10000}$ | $\frac{7999}{10000}$ |

$$E(x) = \sum x_i \cdot p(x_i)$$
$$= -\frac{9999}{10000} + \frac{7999}{10000}$$
$$= \frac{-2000}{10000}$$
$$= -0.20.$$

**Illustration 14 : There are 10 electric bulbs in a box in which 3 are defective bulbs. If 3 bulbs are selected at random from the box, find the expected number of defective bulbs.**

**Ans. :**

| Number of Defective bulbs $x_i$ | Probability $p(x_i)$ | $x_i \cdot p(x_i)$ |
|---|---|---|
| 0 | $\frac{^7C_3}{^{10}C_3} = \frac{35}{120}$ | 0 |
| 1 | $\frac{^3C_1 \times {^7C_2}}{^{10}C_3} = \frac{63}{120}$ | $\frac{63}{120}$ |
| 2 | $\frac{^3C_2 \times {^7C_1}}{^{10}C_3} = \frac{21}{120}$ | $\frac{42}{120}$ |
| 3 | $\frac{^3C_3}{^{10}C_3} = \frac{1}{120}$ | $\frac{3}{120}$ |
| Total | 1 | $\frac{108}{120}$ |



Expected number of defective bulbs

$$= E(x)$$
$$= \sum x_i \cdot p(x_i)$$
$$= \frac{108}{120}$$
$$= 0.9$$

**Illustration 15 : The probability distribution of a descrete random variable is as follow :**

**$P(x) = 0.1$ when $x = 0$**

**$P(x) = K(x + 1)$ when $x = 1, 2, 3$**

**Find the value of K. Also determine mean and variance of the distribution.**

**Ans. :**

Here the values of descrete variable are 0, 1, 2, 3

When $x = 0$, $P(0) = 0.1$

When $x = 1$, $P(1) = K(1 + 1) = 2K$

When $x = 2$, $P(2) = K(2 + 1) = 3K$

When $x = 3$, $P(3) = K(3 + 1) = 4K$

For a probability distribution

$$\Sigma P(x_i) = 1$$
$$\therefore \quad 0.1 + 2K + 3K + 4K = 1$$
$$\therefore \quad 9K = 1 - 0.1$$
$$\therefore \quad 9K = 0.9$$
$$\therefore \quad K = 0.1$$

Hence the probability distribution will be as follow :

| $x_i$ | $P(x_i)$ | $x_i\, P(x_i)$ | $x_i^2\, P(x_i)$ |
|---|---|---|---|
| 0 | 0.1 | 0 | 0 |
| 1 | 2(0.1) = 0.2 | 0.2 | 0.2 |
| 2 | 3(0.1) = 0.3 | 0.6 | 1.2 |
| 3 | 4(0.1) = 0.4 | 1.2 | 3.6 |
| | 1 | 2.0 | 5.0 |

$$\text{Mean} = E(x)$$
$$= E\, x_i\, P(x_i)$$
$$= 2$$

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$E(x^2) = \Sigma x_i^2 P(x_i)$

$= 5.0$

$V(x) = E(x^2) - (Ex)^2$

$= 5.0 - (2)^2$

$= 1$

**Illustration 16 : For two independent variables $x$ and $y$ $E(x) = 3$, $E(y) = 2$, $V(x) = 1.2$, $V(y) = 1$. Find (i) $E(x + y)$ (ii) $E(x - y)$ (iii) $E(2x + 3y)$ (iv) $E(3x - y)$ (v) $V(5x + 2y)$ (vi) $V(3x - 2y)$ (vii) $V(x + 2y + 5)$**

