Gujarat University

B.Com. (Honours)

Semester - 2

PROBABILITY AND DISCRETE PROBABILITY DISTRIBUTIONS

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Your mind is computer and therefore, programmable.

UNIT - 1

1 PROBABILITY

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1. Introduction :

In our day to day life many time we come cross a situation of uncertainity. For example as we toss a fair coin then we can't know in advance that which outcome is going to occur, but it is definite that either head or tail occurs. In such case, in order to measure that uncertainity in the occurance of any uncertainity result the concept of probability is used. It means that probability theory should not be used in the case when some events are definite to occur or definite not to occur. In such cases we restrict the application of probability by assigning the value zero or one in the respective cases.

The theory of probability is an attempt to measure the degree of uncertainity in the occurance any outcome. There are two broad divisions of probability objective and subjective. Objective probabilities are generally divided into two categories - classical probabilities and relative probabilities. The probabilities calculated on the basis of classical random experiments are called classical probabilities and the probabilities calculated on the basis of past data or frequency distribution are called relative probabilities. Where as the probabilities which indicate the belief of a person regarding the occurance of events are called subjective probabilities. Which may vary from person to person or time to time. In our day – to – day life we usually use subjective probabilities. We know that when we toss a fair coin then there are 50% chances of getting head, so we say that the probability of getting head while tossing a fair coin is 0.5 which we called classical probability because tossing a fair coin is random experiment. Similarly if we have information that a student has attended six classes out of then classes last week then the chances that he attain the classes next week are 60% i.e. the probability that the student attain classes is 0.6. This is called relative probability. In our routine life many time we make a statement that "today 99% I-will go for watching movie" i.e. the probability that the person go to watch movie is 0.99.This is called subjective probability. 99% indicate the willingness of the person to watch movie, here we can not able to explain the total number of cases and favourable number of case. In our study we almost deal with objective probabilities.

2. Basic term of probability :

Before giving definitions of probability, we shall understand certain terms :

1. Random experiment (or trial) :

An experiment which can result in any one of the several possible outcomes is called a random experiment or a trial. e.g.,

(i) Tossing of an unbiased coin is a random experiment.

(ii) Throwing an unbiased die is a random experiment.

Characteristics of a random experiment

(1) The experiment results in any one of the outcomes

(2) All possible outcomes of the experiment can be described in advance but cannot be known in advance.

(3) The experiment can be repeated under same conditions.

2. Sample space :

A set representing all possible outcomes of a random experiment is called a sample space and it is denoted by S or U. Each outcome is called a sample point. The number of sample points in S may be denoted by n(S). If the number of sample points of S is finite, it is known as a finite sample space and if the number of sample points is infinite, it is known as an infinite sample space.

e.g. If a coin is tossed, the sample space will be as follows :

$$S = \{H, T\}$$

Similarly if two coins are tossed the following sample space is generated :

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

When 3 coins are thrown simultaneously, the sample space S is given by.

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

Similarly if two dice are thrown the following sample space is obtained.

$$S = \{ \begin{array}{llllll} (1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\ (2, 1) & (2, 2) & (2, 3) & (2, 4) & (2, 5) & (2, 6) \\ (3, 1) & (3, 2) & (3, 3) & (3, 4) & (3, 5) & (3, 6) \\ (4, 1) & (4, 2) & (4, 3) & (4, 4) & (4, 5) & (4, 6) \\ (5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 5) & (5, 6) \\ (6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5) & (6, 6) \end{array} \}$$

In this experiment the sample space consists of 36 sample points.

If a coin is tossed until head appears we get the following sample space

$$\{H, TH, TTH, TTTH, TTTTH, \ldots\ldots\ldots\}$$

In the first four experiments the sample spaces are finite while in the fifth experiment the sample space is infinite.

3. Events :

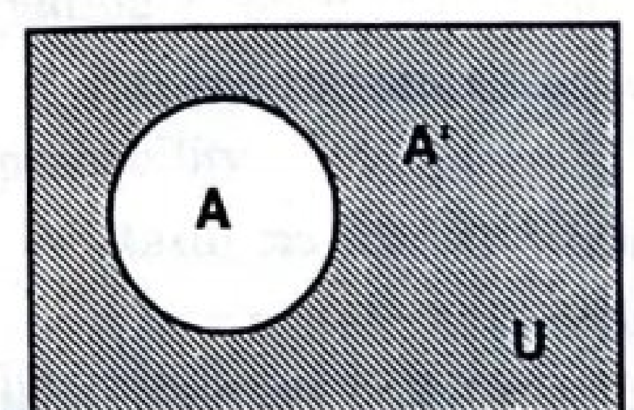
The results of an experiment are known as events :

e.g., (i) If a coin is tossed head (H) and tail (T) are two different events.

(ii) 1,2,3,4,5,6 are different events when a die is thrown.

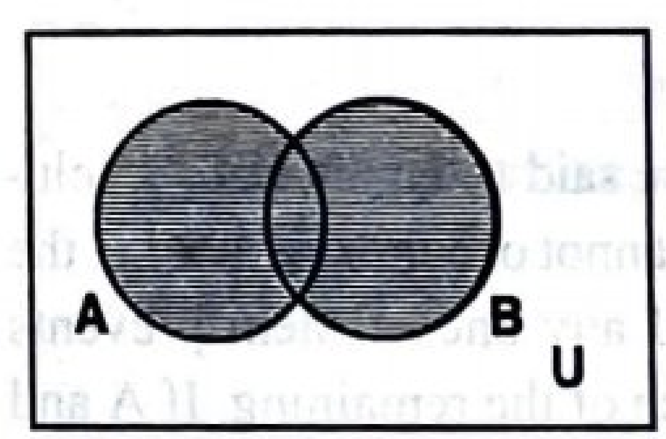
If A is an event and S is a sample space then A is a subset of sample space S. i.e., $A \subset S$.. Generally events are denoted by A, B, C or A_1, A_2, A_3 etc. If $A = \phi$ then A is impossible event and if A = S, then the event A is certain to occur.

4. Complementary event :



The complement of an event A is the aggregate of all the sample points of sample space S which do not belong to A. It is denoted be A' or $\overline{A}$.e.g. If A is an event of getting an odd number when a die is thrown, then event of not getting an odd number i.e. getting an even number is the complement of event A and it is denoted by A' or $\overline{A}$.

5. Union of two events :

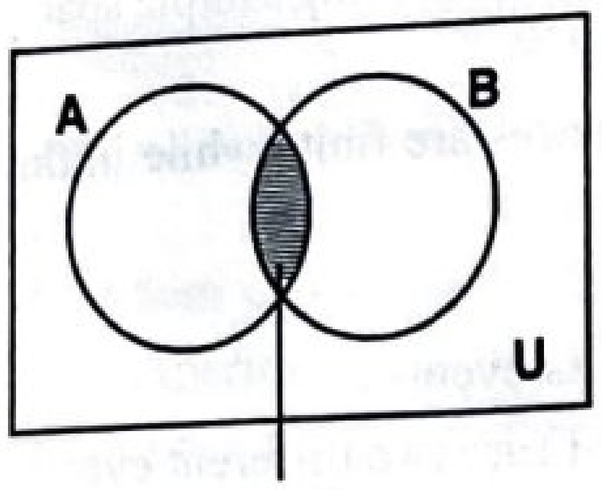


$A \cup B$

The union of two events A and B is denoted by $A \cup B$. It is the aggregate of all sample points belonging to either A or B or both.

e.g. If A is an event that a student plays cricket and B is an event that a student plays hockey then $A \cup B$ represents an event that a student plays either cricket or hockey or both.

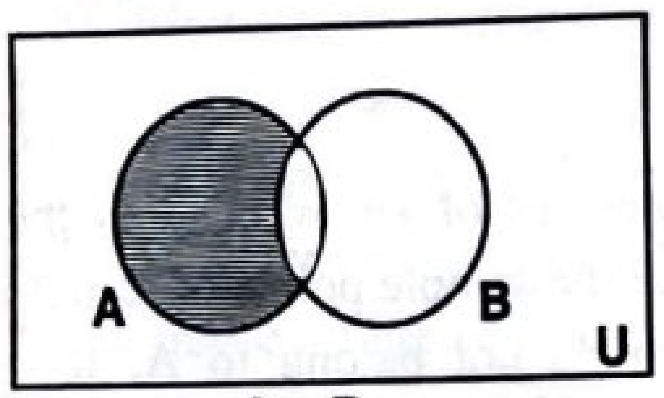
6. Intersection of two events :



A ∩ B

The intersection of two events A and B is denoted by $A \cap B$. It is the aggregate of all sample points belonging to A and B both. When two events A and B occur simultaneously we say that $A \cap B$ has occured e.g., If A is an event that a student plays cricket and B is an event that a student plays hockey, then $A \cap B$ represents an event that a student plays cricket and hockey both.