**Ans. :** (i) $E(x + y)$

$= E(x) + E(y)$

$= 3 + 2$

$= 5$

(ii) $E(x - y)$

$= E(x) - E(y)$

$= 3 - 2$

$= 1$

(iii) $E(2x + 3y)$

$= 2E(x) + 3E(y)$

$= 2(3) + 3(2)$

$= 12$

(iv) $E(3x - y)$

$= 3E(x) - E(y)$

$= 3(3) - 2$

$= 7$

(v) $V(5x + 2y)$

$= 5^2 V(x) + 2^2 V(y)$

$= 25 (1.2) + 4 (1)$

$= 30 + 4$

$= 34$

(vi) $V(3x - 2y)$

$= 3^2 V(x) + 2^2 V(y)$

$= 9 (1.2) + 4(1)$

$= 10.8 + 4$

$= 14.8$



(vii) $V(x + 2y + 5)$

$= 1^2 V(x) + 2^2 V(y)$

$= 1(1.2) + 4(1)$

$= 5.2$

**Illustration 17 : Five dice are tossed simultaneously find the expected value of the total on the dice.**

**Ans. :** Let the numbers turning on the five dice be denoted by $x_1, x_2, x_3, x_4, x_5$

Now $E(x_1) = \Sigma x_i p_i$

$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6}\ 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$

$= \frac{1}{6} (1 + 2 + 3 + 4 + 5 + 6)$

$= \frac{7}{2}$

Similarly $E(x_2) = 7/2$

$E(x_3) = 7/2$

$E(x_4) = 7/2$

$E(x_5) = 7/2$

$\therefore\ E(x_1 + x_2 + x_3 + x_4 + x_5) = E(x_1) + E(x_2) + E(x_3) + E(x_4) + E(x_5)$

$= \frac{7}{2} + \frac{7}{2} + \frac{7}{2} + \frac{7}{2} + \frac{7}{2}$

$= \frac{35}{2}$

**Illustration 18 :** The following is a distribution of students of a class, according to age.

| Age (In years) | 18 | 19. | 20 | 21 | 22 | Total |
|---|---|---|---|---|---|---|
| Number of students | 38 | 42 | 12 | 4 | 4 | 100 |

A student is selected at random from the class. Find his expected age.

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**Ans. :**

| Age $x_i$ | Number of students $fi$ | $p(x_i) = \frac{fi}{N}$ | $x_i\, p(x_i)$ |
|---|---|---|---|
| 18 | 38 | 0.38 | 6.84 |
| 19 | 42 | 0.42 | 7.98 |
| 20 | 12 | 0.12 | 2.40 |
| 21 | 4 | 0.04 | 0.84 |
| 22 | 4 | 0.04 | 0.88 |
| | N = 100 | | 18.94 |

Expected age of the student = $\Sigma x_i p(x_i)$ = 18.94 years.

**Illustration 19 : $x$ and $y$ are two independent variables and their variances are respectively 3 and 5 find V $(3x + 4y)$.**

**Ans. :** $V(3x + 4y)$

$= 3^2\, V(x) + 4^2 V(y)$
$= 9V(x) + 16.V(y)$
$= 9(3) + 16(5)$
$= 27 + 80$
$= 107$

**Illustration 20 : The different values of two variables $x$ and $y$ alongwith their probabilities are given in the following table. Find E($x$), E($y$), E($x$+$y$), E($xy$), V($x$), V($y$), Cov. ($x$, $y$)**

| y \ x | 1 | 2 | 3 | Total |
|---|---|---|---|---|
| 1 | 0.1 | 0.1 | 0.1 | 0.3 |
| 2 | 0.1 | 0.2 | 0.1 | 0.4 |
| 3 | 0.1 | 0.1 | 0.1 | 0.3 |
| Total | 0.3 | 0.4 | 0.3 | 1.0 |

**Ans. :** (i) $E(x) = \Sigma x_i p_i$

$= 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.3$
$= 2.0$

$E(y) = \Sigma y_j p_j$
$= 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.3$
$= 2.0$

$E(x + y) = E(x) + (E(y)$
$= 2.0 + 2.0$
$= 4.0$



$E(xy) = \Sigma x_i\, y_j\, p_{ij}$
$= (1)(1)(0.1) + (2)(1)(0.1) + 3(1)(0.1)$
$+ (1)(2)(0.1) + (2)(2)(0.2) + (3)(2)(0.1)$
$+ (1)(3)(0.1) + (2)(3)(0.1) + (3)(3)(0.1)$
$= 0.1 + 0.2 + 0.3 + 0.2 + 0.8 + 0.6 + 0.3 + 0.6 + 0.9$
$= 4.0$

$V(x) = Ex^2 - [E(x)]^2$

We shall find $E(x^2)$

$E(x^2) = \Sigma x^2_i p_i$
$= 1^2(0.3) + 2^2(0.4) + 3^2(0.3)$
$= 0.3 + 1.6 + 2.7$
$= 4.6$

Now $V(x) = E(x^2) - [E(x)]^2$
$= 4.6 - (2.0)^2$
$= 4.6 - 4.0$
$= 0.6$

Similarly $V(y) = E(y^2) - [E(y)]^2$
$= 4.6 - 4$
$= 0.6$

Cov. $(x, y) = E(xy) - E(x) . E(y)$
$= 4 - 2 \times 2$
$= 0$

Thus $x$ and $y$ are proved to be independent variables.

**Illustration 21 : The probability distribution of a random variable is as follows.**

Find (i) K (ii) E $(x + 3)$ (iii) V $(2x + 3)$

| $x_i$ | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|---|
| $P(x_i)$ | K | 0.2 | 0.1 | K | 0.05 | 0.05 | 1 |

*(G. U. April, 2001)*

**Ans. :** For a probability distribution $\Sigma P(x_i) = 1$

$\therefore$ K + 0.2 + 0.1 + K + 0.05 + 0.05 = 1
$\therefore$ 2K + 0.4 = 1
$\therefore$ 2K = 0.6
$\therefore$ K = 0.3

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The following table can be prepared

| $x_i$ | $p(x_i)$ | $x_i p(x_i)$ | $x_i^2 P(x_i)$ |
|---|---|---|---|
| 0 | 0.3 | 0 | 0 |
| 1 | 0.2 | 0.20 | 0.20 |
| 2 | 0.1 | 0.20 | 0.40 |
| 3 | 0.3 | 0.90 | 2.70 |
| 4 | 0.05 | 0.20 | 0.80 |
| 5 | 0.05 | 0.25 | 1.25 |
| Total | 1.0 | 1.75 | 5.35 |

$E(x) = \Sigma x_i p(x_i)$

$= 1.75$

(ii) $E(x + 3) = E(x) + E(3)$

$= 1.75 + 3$

$= 4.75$

Now $E(x)^2 = Ex_i^2 P(x_i)$

$= 5.35$

$V(x) = E(x^2) - [E(x)]^2$

$= 5.35 - (1.75)^2$

$= 5.35 - 3.0625$

$= 2.2875$

$\therefore V(2x + 3) = 4V(x)$

$= 4(2.2875)$

$= 9.15$

## EXERCISE

### SECTION-A

**Answer in short :**

1. Which measures are obtain for statistical analysis of data ?
2. For what purpose the concept of mathematical expectation is used?
3. Define random variable.
4. Give one example of discrete random variable.
5. Give one example of continuous random variable.
6. Define probability distribution.
7. Check whether the following distribution is a probability distribution or not.

| x | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| P(x) | 0.12 | 0.18 | 0.30 | 0.12 | 0.18 |



8. A random variable x takes value 1 with probability 0.4 and value 2 with probability 0.6 Then what is its expected value.
9. If $E(x) = 5$ then what is the value of $E(2x + 3)$ ?
10. If $E(x) = 10$ and $V(x) = 15$ then what is the value of $E(x - 2)^2$ ?
11. For a bivariate data $E(x) = 5$, $E(y) = 4$ and $E(x, y) = 20$. Can we say that $x$ and $y$ are independent events ?
12. If $E(x) = 8.5$ and $E(y) = 06$ then find $E(x + y)$.
13. For a data a student has calculated that $E(x) = 15$ and $E(x^2) = 200$. Is this calculations are correct ?
14. for a data $E(x) = 15$ and $E(x^2) = 250$. Find $V(x)$.
15. For a data $V(x) = 3$. Then find $V(2x - 3)$
16. For a data $V(x) = 5$. Find $V(3 - 2x)$
17. If for a data S.D. of $x$ is 5 then find S.D. $(4 + 3x)$
18. For a data, $Sx = 10$ then find S.D. of $(3 - 5x)$
19. The random variable x assumes values –1, 0 and 1 with respective probabilities 0.3, 0.4 and 0.3. Find the mean and variance of $x$.
20. In a certain business, there are 70% chances to have a profit of Rs. 5000 and 30% chances to have a loss of Rs. 12000. Can you advise the person to do the business ?
21. If $E(x) = 15$, Standard deviation is 5 then find $E(x^2)$.