7. Difference event :



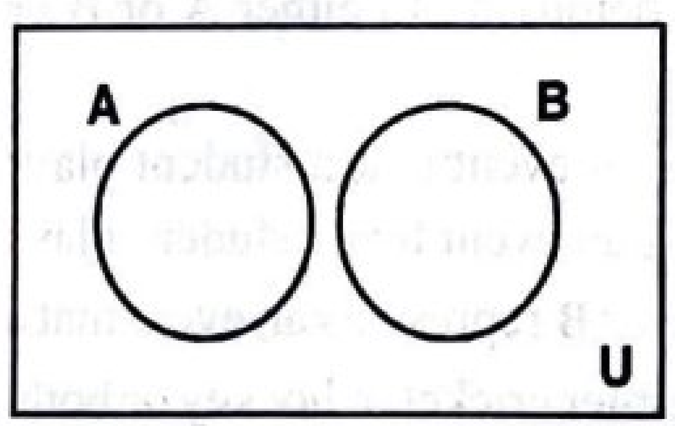
A – B

The difference of two events A and B is the event that A happens and B does not happen. It is denoted by $A - B$ or $A \cap B'$.

8. Exhaustive events :

If all possible outcomes of an experiment are considered, the outcomes are said to be exhaustive. The exhaustive events are nothing but all the sample points in the sample space. In throwing a die 1, 2, 3, 4, 5, 6 are exhaustive events.

9. Mutually exclusive events :



$A \cap B = \phi$

Events are said to be mutually exclusive, if they cannot occur to-gether i.e. the occurrence of any one of them prevents the occurrence of the remaining. If A and B are two mutually exclusive events, then $A \cap B = \phi$. Head and Tail are mutually exclusive events when a coin is tossed.

10. Equally likely events :

Events are said to be equally likely if we have no reason to believe that one event is preferable to the others. Head and Tail are equally likely events in tossing a coin.

11. Favourable cases :

The number of sample points favourable to the happening of an event A are known as favourable cases of A e.g, in drawing a card from a pack of cards, the favourable cases for getting a spade are 13.

12. Independent events :

Events are said to be independent if the happening of one event does not depend upon the happening or non-happening of other events.

e.g. When a coin is tossed two times, the event of getting head in the first throw and that of getting head in the second throw are independent events. Here the result of the second throw does not depend upon the result of the first throw. Similarly the event of getting 3 when a die is thrown and getting a spade from a pack of cards are independent events.

Having defined some basic terms we are now in a position to define probability.

Mathematical or Classical or Apriori definition of probability :

If an experiment can result in n exhaustive, mutually exclusive and equally likely ways, and if m of them are favourable to the happening of an event A, then the probability of happening of an event A is defined as the ratio of m to n. The probability of happening of an event A is denoted by P(A).

i.e.,

$$P(A) = \frac{\text{Favourable cases for happening an event A}}{\text{Total exhaustive, mutually exclusive and equally likely cases}}$$

$$= \frac{m}{n}.$$

Generally the probability of an event is denoted by P(A) and the probability of not happening an event is denoted by P(A').

$$\therefore \quad P(A) = \frac{m}{n}$$

The number of favourable cases are always less than or equal to the total number of exhaustive cases, and also they cannot be negative.

$$\therefore \quad 0 \le m \le n$$

dividing by n

$$0 \le \frac{m}{n} \le 1$$

$$\therefore \quad 0 \le P(A) \le 1$$

Thus the probability of an event is always between 0 and 1. When **P(A) = 0, the event is impossible and when P(A) = 1, the event is certain to happen.** Moreover if m cases are favourable to the happening of an event A, then $n - m$ are the cases not favourable to the happening of an event A. If we denote the probability of not happening an event A by P(A') then

$$\therefore \quad P(A') = \frac{n-m}{n}$$

$$\therefore \quad P(A') = \frac{n}{n} - \frac{m}{n}$$

$$\therefore \quad P(A') = 1 - P(A)$$

$$\therefore \quad P(A') + P(A) = 1$$

Thus the **total probability of happening an event and not happening an event is 1.**

We shall now discuss few Illustrations.

Illustration 1 : A bag contains 4 white and 3 black balls. Find the probability of drawing a white ball from it.

Ans. : Here total exhaustive cases = 4 + 3 = 7

Favourable cases for getting a white ball = 4.

∴ Probability of getting a white ball.

$$= \frac{\text{Favourable cases}}{\text{Total cases}}$$

$$= \frac{4}{7}$$

Illustration 2 : Find the probability of getting an odd number when a cubical die is thrown.

Ans. : When a die is thrown there are six exhaustive cases i.e, 1, 2, 3, 4, 5, 6. The cases favourable in getting an odd number are 1, 3, 5, i.e. 3 in all.

∴ Probability of getting an odd number,

$$= \frac{m}{n}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Illustration 3 : Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that both are kings.

Ans. : The total exhaustive cases of drawing 2 cards from 52 cards are ${}_{52}C_2$. The favourable cases of drawing 2 kings from 4 kings are ${}_4C_2$.

∴ Probability of getting 2 kings.

$$= \frac{m}{n}$$

$$= \frac{{}^4C_2}{{}_{52}C_2}$$

$$= \frac{6}{1326}$$

$$= \frac{1}{221}$$

Illustration 4 : Two dice are thrown simultaneously. Find the probability of obtaining total 10 on the two dice.

Ans. : If two dice are thrown, the sample space will consist of the following 36 sample points :

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

The subset of sample space S favourable for getting total 10 is

{(4,6), (5,5), (6,4)}

It contains 3 sample points.

∴ Probability of getting total 10

$$= \frac{3}{36}$$

$$= \frac{1}{12}$$

Illustration 5 : Three coins are thrown simultaneously, find the probability of getting two heads and one tail.

Ans. : If we denote head by 'H' and tail by 'T' then the sample space will be as follows :

S = {HHH, HHT, HTH, THH, TTH, HTT, THT, TTT}

∴ Total sample points $n = 8$.

The subset of S for getting two heads and one tail will be.

{HHT, HTH, THH} i.e. $m = 3$

$$\therefore \text{Probability} = \frac{3}{8}$$

Limitations of mathematical definition : The following are the limitations of the mathematical definition of probability :

(1) If the total number of exhaustive cases n is not known, probability cannot be obtained.

(2) If the exhaustive cases are infinite, probability cannot be found out.

(3) This definition can be used only when the cases are equally likely. If they are not equally likely, the definition cannot be used.

(4) In this definition of probability the word equally likely is used. Events are said to be equally likely if they have the same chance of occurrence i.e., the same probability of occurrence. Thus, in defining probability the word probability is indirectly used. The definition is therefore, circular in nature, and hence cannot be regarded as a good definition.

Statistical or empirical or Aposteriori definition of probability : If an experiment is repeated under essentially, the same conditions for a great number of times then the limit of the ratio of number of times the event happens to the total number of trials, is defined as the probability of the event. Here it is assumed that the limit exists and it is unique,

i.e., $$P(A) = \lim_{n \to \infty} \left(\frac{m}{n}\right)$$

Modern or Axiomatic definition of probability :

The modern concept of probability was introduced by a Russian mathematician Kolomogorov with the help of set theory.

If P(A) is a real number assigned to a subset A of a sample space S, then it is called the probability of an event A, provided P(A) satisfies the following postulates :

Postulate (1) $0 \le P(A) \le 1$

Postulate (2) $P(S) = 1$

Postulate (3) If $A_1, A_2, A_3, \ldots\ldots$ is finite or infinite sequence of disjoint events i.e, subsets of S then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots.$$

In developing theorems on probability, certain notations are used. Let us now understand their meanings :

Conditional probability :

Suppose there are 100 persons in a group, 60 males and 40 famales. Suppose 15 of them use spectacles and out of them 10 are males. If A denotes male and B denotes a person using spectacles then,

$$P(A) = \frac{60}{100};\ P(B) = \frac{15}{100}.$$

Now, the probability of a person using spectacles given that he is a male $= \frac{10}{60}$.

This probability of occurrence of an event B when it is known that A has occurred is known as conditional probability of B under the condition that A has occurred and it is denoted by P(B/A).

$$\therefore P(B/A) = \frac{10}{60}$$

Thus the probability of happening an event B, when A has happened is defined as conditional probability of B under A and it is denoted by P(B/A.) and is given as $P(B/A) = \frac{P(A \cap B)}{P(A)}$; $P(A) \neq 0$

3. Important Results and meanings :

1. $P(A)$ = Probability of happening of an event A

   $P(A) = \frac{m}{n}$

2. $P(A')$ = Probability of non-happening of an event A $P(A') = 1 - P(A)$.
3. **Addition Rule on Probability.**
   If A and B are any two events of the sample space then the probability that at least one of the event will occur is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. If events A and B are mutually independent then $P(A \cap B) = P(A) . P(B)$.
5. If two events A and B are independent then (i) A' and B' (ii) A' and B (iii)A and B' are also independent and vice-versau.
6. If events A and B are mutually exclusive then $P(A \cap B) = P(\phi) = 0$.
7. For any two events A and B of the sample space, the probability that only event A occurs is given as $P(A - B) = P(A \cap B') = P(A) - P(A \cap B) = P(A \cup B) - P(B)$
8. Probability that at least one of event A or B will not occurs is $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$.
9. Probability that both the events A and B does not occur is $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
10. Addition rule of probability for three events. If A, B and C are any three events of the sample space then probability that at least one of the event will occur is $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
11. If three events A, B and C are mutually exclusive and exhaustive then $P(A \cup B \cup C) = P(A) + P(B) + P(C) = P(\cup) = 1$
12. If A and B are any two events of the sample space then probability that event A occurs when it is known that event B is already occurred is called conditional probability and is defined as

    $P(A/B) = \frac{P(A \cap B)}{P(B)}$; $P(B) \neq 0$.