| [Ans. : | 7. Not | 8. 1.6 | 9. 13 | 10. 79 |
|---|---|---|---|---|
| | 11. Yes | 12. 14.5 | 13. Not Correct | 14. 25 |
| | 15. 12 | 16. 20 | 17. 15 | 18. 50 |
| | 19. 0, 0.6 | 20. Not | 21. 240 | |

### SECTION-B

1. What is a probability distribution ? State its main properties.
2. Define mathematical expectation. State the characteristics of mathematical Expectation.
3. Define variance of a random variable $x$ and prove that

$$\text{Variance } \sigma^2 = E(x^2) - [E(x)]^2$$

4. The probability distribution of a random variable $x$ is as follows :

| $x_i$ | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $p(x_i)$ | 0.15 | 0.30 | 0.30 | 0.15 | 0.10 |

Find the values of the followings :

(i) $E(x)$ (ii) $E(3x + 1)$ (iii) $E(x^2)$

(iv) $E(x + 1)^2$ (v) $V(x)$ (vi) $V(2x + 3)$

[Ans. : (i) – 0.25 (ii) 0.25 (iii) 1.45

(iv) 1.95 (v) 1.3875 (vi) 5.55]

5. For a random variable $x$, $E(x) = 2$, find the values of the following :

(i) $E(3x)$; (ii) $E(x + 5)$; (iii) $E(2x + 3)$

[Ans. : (i) 6, (ii) 7, (iii) 7]

6. The probability distribution of a random variable $x$ is as follows :

| $x_i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $p(x_i)$ | 0.05 | .10 | .30 | .20 | .05 | .10 | .05 | .10 | .05 |

Find mean and S.D of $x$. [Ans. : Mean = 5.4; S.D = 2.18]

7. The probability distribution of a random variable is as follows :

| $x_i$ | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|
| $p(x_i)$ | .04 | .19 | $3p$ | .26 | $p$ | .07 |

Find the value of $p$ and hence obtain the expected value of $x$.

[Ans. : $p = 0.11$, $E(x) = 17.42$]

8. The probability distribution of a random variable $x$ is as follows :

| $x_i$ | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $p(x_i)$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $p$ | $p$ | $\frac{1}{12}$ | $\frac{1}{12}$ |

Find the value of $p$ and also obtain mean and variance of $x$.

[Ans. : $p = \frac{1}{6}$, mean $= \frac{11}{12}$, variance $= \frac{323}{144}$]

9. There are 5 white and 3 black balls in a box. 3 balls are taken at random from the box. Find the expected number of black balls.

[Ans. : $\frac{9}{8}$]

10. There are 8 screws in a packet of which 2 are defective. If 2 screws are taken at random, find the expected number of defective screws and also obtain its variance.

[Ans. : Expected value $= \frac{1}{2}$; variance $= \frac{9}{28}$ ]



11. The distribution of demand of an item for different number of days is given below. Find the expected demand.

| Demand | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|
| Number of days | 10 | 20 | 20 | 40 | 10 |

[Ans. : 22.2]

12. The record showing the demand of trucks for last 50 days is given below. Find the expected demand and variance of the demand.

| Number of trucks demanded | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Number of days | 5 | 10 | 15 | 10 | 10 |

[Ans. : $E(x) = 2.2$, $V(x) = 1.56$]

13. There are 100 tickets in a lottery of Re. 1 each. There is only one ticket in the lottery bearing a prize of Rs. 80. A person purchases 1 ticket. Find his expectation. [Ans. : – 0.20]