13. **Multiplication Rule : If A and B are any two events of the sample space then the probability that both the events occurs together is $P(A \cap B) - P(A) . P(B/A)$ OR**
    $P(A \cap B) = P(B) . P(B/A)$

Illustration 6 : Two cards are drawn at random from a pack of 52 cards. Find the probabilities that

(i) One is king and the other is queen.

(ii) both are spade

(iii) both are of the same suit.

Ans. : The total number of exhaustive ways of drawing two cards out of 52 cards is ${}_{52}C_2 = 1326$.

(1) The number of favourable ways of drawing 1 king and one queen

$$= {}_4C_1 \times {}_4C_1 = 4 \times 4 = 16$$

$$\therefore \text{ Probability } = \frac{16}{1326} = \frac{8}{663}$$

(2) Favourable ways of getting both spades

$$= {}_{13}C_2 = 78$$

$$\therefore \text{ Probability } = \frac{78}{1326} = \frac{13}{221} = \frac{1}{17}$$

(3) The probability that both the cards are of the same suit = probability that both are spades or both are hearts or both are clubs or both are diamonds,

$$= \frac{13}{221} + \frac{13}{221} + \frac{13}{221} + \frac{13}{221} = \frac{52}{221} = \frac{4}{17}$$

Illustration 7 : There are 5 red and 7 black balls in an urn. Two balls are drawn at random one after the other. If they are drawn (i) with replacement (ii) without replacement, find the probability that both the balls are red.

Ans. : Here total number of balls = 5 red + 7 black = 12

(i) Drawing with replacement

Probability that both the balls are red

$$= P(R_1 \cap R_2) = P(R_1) \cdot P(R_2).$$

$$= \frac{5}{12} \times \frac{5}{12}$$

$$= \frac{25}{144}$$

(ii) Drawing without replacement

Probability that both the balls are red

$= P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$

$= \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

Illustration 8 : If $P(A) = \frac{1}{3}$, $P(B') = \frac{1}{4}$, and $P(A \cap B) = \frac{1}{6}$, find $P(A \cup B)$, $P(A' \cap B')$ and $P(A'/B')$.

(Here E denotes the event of not happening of E)

Ans. :

$P(A) = \frac{1}{3}$ $\quad \therefore P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B') = \frac{1}{4}$ $\quad \therefore P(B) = 1 - P(B')$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{3}{4} - \frac{1}{6}$

$= \frac{4 + 9 - 2}{12} = \frac{11}{12}$

For events A and B

$P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - \frac{11}{12}$

$= \frac{1}{12}$

Now, $P(A'/B') = \frac{P(A' \cap B')}{P(B')}$

$= \frac{1/12}{1/4}$

$= \frac{4}{12}$

$= \frac{1}{3}$

Illustration 9 : If $P(A_1) = 2 \cdot P(A_2) = P(A_1/A_2) = 0.4$ find the following probabilities :

(i) Both A_1 and A_2 happen

(ii) Only A_2 happens

(iii) At least one happens.

(iv) Neither of A_1 and A_2 happen.

Ans. :

$P(A_1) = 0.4$

$2 \cdot P(A_2) = 0.4$

$\therefore P(A_2) = 0.2$

$P(A_1/A_2) = 0.4$

(i) Now the probability that both A_1 and A_2 happen

$P(A_1 \cap A_2) = P(A_2) \cdot P(A_1/A_2)$

$= (0.2)(0.4) = 0.08$

(ii) The probability that only A_2 happens

$= P(A'_1 \cap A_2)$

Now, $P(A_2) = P(A_1 \cap A_2) + P(A'_1 \cap A_2)$

$0.2 = 0.08 + P(A'_1 \cap A_2)$

$\therefore P(A'_1 \cap A_2) = 0.2 - 0.08 = 0.12$

(iii) Probability that atleast one happens :

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= 0.40 + 0.20 - 0.08$

$= 0.52$

(iv) Probability that neither of A_1 and A_2 happen

$P(A'_1 \cap A'_2)$

$P(A'_1 \cap A'_2) = P(A_1 \cup A_2)' = 1 - P(A_1 \cup A_2)$

$= 1 - 0.52$

$= 0.48$

4. Inverse probability Rule or Bayes' Theorem :

The rule of finding out Inverse probability was given by British mathematician Bayes. This rule is used to modify prior or postirior probability of the event. On the basis of current information regarding the event. This rule is generally used when the random expcriment is over i.e. the out come of random experiment is known and on the basis of it we want to modify the probability of the event related to that random expcriment. This rule has very wide applications for decision making in business and industry.

Bayes' theorem Statement :

If an event D can happen with mutually exclusive and exhaustive events $A_1, A_2, A_3, ...A_n$ and if the probabilities $P(A_1), P(A_2), P(A_3),....P(A_n)$ and also the conditional probabilities $P(D/A_1)$, $P(D/A_2)$, $P(D/A_3),.....P(D/A_n)$ are known then the inverse probability of happening an event Ai under the condition that the event D has happened is given by

$$P(A_i/D) = \frac{P(A_i \cap D)}{P(D)}$$

$$= \frac{P(A_i \cap D)}{P(A_1 \cap D) + P(A_2 \cap D) + P(A_3 \cap D) + ...+ P(A_n \cap D)}$$

$$= \frac{P(A_i) \quad P(D/A_i)}{P(A_1) \cdot P(D/A_1) + P(A_2)\, P(D/A_2) + P(A_3) \cdot P(D/A_3) .... + P(A_n) \cdot P(D/A_n)}$$

We shall study the use of Bayes' Theoem for three mutually exclusive and exhaustive events only.

Illustration 10 : There are three urns containing respectively 3 white and 4 blackballs; 2 white and 2 black balls; 1 white and 3 black balls. One urn is selected at random and a ball is drawn from it. The ball is found to be white. Find the probabilities that this ball comes from (1) first urn (2) third urn.

Ans. : Let D denote the event of getting a white ball and A_1, A_2, A_3 the events of selecting the urns.

$P(A_1) = \frac{1}{3}$	$P(D/A_1) = \frac{3}{7}$	$P(A_1) \cdot P(D/A_1) = \frac{1}{7}$
$P(A_2) = \frac{1}{3}$	$P(D/A_2) = \frac{2}{4}$	$P(A_2) \cdot P(D/A_2) = \frac{1}{6}$
$P(A_3) = \frac{1}{3}$	$P(D/A_3) = \frac{1}{4}$	$P(A_3) \cdot P(D/A_3) = \frac{1}{12}$
		$\frac{1}{7} + \frac{1}{6} + \frac{1}{12} = \frac{33}{84}$

Now,

$$\text{(i)}\quad P(A_1/D) = \frac{P(A_1) \cdot P(D/A_1)}{P(A_1) \cdot P(D/A_1) + P(A_2) \cdot P(D/A_2) + P(A_3) \cdot P(D/A_3)}$$

$$= \frac{\frac{1}{7}}{\frac{33}{84}}$$

$$= \frac{1}{7} \times \frac{84}{33}$$

$$= \frac{4}{11}$$

$$\text{(ii)}\quad P(A_3/D) = \frac{P(A_3) \cdot P(D/A_3)}{P(A) \cdot P(D/A_1) + P(A_2) \cdot P(D/A_2) + P(A_3) \cdot P(D/A_3)}$$

$$= \frac{\frac{1}{12}}{\frac{33}{84}}$$

$$= \frac{7}{33}$$

Illustration 11 : In a factory there are three machines and they produce respectively 200, 300, 500 units of an item daily. The proportions defectives of these machines are 2%, 4% and 3% respectively. An item is taken at random from the day's production and it is found to be defective. Find the probability that the item is produced by second machine.

Ans. : Let D denote the defective item and A_1, A_2, A_3 denote three different machines.

$P(A_1) = \frac{200}{1000} = \frac{2}{10}$	$P(D/A_1) = \frac{2}{100}$	$P(A_1) \cdot P(D/A_1) = \frac{4}{1000}$
$P(A_2) = \frac{300}{1000} = \frac{3}{10}$	$P(D/A_2) = \frac{4}{100}$	$P(A_2) \cdot P(D/A_2) = \frac{12}{1000}$
$P(A_3) = \frac{500}{1000} = \frac{5}{10}$	$P(D/A_3) = \frac{3}{100}$	$P(A_3) \cdot P(D/A_3) = \frac{15}{1000}$
		$\frac{31}{1000}$

Now,

$$P(A_2/D) = \frac{P(A_2)\cdot P(D/A_2)}{P(A_1)\cdot P(D/A_1) + P(A_2)\cdot P(D/A_2) + P(A_3)\cdot P(D/A_3)}$$

$$= \frac{\frac{12}{1000}}{\frac{31}{1000}}$$

$$= \frac{12}{31}$$

Illustration 12 : Find the probability of 53 Sundays in a leap year.

Ans. : There are 366 days in a leap year. Hence there are 52 complete weeks and 2 additional days. These 2 days can be as one of the following pairs :

Sunday – Monday	Monday – Tuesday
Tuesday – Wednesday	Wednesday – Thursday
Thursday – Friday	Friday – Saturday
Saturday – Sunday	

Thus, there are 7 equally likely pairs of which 2 are favourable for getting an additional Sunday.

∴ The probability of 53 Sundays in a leap year $= \frac{2}{7}$.