14. In a lottery the prize of each ticket is Rs. 100. The probability of winning the prize of Rs. 1,00,000 is 0.00001, the probability of winning the prize of Rs. 50,000 is 0.0001 and the probability of winning the prize of Rs. 10 is 0.4. If one ticket is purchased find the expectation. [Ans. : –90]

15. A person takes an insurance of Rs. 1000 and pays premium of Rs. 20. The probability that any person of his age group dies within a year is 0.01. Find the expected gain of the insurance company. [Ans. : Rs. 10]

16. There are 5 tickets in a box numbered 1, 1, 2, 2, 2 respectively. Two tickets are taken at random from it, find the expectation of the total of the numbers on the tickets.

[Ans. : $\frac{16}{5}$]

17. Two tickets are taken at random from 5 tickets numbered from 1 to 5. Find the expected value of the sum obtained on the two tickets. [Ans. : 6]

18. 4 coins are tossed simultaneously, find the expected number of heads and its variance.

[Ans. : Expected number of heads = 2, Variance = 1]

19. Two coins are tossed simultaneously. A person receives Rs. 8 for each head and loses Rs. 10 for each tail. Find the expected value of the amount gained by him. [Ans. : –2]

20. There are 4 black and 2 white balls in a box and 2 balls are taken at random from it. If a person receives Rs. 4 for each white ball

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and loses Rs. 2 for each black ball, find the mathematical expectation of the amount received by him.

[**Ans.** : Expectation = 0]

**21.** There are 3 black and 2 white balls in a box. 2 balls are taken from it. Rs. 24 is given for each black ball. What amount should be charged for each white ball so that the game is fair ?

[**Ans.** : As the game is fair, $E(x) = 0$, Rs. 36 should be charged for each white ball.]

**22.** The probability mass function of a random variable $x$ is given below,

$$p(x) = k(x+2), \quad x = -2, -1, 0, 1, 2$$
$$= 0, \quad \text{elsewhere}$$

Find the value of constant $k$ and obtain mean and variance of the distribution. *(G. U., F.Y. B.Com., April, 1998)*

[**Ans.** : $k = 0.1$, Mean = 1, Vriance = 1]

**23.** The probability distribution of a random variable $x$ is as follows :

| $x_i$ | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $p(x_i)$ | 0.04 | 0.16 | $3p$ | 0.29 | $p$ | 0.07 |

obtaining the value of $p$, find $E(3x+5)$, and $V(2x-3)$

*(G.U., F. Y. B. Com., April, 2000)*

[**Ans.** : $p = 0.11$ ; $E(3x+5) = 9.44$, $V(2x-3) = 5.7984$]

**24.** If $E(x) = 2$, $V(x) = 1$, find $E(x+1)^2$ and $V(5x+3)$

*(G. U., F. Y. B.Com., April, 2002)*

[**Ans.** : $E(x+1)^2 = 10$; $V(5x+3) = 25$]

**25.** $x$ and $y$ are two independent variables, for which $E(x) = 2.8$, $E(y) = 5.3$, $V(x) = 18.6$ ; $V(y) = 41.3$

Compute (i) $E(2x+3y)$ (ii) $V(3x-2y)$ (iii) $V(7x+5y+11)$

*(G. U., F. Y. B.Com., April, 1998)*

[**Ans.** : (i) 21.5, (ii) 332.6, (iii) 1943.9]

**26.** (a) Define mathematical expectation and state its characteristics.

(b) The probability distribution of a random variable $x$ is as follow. Find the constant $k$, mean and variance of $x$

| $X = x_i$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | $k$ | $\frac{2}{15}$ | $2k$ | $\frac{4}{15}$ | $3k$ | $\frac{1}{5}$ |

*(G.U., March, 2004)*

[**Ans.** : (b) $k = \frac{1}{15}$, Mean = 1, Variance = $\frac{34}{15}$]



**27.** (a) Prove that $V(x) = E(x^2) - [E(x)]^2$

(b) The probability distribution of a random variable $x$ is as follow :

| $X = x_i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(x_i)$ | $\frac{1}{10}$ | $p$ | $\frac{3}{10}$ | $p$ | $\frac{1}{10}$ |