Illustration 13 : There are 4 red and 6 green balls in one bag and 5 red and 4 green balls in another bag.

One bag is selected at random and 2 balls are drawn from it. Find the probability that both the balls are red.

Ans. : Suppose two bags are denoted by A and B and the event of drawing two red balls is denoted by R.

Here $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, and we have to find out P(R)

Now, $P(R) = P(A \cap R) + P(B \cap R)$

$$= P(A)\,(R/A) + P(B)\,P(R/B)$$

$$= \frac{1}{2} \times \frac{^4C_2}{^{10}C_2} + \frac{1}{2} \times \frac{^5C_2}{^9C_2}$$

$$= \frac{1}{2} \times \frac{6}{45} + \frac{1}{2} \times \frac{10}{36}$$

$$= \frac{1}{15} + \frac{5}{36}$$

$$= \frac{12+25}{180}$$

$$= \frac{37}{180}$$

Illustration 14 : Three dice are thrown simultaneously, find the probability of getting total atleast 16.

Ans. : The total exhaustive cases $= 6^3 = 216$.

The favourable cases for getting total atleast 16 are :

For getting total '16' :

(4, 6, 6), (5, 5, 6), (5, 6, 5)

(6, 4, 6), (6, 5, 5), (6, 6, 4) = in all 6.

For getting total '17'

(5, 6, 6), (6, 5, 6), (6, 6, 5) = in all 3.

For getting total '18' :

(6, 6, 6) = 1

Thus the favourable cases for getting total atleast '16'

= 6 + 3 + 1 = 10

$$\text{Required probability} = \frac{10}{216}$$

$$= \frac{5}{108}$$

Illustration 15 : Three families have respectively 2 boys and 3 girls; 3 boys and 2 girls; 2 boys and 2 girls. One child is selected at random from each family. Find the probabilities that the selected group of 3 children will have (i) all boys (ii) all girls (iii) 2 boys and 1 girl.

Ans. :

Family	Boys	Girls	Total
I	2	3	5
II	3	2	5
III	2	2	4

(i) The probability that all the three are boys

$= P(B_1 \cap B_2 \cap B_3)$

$= P(B_1) \cdot P(B_2) \cdot P(B_3)$

$= \frac{2}{5} \times \frac{3}{5} \times \frac{2}{4}$

$= \frac{12}{100}$

$= 0.12$

(ii) The probability that all the three are girls

$= P(G_1 \cap G_2 \cap G_3)$

$= P(G_1) \cdot P(G_2) \cdot P(G_3)$

$= \frac{3}{5} \times \frac{2}{5} \times \frac{2}{4}$

$= \frac{12}{100}$

$= 0.12$

(iii) 2 boys and 1 girl can be selected in the following mutually exclusive ways :

$(B_1 \cap B_2 \cap G_3) \cup (B_1 \cap G_2 \cap B_3) \cup (G_1 \cap B_2 \cap B_3)$

$\therefore$ Required probability

$= P(B_1 \cap B_2 \cap G_3) + P(B_1 \cap G_2 \cap B_3) + P(G_1 \cap B_2 \cap B_3)$

$= P(B_1)\, P(B_2)\, P(G_3) + P(B_1)\, P(G_2)\, P(B_3) + P(G_1)\, P(B_2)\, P(B_3)$

$= \frac{2}{5} \times \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{5} \times \frac{2}{4} + \frac{3}{5} \times \frac{3}{5} \times \frac{2}{4}$

$= \frac{12}{100} + \frac{8}{100} + \frac{18}{100}$

$= 0.38$

Illustration 16 : A machine is made up of two parts A and B. The probability that part A is defective is 0.05 and the probability that part B is defective is 0.07. Find the probability that the entire machine is not defective.

Ans. : The probability that part A is not defective

$= 1 - P(A)$

$= 1 - 0.05$

$= 0.95$

Similarly the probability that the part B is not defective

$= 1 - P(B)$

$= 1 - 0.07$

$= 0.93$

The entire machine is not defective if both the parts are not defective.

$\therefore$ Probability that the machine is not defective = probability that part A is not defective $\times$ probability that part B is not defective.

$= P(A' \cap B')$

$= P(A') \cdot P(B')$

$= 0.95 \times 0.93$

$= 0.8835$

Illustration 17 : A, B and C are given an example. The probabilities that they will solve the example correctly are respectively $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{4}$. Find the probability that at least one of them will solve the example correctly.

Ans. : The probability that A solves the example correctly $P(A) = \frac{1}{2}$

$\therefore$ Probability that A does not solve the example correctly

$P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

Similarly the probability that B does not solve the example correctly

$P(B') = 1 - \frac{1}{3} = \frac{2}{3}$

And the probability that C does not solve the example correctly

$P(C') = 1 - \frac{3}{4} = \frac{1}{4}$

Now, the probability that at least one will solve the example correctly

$= P(A \cup B \cup C)$

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A')\ P(B')\ P(C')$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$

$= 1 - \frac{1}{12}$

$= \frac{11}{12}$

Illustration : 18 : An urn contains 3 red and 7 white balls. A ball is drawn at random from the urn and in its place a ball of other colour is put. If now one ball is drawn from the urn, find the probability that it is red.

Ans. : Let R_1 denote the event of getting a red ball in the first trial and W_1 denote the event of getting a white ball in the first trial. Let R_2 denote an event of getting a red ball in the second trial. The event of getting a red ball in the second trial can happen in one of the following mutually exclusive ways :

A red ball is drawn in the first trial and a red ball is drawn in the second trial.

OR

A white ball is drawn in the first trial and a red ball is drawn in the second trial.

$$\therefore\ P(R_2) = P(R_1 \cap R_2) + P(W_1 \cap R_2)$$
$$= P(R_1)\cdot P(R_2/R_1) + P(W_1)\cdot P(R_2/W_1)$$
$$= \frac{3}{10}\cdot\frac{2}{10} + \frac{7}{10}\cdot\frac{4}{10}$$
$$= \frac{6}{100} + \frac{28}{100}$$
$$= \frac{34}{100}$$
$$= 0.34$$

Illustration 19 : The probability that A speaks the truth is 0.6 and the probability that B speaks the truth is 0.7. They both agree about a statement. Find the probability that the statement is true.

Ans. : The probability that A speaks the truth = P(A) = 0.6.

$\therefore$ Probability that A does not speak the truth $P(A') = 1 - 0.6 = 0.4$

Similarly, the probability that B does not speak the truth $P(B') = 1 - 0.7 = 0.3$. When both A and B agree, either both speak the truth or both do not speak the truth.

$$= P(A \cap B) + P(A' \cap B')$$
$$= P(A)\cdot P(B) + P(A')\cdot P(B')$$
$$= 0.6 \times 0.7 + 0.4 \times 0.3$$
$$= 0.42 + 0.12$$
$$= 0.54$$

The probability that the statement is true when both agree.

$$= \frac{0.42}{0.54}$$
$$= \frac{7}{9}$$

Illustration 20 : The probability that a person will get promotion in his job is 0.6 and the probability that he will be transferred is 0.7. The probability that he will get promotion and also he will be transferred is 0.3. Find the probability that (i) he will get promotion given that he is transferred (ii) he will be transferred given that he is promoted.

Ans. : Let A be an event that the person gets promotion in his job and B be an event that he is transferred.

$\therefore$ P(A) = 0.6, P(B) = 0.7, $P(A \cap B) = 0.3$

(i) The probability that the person will get promotion, given that he is transferred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$$

(ii) The probability that the person will be transferred, given that he is promoted.

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.6} = \frac{1}{2}$$

Illustration 21 : 6 books are to be arranged on a shelf in one row. Find the probability that two particular books are not side by side.

Ans. : The number of ways of arranging 6 books in a row = 6! = 720.

Considering the two particular books as one unit the 5 units can be arranged in 5! ways. In each arrangement the two particular books can be permutted in 2! ways.

$\therefore$ Number of ways in which the two particular books will be side by side = 5! × 2! = 240.

$\therefore$ Favourable cases in which two particular books are not side by side = 720 − 240 = 480

∴ Probability that the two particular books are not side by side.

$$= \frac{480}{720}$$

$$= \frac{2}{3}$$

Illustration 22 : 6 boys and 3 girls are arranged randomly in one row. Find the probability that all the girls are together.

Ans. : The total number of arranging 6 boys and 3 girls (9 in all) = 9! considering 3 girls as a single unit 6 + 1 = 7 units can be arranged in 7! ways. In each arrangement the three girls can be internally permutted in 3! ways.

∴ Number of ways in which 3 girls will always be together = 7! × 3!

∴ Probability that all the three girls are together

$$= \frac{7! \times 3!}{9!}$$

$$= \frac{1 \times 2 \times 3}{8 \times 9}$$

$$= \frac{1}{12}$$

Illustration 23 : If P(A) = 0.4, P(B) = 0.6, P(A ∪ B) = 0.8 find (i) P (A/B) (ii) P (A ∩ B') (iii) P (B/A') (iv) P (A' ∩ B')

Ans. :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore \quad 0.8 = 0.4 + 0.6 - P(A \cap B)$$

$$\therefore \quad P(A \cap B) = 0.2$$

(i) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.6} = 0.33$$

(ii) $$P(A \cap B') = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2$$

(iii) $$P(B/A') = \frac{P(B \cap A')}{P(A')}$$

Now, $P(B \cap A') = P(B) - P(A \cap B) = 0.6 - 0.2 = 0.4$

and $P(A') = 1 - P(A) = 1 - 0.4 = 0.6$

$$\therefore P(B/A') = \frac{0.4}{0.6} = 0.67$$

(iv) $$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$$

Illustration 24 : The odds in favour of A going to America is 2 : 3 and the odds against B going to America is 4 : 3. Find the probability that atleast one of them will go to America.