(i) Find the value of $p$

(ii) Find $E(x+1)$ *(G.U., March, 2005)*

[**Ans.** : (b) (i) $p = \frac{1}{4}$, (ii) $E(x) = 2$, $E(x+1) = 3$]

**28.** (a) Define mathematical expectation and state its characteristics.

(b) For two independent variables $x$ and $y$, $E(x) = 2.5$; $E(y) = 5.2$; $V(x) = 14.2$; $V(y) = 35.5$

Find (i) $E(2x+9)$ (ii) $E(5+3y)$

(iii) $E(4x-y)$ (iv) $E(x^2)$

(v) $V(4x)$ (vi) $V(x-y)$ *(G.U., March, 2006)*

[**Ans.** : (i) 14 (ii) 20.6 (iii) 4.8 (iv) 20.45 (v) 227.2 (vi) 49.7]

**29.** (a) Define mathematical expectation. State the characteristics of the mathematical expectation.

(b) There are 5 tickets in a box and number 1, 1, 2, 2 and 3 are written on tickets. Two tickets are taken at random from the box. Find the expected value of the sum of the numbers on the tickets.

(c) Probability distribution $P(x)$ of a random variable $x$ is as follows : $P(x) = K \cdot x^3$, where $x = 1, 2, 3$.

Find the constant K. Find the expected value of $x$.

*(G.U., March, 2007)*

[**Ans.** : (b) 3.6 (c) $K = \frac{1}{36}$, $E(x) = \frac{98}{36}$]

**30.** (a) Define methematical expectation and state its properties.

(c) If $x$ and y are two independent random variables and $E(x) = 8$, $E(y) = 3$, $E(xy), = 24$, $V(x) = 3$, $V(y) = 2$, then obtain the value of (i) $E(2x+y)$ (ii) $V(2x-3y)$ (iii) $V(3x-10)$ (iv) $E(3x+2)^2$

*(G.U., April, 2008)*

[**Ans.** : (b) (i) 19 (ii) 30 (iii) 27 (iv) 703]

**31.** (a) Prove that $V(x) = E(x^2) - [E(x)]^2$

(b) The Probability distribution of a random variable $x$ is as follows:

| $x_i$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | p | 0.1 | $6p$ | 0.4 | $2p$ | 0.05 |

Obtaining the value of $p$, find $E(2x+3)$ and $V(3x-4)$

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**(c)** There are five tickets in a box numbered 2, 2, 2, 5 and 5 respectively. Two tickets are taken at random from it. Find the expectation of the total of the numbers on the tickets.

***(Guj. Uni., March, 2009)***

**[Ans. :** (b) $p = 0.05$, $E(2x + 3) = 4.10$, $V(3x - 4) = 11.2275$
**(c)** $E(x) = 6.4$**]**

**32. (a)** Give the definition of the following :
(1) Probability mass function
(2) Mathematical expectation of a discrete random variable

**(b)** A discrete random variable has following probability distribution.

| $x_i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(x_i)$ | p | 0.1 | 0.5 | p |

Find : (i) value of p
(ii) mean
(iii) variance of this distribution

**(c)** If E(X) = 2.5, E(Y) = 1.5, V(X) = 2.55 and V(Y) = 1.75, then find the values of
(i) E(X – 2Y + 5)
(ii) V(5X + 2Y + 3)

**(d)** Find the values of (i) E(X) and (ii) V(X) for the following frequency distribution.

| $x_i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f_i$ | 5 | 20 | 50 | 20 | 5 |

**(e)** If E(X) = 0.75 and $E(X^2) = 2.44$ then find the value of standard deviation for X.

***(Guj. Uni., Dec., 2012)***

**[Ans.:** **(b)** $p = 0.2$, Mean = 1.7, Variance = 1.01
**(c)** (i) 4.5 (ii) 70.75 **(d)** (i) 2 (ii) 0.8
**(e)** 1.37**]**

**33. (a)** Define Mathematical Expectation of a discrete variable and write any three properties of expected value.

**(b)** There are 20 screws in a packet of which 2 screws are defective. If 2 screws are taken at random, find the expected number of defective screws and also obtain its variance.

**(c)** If E(x) = 4, V(x) = 1, find $E(x + 1)^2$, V(3x + 5) & V(3x – 5).



**(d)** The probability mass function of a random variable x is $p(x) = k(x + 3)$, x = –1, 0, 1 = 0 elsewhere than value of K = ........

(a) $\frac{1}{9}$ (b) 0.1 (c) 10 (d) None of above

***(Guj. Uni., Dec., 2013)***

**[Ans.:** **(b)** $\frac{18}{45}$, 0.2844; **(c)** 26, 9, 9 **(d)** $\frac{1}{9}$**]**

**34. (a)** Define Mathematical expectation of a descrete variable and write its properties.

**(b)** For the following probability distribution find
(i) K
(ii) Mean
(iii) Standard deviation

| $x_i$ | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | K | 2K | 3K | 2K | 0.4 | 0.2 |

**(c)** Two random variables X and Y are independent and E(X) = 3, E(Y) = 5, V(X) = 6, V(Y) = 4 then find the value of
(i) $E(X^2)$
(ii) $E(Y - 1)^2$
(iii) V(3 – X)
(iv) V(2Y – 3)
(v) V(2X – Y – 5)

**(d)** If E(X) = 4 and V(X) = 9 then find $E(x - 1)^2$.

***(Guj. Uni., Dec., 2014)***

**[Ans.:** **(b)** (i) 0.05 (ii) 2.3 (iii) 1.45
**(c)** (i) 15 (ii) 20 (iii) 6 (iv) 16 (v) 28 **(d)** 18**]**

**35. (a)** Write characteristics of Mathematical Expectation of a discrete variable.

**(b)** The probability density function of a random variable x is as given below :

| x | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|
| P(x) | 0.1 | P | 2P | 0.3 | 2P | 0.1 |

Find the
(i) Value of P
(ii) Variance
(iii) V(5 – 2x).



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**(c)** Following results were obtain for two independent random variable $x$ and $y$.

$E(x) = 4$, $E(y) = 6$, $V(x) = 5$ and $V(y) = 4$

Find the values of

(i) $E(2x - y)^2$

(ii) $V(7 - 2x - 5y)$

*(Guj. Uni., Dec., 2015)*

**[Ans.: (b)** (i) P = 0.1 (ii) V(x) = 2.01 (iii) 8.04; **(c)** (i) 60 (ii) 120 **]**

**36. (a)** Explain mathematical expectation of a discret random variable and state its properties.

**(b)** Explain covariance of a discrete random variable and prove the formula to obtain it.

**(c)** The probability distribution of a random variable $x$ is as follows :

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(x)$ | 0.1 | 0.1 | 2P | 0.3 | 3P |

Find

(i) The value of P

(ii) Mean and

(iii) Variance

**(d)** Two tickets are taken at random from 6 tickets numbered from 10 to 15. Find the expected value of the sum obtained on the two tickets. Also find its variance.

*(Guj. Uni., Dec., 2016)*

**[Ans.: (c)** (i) 0.1 (ii) 2.6 (iii) 1.64; **(d)** 25, 4.67**]**

**37. (a)** Define mathematical expectation for discrete random variable. State its properties.

**(b)** The probability distribution for random variable X is follow:

| $X_i$ | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $P(X_i)$ | 0.04 | 0.016 | 0.33 | k | 0.11 | 0.07 |

Find (i) '$k$', (ii) $E(5x + 3)$, (iii) $V(3x - 5)$

**(c)** There are 10 screws in a packet of which 3 screws are defective. If 3 screws are taken at random. Find the expected number of defective screws. *[Guj. Uni., Dec., 2017]*

**[Ans. : (b)** (i) $k = 0.29$ (ii) 10.4 (iii) 13.04 **(c)** 0.9**]**



**38. (a)** Define Mathematical expectation and variance of a discrete random variable. State the characteristics of mathematical expectation.