Ans. : The probability that A will go to America $= \frac{2}{2+3} = \frac{2}{5}$

The probability that B will go to America $= \frac{3}{4+3} = \frac{3}{7}$

Probability that atleast one of them will go to America is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{5} + \frac{3}{7} - \frac{2}{5} \times \frac{3}{7} \quad (\because \text{A and B are independent})$$

$$= \frac{14 + 15 - 6}{35}$$

$$= \frac{23}{35}$$

Illustration 25 : A, B and C are three mutually exclusive and exhaustive events. Examine the correctness of the following statements :

(i) $P(A) = \frac{2}{5}$; $P(B) = \frac{3}{7}$; $P(C) = \frac{6}{35}$

(ii) $P(A) = \frac{2}{3}$; $P(B) = \frac{3}{5}$; $P(C) = \frac{1}{9}$

(iii) $P(A) = 0.27$; $P(B) = 0.52$; $P(C) = 0.21$

(iv) $P(A) = 0.42$; $P(B) = 0.60$; $P(C) = -0.02$

Ans. : As, A, B, C are three mutually exclusive and exhaustive events.

$$P(A) + P(B) + P(C) = 1$$

(i) Here $P(A) + P(B) + P(C)$

$$= \frac{2}{5} + \frac{3}{7} + \frac{6}{35}$$

$$= 1$$

∴ The statement is correct

(ii) Here, $P(A) + P(B) + P(C)$

$$= \frac{2}{3} + \frac{3}{5} + \frac{1}{9}$$

$$= \frac{30 + 27 + 5}{45}$$

$$= \frac{62}{45}$$

$$> 1$$

∴ The statement is not correct

(iii) P(A) = 0.27; P(B) = 0.52; P(C) = 0.21

$\therefore$ $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= 0.27 + 0.52 + 0.21$

$= 1.0$

$\therefore$ The statement is correct

(iv) P(A) = 0.42; P(B) = 0.60; P(C) = – 0.02

Here P(C) is negative which is not possible. Hence the statement is not correct.

Illustration 26 : A and B are two independent events and $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$ find $P(A \cup B)$.

Ans. : As A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{2} \times \frac{1}{5}$$

$$= \frac{1}{10}$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{5} - \frac{1}{10}$$

$$= \frac{6}{10}$$

$$= 0.6$$

Illustration 27 : If A, B and C are three mutually exclusive and exchustive events and if $3P(A) = 2 \cdot P(B) = 6P(C)$ find $P(A \cup B)$.

Ans. : A, B and C are three mutually exclusive and exhaustive events.

$\therefore$ $P(A \cup B \cup C) = 1$

$\therefore$ $P(A) + P(B) + P(C) = 1$

$\therefore$ $P(A) + \frac{3}{2} P(A) + \frac{3}{6} P(A) = 1$

$\therefore$ $P(A)\left[1 + \frac{3}{2} + \frac{3}{6}\right] = 1$

$\therefore$ $3 P(A) = 1$

$\therefore$ $P(A) = \frac{1}{3}$

$\therefore$ $P(B) = \frac{3}{2} P(A) = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{2} - 0$ ($\because$ A and B are Matually exclusive)

$= \frac{5}{6}$

Illustration 28 : There are 6 black and some white balls in an urn. The probability of drawing 2 black balls from it is $\frac{1}{3}$. Find the number of white balls in the urn.

Ans. : Suppose there are x white balls in the urn.

Total number of balls in the urn = $6 + x$

Now, the probability of getting 2 black balls from the urn.

$$\frac{{}_6C_2}{{}_{(6+x)}C_2} = \frac{1}{3}.$$

$\therefore$ $\frac{15 \times 2}{(6+x)(5+x)} = \frac{1}{3}$

$\therefore$ $15 \times 2 \times 3 = (6 + x)(5 + x)$

$\therefore$ $90 = 30 + 11x + x^2$

$\therefore$ $x^2 + 11x - 60 = 0$

$\therefore$ $(x + 15)(x - 4) = 0$

$\therefore$ $x = -15$ or $x = 4$

As number of balls cannot be negative, $x \neq -15$.

$\therefore$ Number of white balls in the urn is 4.

Illustration 29 : There are 1000 people in a locality. Three news papers A, B and C are available to them. 500 people read A, 400 read B, and 400 read C. 100 people read both A and B, 150 read both B and C and 200 read both A and C. 40 people read all the three newspapers. Find the probability that a person selected at random from that locality reads atleast one of the three papers.

Ans. : Here, we are given

$$P(A) = \frac{500}{1000} = 0.50, \; P(B) = \frac{400}{1000} = 0.40$$

$$P(C) = \frac{400}{1000} = 0.40, \; P(A \cap B) = \frac{100}{1000} = 0.10$$

$$P(B \cap C) = \frac{150}{1000} = 0.15;\ P(A \cap C) = \frac{200}{1000} = 0.20$$

and $$P(A \cap B \cap C) = \frac{40}{1000} = 0.04$$

Now, the probability that a person reads atleast one of the three papers is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= 0.50 + 0.40 + 0.40 - 0.10 - 0.15 - 0.20 + 0.04$$

$$= 0.89$$

Illustration 30 : The probability of not happening an event A is 0.3 and the probability of happening an event B is 0.5. The probability of happening at least one of the two events A and B is 0.8. Find the probability of happening both the events A and B. What is the probability that none of the events will happen ?

Ans. : $P(A') = 0.3$;

$\therefore$ $P(A) = 1 - P(A') = 1 - 0.3 = 0.7$

$P(B) = 0.5$

$P(A \cup B) = 0.8$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ $0.8 = 0.7 + 0.5 - P(A \cap B)$

$\therefore$ $P(A \cap B) = 0.4$

Now, $P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - 0.8$

$= 0.2$

Illustration 31 : An urn contains 2 white and 3 black balls. A, B, C and D draw one ball each in turn without replacing them. One who gets a white ball first, wins, find their probabilities of winning.

Ans. : Probability that A wins $= \frac{2}{5}$.

B can win, if A fails, i.e, A gets a black ball.

$\therefore$ Probability that B wins $= P(A' \cap B)$

$$= \frac{3}{5} \times \frac{2}{4}$$

$$= \frac{3}{10}$$

Similarly the probability that C wins

$$= P(A' \cap B' \cap C)$$

$$= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$$

$$= \frac{1}{5}$$

And the probability that D wins

$$= P(A' \cap B' \cap C' \cap D)$$

$$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$$

$$= \frac{1}{10}$$

Illustration 32 : In one urn there are 3 red and 2 black balls. In second urn there are 4 red and 1 black ball and in the third urn there are 2 red and 3 black balls. An urn is selected at random and a ball is drawn at random from it. Find the probability that the ball is red.

Ans. : Let the three urns be denoted by A, B and C and the event of drawing a red ball by R.

A red ball can be drawn in one of the following three mutually exclusive and exhaustive ways :

(i) The first urn is selected and a red ball is drawn from it. or

(ii) The second urn is selected and a red ball is drawn from it. or

(iii) The third urn is selected and a red ball is drawn from it.

$\therefore$ $P(R) = P(A \cap R) + P(B \cap R) + P(C \cap R)$

$$= P(A) \cdot P(R/A) + P(B) \cdot P(R/B) + P(C) \cdot P(R/C)$$

$$= \frac{1}{3} \cdot \frac{3}{5} + \frac{1}{3} \cdot \frac{4}{5} + \frac{1}{3} \cdot \frac{2}{5}$$

$$= \frac{9}{15} = \frac{3}{5}$$

Illustration 33 : In an industry a managing director is to be appointed from three persons A, B and C. The chance of selection of A is twice than that of B, while the chance of selection of B is twice than that of C. The probabilities that these persons, if selected as managing director will increase the bonus of the workers are respectively 0.2, 0.3 and 0.4. If the bonus has increased in the industry, find the probability that A is selected as managing director.

Ans. : Let the event of increase in the bonus be denoted by D

Here, $P(A) = 2.P(B)$ and $P(B) = 2P(C)$

$\therefore\ P(A) = 2P(B) = 4P(C)$

Now, $P(A) + P(B) + P(C) = 1$

$\therefore\ 4.P(C) + 2.P(C) + P(C) = 1$

$\therefore\ 7.P(C) = 1$

$\therefore\ P(C) = \frac{1}{7}$

and $P(B) = \frac{2}{7}, P(A) = \frac{4}{7}$

$P(A) = \frac{4}{7}$	$P(D/A) = 0.2$	$P(A) \cdot P(D/A) = \frac{8}{70}$
$P(B) = \frac{2}{7}$	$P(D/B) = 0.3$	$P(B) \cdot P(D/B) = \frac{6}{70}$
$P(C) = \frac{1}{7}$	$P(D/C) = 0.4$	$P(C) \cdot P(D/C) = \frac{4}{70}$
		$= \frac{18}{70}$

Now, $P(A/D) = \frac{P(A) \cdot P(D/A)}{P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$

$= \frac{\frac{8}{70}}{\frac{18}{70}}$

$= \frac{8}{18}$

$= \frac{4}{9}$

EXERCISE

SECTION-A

Answer in short :

1. Why probability theory should be studied ?
2. Give one example where we can not apply probability theory.
3. Give an example of classical probability.
4. Give an example of relative probability.
5. Give an example of subjective probability.
6. Define random expriment.
7. What do you mean by finite sample space ?
8. Give one example of infinite sample space.
9. State complementary law of probability.
10. Define union of two events
11. Define intersection of two events.
12. If a coin and a die is tossed simultaneously then obtain its sample space.
13. Define difference of event.
14. What is the condition for the events to be mutually exhaustive ?
15. What is the condition for the events to be mutually exclusive ?
16. Give one example for the mutually exclusive events.
17. Give one example of mutually exhaustive events.
18. Give one example, illustrating mutually independent events.
19. If the events are mutually exclusive then can, they are mutually independent ?
20. Give limitations of mathematical definition of probability.
21. Define conditional probability P (A/B).
22. What do you mean by equi-probable events ?
23. Give definition of favourable out comes.
24. For what purpose Bayes' Theorem is used ?
25. When the conditional probability P (A/B') is defined ?
26. If A, B and C are mutually exclusive and ehaustive events and P(A) = 0.3, P(C) = 0.24 then find P(B). *[Guj. Uni., Dec., 2017]* **[Ans. : 0.46]**

SECTION-B

1. Give the following definitions of probability (i) Mathematical (ii) Empirical (ii) Axiomatic
2. State Bayes' Theorem of inverse probability.
3. Give mathematical definition of probability and from it prove that $0 \le P(A) \le 1$.

4. Define complement of an event A and prove that P(A') = 1 – P(A).
5. A card is drawn at random from a pack of 52 cards, find the probabilities of getting : (1) a club (2) a queen (3) a club queen (4) a club or a queen.

[Ans. : $\frac{1}{4}, \frac{1}{13}, \frac{1}{52}, \frac{4}{13}$

6. If A, B, C are exhaustive and mutually exclusive events, examine whether the following statements are true or false.

(1) P(A) = 0.5; P(B) = 0.4; P(C) = 0.2

(2) P(A) = 0.62; P(B) = 0.48; P(C) = – 0.10

(3) P(A) = 0.5; P(B) = 0.4; P(A ∩ B) = 0.6

[Ans. : (1) False (2) False (3) False

7. A box contains 6 black and 4 white balls. Two balls are drawn at random from it. Find the probabilities that (1) both are black (2) both are white (3) both are of different colour.

[Ans. : $\frac{1}{3}, \frac{2}{15}, \frac{8}{15}$

8. (a) Two cubical dice are thrown simultaneously. Find the probabilities of getting :

(1) Total '9' (2) Total at least '9' [Ans. : $\frac{1}{9}, \frac{5}{18}$

(b) One bag contains 5 black and 3 white balls. Another bag contains 4 black and 5 white balls. One ball is drawn from each bag, find the probability that they are of different colour.

[Ans. : $\frac{37}{72}$

9. Two dice are thrown simultaneously. Find the probability that the sum of the numbers is divisible by 3 or 4. [Ans. : $\frac{5}{9}$

10. Find the probability of getting total at the most '6' when three cubical dice are thrown.

[Ans. : $\frac{5}{54}$

11. An urn contains 5 white, 4 red and 3 black balls. Three balls are drawn at random from it. Find the probabilities that (i) all are black (ii) all are of different colour (iii) two are of the same colour.

[Ans. : $\frac{1}{220}, \frac{3}{11}, \frac{29}{44}$

12. A card as drawn from a pack of 52 cards and it is thrown away. Then another card is drawn. Find the probability that it is a queen.

[Ans. : $\frac{1}{13}$

13. Three persons A, B and C aim a target. The probabilities of their hitting the target are respectively $\frac{2}{3}, \frac{1}{4}, \frac{1}{2}$. Find the probability that the target will be hit.

[Ans. : $\frac{7}{8}$]

14. An example of statistics is given to three students A, B and C. Their probabilities of solving the example correctly are respectively $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$. Find the probability that the example will be solved.

[Ans. : $\frac{29}{32}$]

15. The present age of a person A is 35. The odds in favour of his living upto the age of 65 is 3 : 2. The age of another person B is 40 at present. The odds against his living upto the age of 70 is 4 : 1. Find the probability that atleast one of them will be alive after 30 years.

[Ans. : $\frac{17}{25}$]

16. In three different families, there are respectively 3 boys and 2 girls, 2 boys and 3 girls, 4 boys and 1 girl. A family is selected at random and from it 2 children are taken at random find the probability that both are boys.

[Ans. : $\frac{1}{3}$]

17. A group consists of 7 men and some women. The probability of selecting 2 women from them is $\frac{1}{15}$. Find the number of women in the group. [Ans. : 3]

18. A number is taken at random from the numbers 1 to 100. Find the probabilities that the number is divisible by (i) '3' (ii) '7' (iii) '3' or '7'

[Ans. : $\frac{33}{100}, \frac{14}{100}, \frac{43}{100}$]

19. There are 5 red and 4 black balls in a bag. Two draws of two balls are made from it. Find the probabilities that the first drawing gives 2 red balls and the second drawing gives 2 black balls if (1) the balls are not replaced (ii) they are replaced.

[Ans. : $\frac{5}{63}, \frac{5}{108}$]

20. In a locality 65% can read Gujarati, 36% can read Hindi and 30% can read English. 18% can read Gujarati and Hindi. 17% can read Gujarati and English and 13% can read Hindi and English. 5% can read all the three languages. Find the probability that a person

selected at random can read, atleast one of the three languages. [Ans. : 0.88]

21. An urn contains 9 red, 7 black and 5 white balls. If three balls are drawn one after the other without replacement find the probability that they are red, black and white respectively.

[Ans. : $\frac{3}{76}$]

22. In a group of 20 persons, there are 5 graduates. If 3 persons are selected at random from the group, find the probabilities that (i) all are graduates, (ii) atleast one is graduate.

[Ans. : $\frac{1}{114}, \frac{137}{228}$]

23. One urn contains 4 red and 5 white balls and the second urn contains 6 red and 3 white balls. One of the urns is selected at random and two balls are drawn from it. Find the probability that both the balls are red.

[Ans. : $\frac{7}{24}$]

24. A can hit a target 3 times out of 5 trials, B can hit the target 2 times out of 5 trials; C can hit the target 3 times out of 4 trials. If all the three try simultaneously find the probability that at least 2 will hit the target. [Ans. : 0.63]

25. 4 boys and 2 girls are randomly arranged in one row. Find the probability that both the girls are side by side.

[Ans. : $\frac{1}{3}$]

26. 5 boys and 3 girls are randomly arranged in one row. Find the probability that no girls are together.

[Ans. : $\frac{5}{14}$]

27. 3 books of English and 5 books of Hindi are arranged randomly on a shelf, in one row. Find the probability that the books of the same language are together.

[Ans.: $\frac{1}{28}$]

28. The probability that a doctor makes correct diagnosis of a patient is 0.6. The probability that the patient will die after correct diagnosis is 0.4. The probability that the patient will die because of incorrect diagnosis is 0.7. If the patient dies. Find the probability that the doctor was correct in the diagnosis. [Ans. : 0.4615]

29. Three dice are throw n simultaneously. Find the probability that 6 will appear on atleast one of the dice.

[Ans. : $\frac{91}{216}$]

30. The odds in favour of three critics independently reviewing a book favourably are 3 : 2; 4 : 3 and 2 : 3. Find the probability that majority will review the book favourably.

[Ans. : $\frac{94}{175}$]

31. If P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.7, find (i) $P\left(\frac{A}{B'}\right)$ (ii) P(A' ∪ B). Are A and B independent events.

[Ans. : (i) 0.5 (ii) 0.7, yes]

32. In an experiment two events A and B can happen. If P(A) = 0.4, P(B) = x and P(A ∪ B) = 0.7, find the value of x when (i) A and B are mutually exclusive. (ii) A and B are independent.

[Ans. : (i) 0.3 (ii) 0.5]

33. A bag contains 5 white and 7 black balls. Another bag contains 7 white and 8 black balls. A ball is taken at random from the first bag and placed it into the second bag without seeing the colour. Then one ball is drawn at random from the second bag. Find the probability that it is a white ball.

[Ans. : $\frac{89}{192}$]

34. A bag contains 12 white and 8 red balls. If four balls are taken one after the other from the bag (i) without replacement (ii) with replacement, find the probabilities that they are alternatively of different colour.

[Ans. : (i) $\frac{616}{4845}$ (ii) $\frac{72}{625}$]

35. In a bag there are 3 red and 7 white balls. One ball is drawn at random from it and in its place a ball of the other colour is placed. Then one ball is drawn at random from that urn. Find the probability that it is red.

[Ans. : $\frac{34}{100}$]

36. An urn contains four tickets numbered 1, 2, 3, 4 and another urn contains six tickets numbered 2, 4, 6, 7, 8, 9. If one of the two urns is chosen at random and a ticket is drawn at random from it, find the probability that the ticket drawn bears the number.
(i) 2 or 4 (ii) 3 (iii) 1 or 9.