**(b)** The probability distribution of a random variable $x$ is as follow :

| $X = x_i$ | – 2 | – 1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | $k$ | $2k$ | $\frac{2}{15}$ | $\frac{4}{15}$ | $3k$ | $\frac{3}{15}$ |

Find K, $E(3x + 5)$ and $V(2x - 1)$.

**(c)** There are 9 screws in a packet of which 3 screws are defective. If 3 screws are taken at random, find the expected number of defective screws.

**(d)** Two random variables X and Y are independent and $E(x) = 3$, $E(y) = 5$, $V(x) = 6$, $V(y) = 4$ then find the value of $E(x - 1)^2$ and $V(3x - 2y + 7)$.

**(e)** Define probability distribution.

**(f)** If $E(x) = 8.5$ and $E(x + y) = 14.5$ find $E(y)$.

**(g)** If S.D. of $x$ is 10, then find $V(3 + 5x)$.

*[Guj. Uni., Nov., 2018]*

**[Ans. : (b)** $K = \frac{1}{15}$, $E(3x + 5) = 8$, $V(2x - 1) = \frac{136}{15}$ **(c)** $E(x) = 1$ **(d)** $E(x - 1)^2 = 10$, $V(3x - 2y + 7) = 70$ **(f)** $E(y) = 6$ **(g)** $V(3 + 5x) = 2500$**]**

**39. (a)** Define :

(a) Mathematical expectation

(b) Central moments and raw moments

**(b)** The probability distribution of a variable x is as follows :

| $x_i$ | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | $\frac{2}{12}$ | $\frac{2}{6}$ | t | t | $\frac{2}{24}$ | $\frac{2}{24}$ |

Find the value of t and also obtain mean and variance of $x$

**(c)** Define variance of random variable and in usual notations prove that, $V(x) = E(x^2) - (E(x))^2$.

**(d)** If $E(x) = -3$ and $V(x) = 5$ find $E(x^2)$.

*[Guj. Uni., Nov., 2019]*

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[**Ans. : (b)** $t = \frac{1}{6}$, Mean $= \frac{11}{12}$, Variance $= \frac{323}{144}$
**(d)** $E(x^2) = 14$]

**40 (a)** Define Mathematical expectation of discrete random variable. Also, state the characteristics of mathematical expectation.

**(b)** For the following probability distribution, find

(1) P

(2) Mean

(3) $V(3x - 2)$

| $x_i$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $P(x_i)$ | 0.4 | 0.2 | P | 2P | 3P | 4P |

**(c) Answer the follwing :**

(i) If $E(x) = 2, V(x) = 1$, find $E(x + 2)^2$

(ii) If $E(x) = 2.5$, $E(y) = 1.5$, $V(x) = 2.55$ and $V(y) = 4$, find the values of $V(4x - 3y + 5)$.

***[Guj. Uni., Jan., 2021]***

[**Ans. : (b)** (1) $P = 0.04$, (2) Mean $= E(x) = 0.8$
(3) $V(3x - 2) = 33.84$
**(c)** (i) $E(x + 2)^2 = 17$ (ii) $V(4x - 3y + 5) = 36.8$]

**41 (a)** Define mathematical expectation and write any two properties of it.

**(b)** Using the following probability distribution, find $E(3x + 2)$ and $V(3x + 2)$.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(x)$ | 0.1 | 0.2 | 0.3 | 0.3 | 0.1 |

**(c) Answer the follwing :**

(1) Find $E(x^2 + x + 1)$ if $V(x) = 1$ and $E(x) = 1$

(ii) Find k from the following probability distribution :

| $x$ | 0 | 1 |
|---|---|---|
| $P(x)$ | k | 1.5 k |

***[Guj. Uni., Dec., 2021]***

[**Ans. : (b)** $E(x) = 2.1$, $E(x^2) = 5.7$, $E(3x + 2) = 8.3$
$V(x) = 1.29$, $V(3x + 2) = 11.61$
**(c)** (1) $E(x^2) = 2$, $E(x^2 + x + 1) = 4$ (2) $K = 0.4$]

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