[Ans. : (i) $\frac{5}{12}$ (ii) $\frac{1}{8}$ (iii) $\frac{5}{24}$]

37. If A, B, C, are three mutually exclusive and exhaustive events and $2P(A) = 3P(B) = 4P(C)$, find $P(B \cup C)$.

[Ans. : $\frac{7}{13}$]

38. Answer the following questions :
(i) If A and B are two mutually exclusive events and $P(A) = 0.6$, $P(B) = 0.3$, find $P(A \cap B')$.
(ii) If A, B and C are mutually exclusive events and $P(A) = 0.2$, $P(B) = 0.3$, $P(C) = 0.4$, find $P(A \cup B \cup C)$.
(iii) A and B are independent events $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{5}$ find $P(A \cap B)$.
(iv) For two events A & B, $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{2}{3}$, find $P(A \cap B)$, $P\left(A/B\right)$, $P\left(B/A\right)$ and $P(A - B)$.
(v) If $P(A) = \frac{2}{3}$, $P(A \cap B) = \frac{4}{15}$, find $P(A \cap B')$ and $P\left(B'/A\right)$.

[Ans. : (i) 0.6 (ii) 0.9
(iii) $\frac{2}{15}$ (iv) $\frac{1}{15}, \frac{1}{5}, \frac{1}{6}, \frac{1}{3}$
(v) $\frac{2}{5}, \frac{3}{5}$]

39. There are three urns containing respectively 5 white and 3 black balls, 3 white and 5 black balls; 2 white and 6 black balls. An urn is selected at random and a ball is drawn from it. Find the probability that the ball is black

[Ans. : $\frac{7}{12}$]

40. In a factory 4000 units of an item are produced. There are three machines A, B and C in the factory. B and C produce equal number of items while machine A produces double items than B. The proportion of defective items produced by A, B and C are 2%, 1% and 3% respectively. An item is selected at random from the total production and it is found to be defective. Find the probability that it comes from machine C. **[Ans. :** 3/8]

41. The probabilities that two independent witnesses speak truth are 0.7 and 0.4. They agree on a statement find the probability that the statement is not true.

[Ans. : $\frac{9}{23}$]

42. There are 7 black and 3 white balls in one bag, and 4 black and 6 white balls in another bag. A die is tossed and if it shows number 1 or 2 two balls are drawn from the first bag and if the number 3, 4, 5, or 6 is shown on the die two balls are drawn from the second bag. If both the balls drawn are black. Find the probability that they come from the second bag.

[Ans. : $\frac{4}{11}$]

43. A principal is to be chosen from three persons A, B and C. Their chances of selection are in the proportion 5 : 2 : 3. The probabilities that the sports activities will be encouraged by these persons are respectively $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$. If it is known that the sports activities are encouraged. Find the probability that C is selected as principal.

[Ans. : $\frac{18}{113}$]

44. Three machines in a factory produce respectively 20%, 50% and 30% of items daily. The percentage of defective items of these machines are respectively 3, 2 and 5. An item is taken at random from the production and is found to be defective. Find the probability that it is produced by machine A.

[Ans. : $\frac{6}{31}$]

45. (a) State and prove addition theorem of probability
(b) If X, Y, Z are three mutually exclusive and exhaustive events and $2.P(X) = 3.P(Y) = 4.P(Z)$, find $P(Y \cup Z)$

(c) Three persons P, Q and R aim a target. The probabilities of their hitting the target are respectively $\frac{2}{3}, \frac{1}{4}, \frac{1}{2}$. Find the probability that the target will be hit.

(G.U., March, 2004)

[Ans. : (b) $P(Y \cup Z) = \frac{7}{13}$ (c) $\frac{7}{8}$]

46. (a) Give different definations of probability

(b) If A, B, C are three mutually exclusive and exhaustive events, and 2.P(A) = 3.P(B) = 4.P(C), then find $P(B \cup C)$

(G.U., March, 2005)

[Ans. : (b) $\frac{7}{13}$]

47. (a) Explain the following terms (any **four**) :

(i) Random experiment (ii) Exhaustive events
(iii) Intersection events (iv) Independent events
(v) Difference events

(b) A group consists of 4 men and some women. If the probability of selecting 2 men from them is $\frac{2}{15}$, find the number of women in the group

(c) Two witnesses A and B are independent. If the probability that A speaks false is $\frac{1}{2}$ and B speaks true is $\frac{2}{3}$, find the probability that the statement is true, if they agree on some statement.

(G.U., March, 2006)

[Ans. : (b) 6 women (c) $\frac{2}{3}$]

48. (a) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{6}$, find the value of $P(A \cup B)$, $P(A' \cap B')$ and $P(A'/B')$

(b) Three machines A, B and C produces 15%, 55% and 30% of items daily in a factory. The percentage of defective items of these machines are respectively 4%, 5% and 6%. An item is taken at random from the production and is found to be defective. Find the probability that it is produced by machine A.

(c) The opinion in favour of a book given by three critics is in the ratio 3 : 5, 2 : 5 and 3 : 4. Find the probability that at least two will give good opinion about a book.

(G.U., March, 2007)

[Ans. : (a) $P(A \cup B) = \frac{5}{12}$, $P(A' \cap B') = \frac{7}{12}$, $P(A'/B') = \frac{7}{9}$

(b) 0.1165 (c) 0.2985]

49. (a) Explain :

(1) Random experiment
(2) Conditional probability
(3) Baye's theorem

(b) Three persons A, B and C aim a target. The probability of hitting the target are respectively $\frac{1}{2}, \frac{1}{3}$ and $\frac{2}{5}$. Find the probability that

(i) at least one person will hit the target.
(ii) majority of persons will hit the target and
(iii) the target will not be hit.

(c) If $3P(A) = 2P(B) = 4P(A \cap B) = \frac{1}{3}$, then find the value of

(i) $P(A \cup B)$
(ii) $P(A' \cap B')$
(iii) $P(B/A')$
(iv) $P(A'/B')$
(v) $P(A \cup B')$

(G.U., April, 2008)

[Ans. : (b) (i) $\frac{4}{5}$ (ii) $\frac{11}{30}$ (iii) $\frac{1}{5}$

(c) (i) $\frac{7}{36}$ (ii) $\frac{29}{36}$ (iii) $\frac{3}{32}$

(iv) $\frac{29}{30}$ (v) $\frac{11}{12}$]

50. (a) Explain :

(1) Conditional Probability
(2) Independent Events
(3) Baye's Theorem

(b) If $P(A') = \frac{2}{3}, P(B) = \frac{1}{4}, P(A \cup B) = \frac{5}{16}$, then find the values of
(1) $P(A \cap B)$ (2) $P(A' \cap B')$ (3) $P(A' / B')$ (4) $P(A - B)$

(c) The opinion in favour of a book given by three critics is in the ratio 3 : 5, 2 : 5 and 3 : 4, find the Probability that
(1) At least one will give good opinion about a book.
(2) Majority will give good opinion about a book.

(Guj. Uni., March ,2009)

[Ans.: (b) (1) $\frac{13}{48}$ (2) $\frac{11}{16}$ (3) $\frac{44}{48}$ (4) $\frac{3}{48}$ **(c)** 0.7449]

51. (a) Probability that a person becomes a Member of Parliament is $\frac{5}{12}$ and he becomes a Minister is $\frac{2}{11}$. Moreover probability that he becomes a Member of Parliament or a Minister is $\frac{19}{44}$. Find the probability that,
(i) A person becomes a member of Parliament and a Minister.
(ii) He becomes a Minister, when it is known that he is a Member of Parliament.

(b) Probabilities for Dr. Parikh , Dr. Hathi, Dr. Singh for being selected as the principal of a college are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$ respectively. If they are selected as principal then probabilities that the attendance becomes compulsory in a college are $\frac{3}{4}, \frac{1}{2}$ and $\frac{2}{3}$ respectively. If it is given that attendance has become compulsory in a college, then find probability that Dr. Parikh is selected as a Principal.

(Guj. Uni., April ,2010)

[Ans.: (a) (i) 0.17 (ii) 0.4 (b) $\frac{27}{47} = 0.57$]

52. (a) Give the definition of following terms :
(1) Random experiment
(2) Events
(3) Exhaustive events

(4) Independent events

OR

Give the definitions of following terms :
(1) Probability
(2) Conditional Probability
(3) Equally likely events
(4) Favourable events

(b) There are 7 red and some white balls in a box. Probability of selection of two white balls is $\frac{1}{15}$. Find the number of white balls in the box.

(c) If P(A) = 2P(B) = P(A/B) = 0.6 then find the values of
(i) $P(A \cap B)$
(ii) $P(A \cup B)$
Obtain the answer as per demand.

(d) If $P(X) = \frac{3}{4}, P(Y) = \frac{1}{3}$ and $P(X \cup Y) = \frac{11}{12}$ then find the values of
(i) $P\left(\frac{Y}{X}\right)$
(ii) $P\left(\frac{Y'}{X}\right)$

(e) There are two machines M_1 and M_2 in a company producing screw respectively 60% and 40% of production. In this production, 3% and 2% screw are defective respectively. A screw is drawn at random from the production and found to be defective. Find the probability that it is produced by machine M_2.

(f) Write the formula of rule of addition for 3 events.

(g) What is the chance that a leap year selected at random will contain 53 Mondays ?

(Guj. Uni., Dec., 2012)

[Ans.: (b) 3 **(c)** (i) 0.18 (ii) 0.72 **(d)** (i) (i) $\frac{2}{9}$ (ii) $\frac{7}{9}$
(e) $\frac{4}{13}$ **(g)** $\frac{2}{7}$]

53. (a) (i) State Bayes' theorem of Inverse Probability.

(ii) Give mathematical definition of probability and prove that $0 \le P(A) \le 1$.

(b) Explain with illustration :

(i) Random Experiment

(ii) Mutually Exclusive Events.

(c) An example of probability is given to three students A, B & C. Their probability of solving the example correctly are respectively $\frac{1}{2}, \frac{3}{4}$ & $\frac{2}{3}$. Find probability that the example will be solved correctly.

(d) If $2P(A) = 3P(B) = 5P(A \cap B) = \frac{1}{3}$, then find value of

(i) $P(A \cup B)$

(ii) $P(B/A')$

(e) A vice-chancellor is to be chosen from three persons A, B & C. Their chances of selection are respectively in the proportion 4 : 2 : 3. The probabilities that the new employment oriented courses will be started by three persons are respectively $\frac{1}{2}, \frac{1}{3}$ & $\frac{1}{5}$. Find the probability that new courses are encouraged if person a is selected.

(f) An urn contains 5 white, 4 red and 3 black balls. Three balls are drawn at random from it. Find the probability that.

(i) All are black balls.

(ii) All are of different colours balls.

(g) If $P(A) = 2\,P(B) = P(A/B) = 0.6$. then $P(A \cap B) =$ ....

(a) 0.3 (b) 0.18 (c) 0.2 (d) None of above

(Guj. Uni., Dec., 2013)

[Ans.: (c) $\frac{23}{24}$ **(d) (i)** $\frac{19}{90}$ **(ii)** $\frac{12}{225}$ **(e)** $\frac{30}{49}$ **OR** 0.6122

(f) (i) $\frac{1}{220}$ **(ii)** $\frac{60}{220}$ **OR** $\frac{3}{11}$ **(g)** 0.18**]**

54. (a) Explain the following terms :

(i) Exhaustive events

(ii) Matually Exclusive events

(iii) Equally likely events

(iv) Independent event

OR

(i) Give mathematical definition of probability.

(ii) State Bayes theorem of inverse probability.

(b) There are 5 black and 4 red balls in the first bag and 4 black and 6 red balls in the second bag. A bag is selected at random and from the bag 2 balls are drawn at random. Find the probability that both the balls are black.

(c) If $P(A) = \frac{1}{9}$, $P(B) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{12}$, then find the following probability :

(i) $P(A \cup B)$

(i) $P(A' \cap B')$

(iii) $P(A'/B')$

(d) A group consist 4 men and some women. If the probability of selecting 2 men from them is $\frac{2}{15}$. Find the number of women in the group.

(e) If $P(A) = \frac{2}{5}$, $P(A \cup B) = \frac{2}{3}$, $P(B') = \frac{1}{2}$ find $P(A \mid B)$ and $P(A \mid B')$

(f) If $P(A \mid B) = 0.5$, $P(B) = 0.6$ then find $P(A \cap B)$.

(Guj. Uni., Dec., 2014)

[Ans.: (b) 0.2056 **(c) (i)** $\frac{7}{36}$ **(ii)** $\frac{29}{36}$ **(iii)** $\frac{29}{30}$

(d) 6 **(e)** $\frac{7}{15}, \frac{1}{3}$ **(f)** 0.30**]**

55. (a) Explain following terms :

(i) Intersection events.

(ii) Union events.

(iii) Disjoint events.

(iv) Equi-Probable events

(b) Explain following terms :

(i) Complementary event

(ii) Primary events

(iii) Independent events

(iv) Exhaustive events

(c) Three persons A, B and C aims a target. The probability of their hitting the target is 1/2, 1/3 and 1/4 respectively. Find the probability that at least 2 persons will hit the target.

(d) There are 4 black and 3 white balls in a bag and another bag contains 5 black and 2 white balls. A bag is selected at random and two balls are drawn randomly from it. Find the probability that both are black balls.

(e) For three mutually Exclusive and exhaustive events A, B and C, If $2P(A) = 3P(B) = 4P(C)$ Find $P(B \cup C)$.

(f) If $P(A) = 1/5$, $P(B) = 1/4$, and $P(A \cap B) = 1/12$ Find $P(A \cup B)$.

(g) If $P(B/A) = 0.4$ and $P(A) = 0.6$ Find $P(A \cap B)$.

(h) If $P(A) = 3/5$, $P(B) = 1/3$ and $P(A \cup B) = 4/5$ Find $P(A \cap B)$.

(Guj. Uni., Dec., 2015)

[Ans. : (c) $\frac{7}{24}$ **(d)** $\frac{8}{21}$ **(e)** $\frac{7}{13}$ **(f)** $\frac{11}{30}$ **(g)** 0.24 **(h)** $\frac{2}{15}$

56. (a) Explain following terms :

(i) Sample space

(ii) Complementary events.

(iii) Union of two events.

(iv) Intersection of two events

(b) Explain following terms :

(i) Event

(ii) Mutually Exclusive Events

(iii) Independent Events

(iv) Exhaustive Events

(c) If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{6}$. Find the values of $P(A \cup B)$, $P(A' \cap B')$ and $P(A'/B')$.

(d) Three persons A, B and C aim a target. The probability of hitting the target are respectively $\frac{1}{3}, \frac{1}{2}, \frac{2}{5}$. Find probability that

(i) At least one person will hit the target.

(ii) At least two person will hit the target.

(iii) The target will not be hit.

(e) If A, B and C are three mutually exclusive and exhaustive events and $2P(A) = 3P(B) = 4P(C)$. Find $P(A \cup B)$.

(f) If $3P(A) = 2P(B) = 5P(A \cap B) = \frac{1}{3}$, then find the value of $P(B/A')$ and $P(A' \cup B')$.

(Guj. Uni., Dec., 2016)

[Ans. : (c) $\frac{5}{12}, \frac{7}{12}, \frac{7}{8}$ **(d)** (i) $\frac{4}{5}$, (ii) $\frac{11}{30}$ (iii) $\frac{1}{5}$ **(e)** $\frac{7}{3}$ **(f)** $\frac{14}{15}$

57. (a) Explain the following terms :

(i) Union of two events.

(ii) Exhaustive event.

(iii) Conditional probability.

(iv) Independent event.

(b) An item is made up of two parts P and Q. The probability that part P is defective is 0.07 and the probability that part Q is non-defective is 0.95. Find the probability that the whole item is non-defective.

(c) Three persons X, Y and Z aim a target. The probabilities of their hitting the target are respectively $\frac{2}{3}, \frac{3}{4}$ and $\frac{4}{5}$. Find the probability that the target will be hit.

(d) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{6}$, then find (i) $P(A \cup B)$, (ii) $P(A' \cap B')$.

(e) Find the probability of 53 Thursday in the year 2017.

(Guj. Uni. Dec. 2017)

[Ans. : (b) 0.8835 **(c)** $\frac{59}{60}$ **(d)** (i) $\frac{5}{12}$ (ii) $\frac{7}{12}$ **(e)** $\frac{1}{7}$]

58. (a) Explain the following terms :

(1) Sample space

(2) Union of two events

(3) Conditional probability

(4) Independent event

(b) Three machines *x*, *y* and *z* produce 35%, 45% of the items daily in a factory. The percentage of defective items of these machines are respectively 3%, 4% and 5%. An item is taken at random form the production and is found to be defective. Find the probability that it is produced by machine *z*.

(c) If $3P(A) = 2P(B) = 4P(A \cap B) = \frac{1}{3}$ then find the value of $P(A \cup B)$, $P(A' \cap B')$ $P(A/B)$ and $P(A'/B)$.

(d) An example of statistics is given to three students X, Y and Z. Their probabilities of solving the example correctly are respectively $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$. Find the probability that the example will be solved.

(e) An urn contains 4 white, 5 red and 6 black balls. Three balls are drawn at random from it. Find the probabilities that two balls are of the same colour and one ball has different colour.

(f) If $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{4}{15}$, find P(B/A).

(g) For two events A and B if P(A) = 0.4. P(B) = x and $P(A \cup B) = 0.8$ find the value of x and (a) A and B are mutually exalusive (b) A and B are independent events.

(h) Find the probability of 53 Monday in the year 2018.

(Guj. Uni. Nov., 2018)

[Ans. : (b) 0.2597 **(c)** $\frac{7}{36}$, $\frac{29}{36}$, $\frac{1}{2}$, $\frac{1}{2}$ **(d)** 0.9667

(e) 0.6615 **(f)** 0.6 **(g)** 0.4 **(h)** $\frac{1}{7}$]

59. (a) Explain mutually exclusive events and independent events with illustration.

(b) There are 7 black and 3 white balls in one bag, and 4 black and 6 white balls in another bag. A die is tossed and if shows number 5 or 6, two balls are drawn from the first bag and if the number 1, 2, 3 or 4 is shown on the die, two balls are drawn from the second bag. If both the balls drawn are black, find the probability that they come from the second bag.

(c) Explain the following terms

(i) Probability

(ii) Difference of events

(iii) Exhaustive events

(d) If A, B, C are three mutually exclusive and exhaustive events, 2P(A) = 3P(B) = 4P(C) then find $P(A \cup B)$ and $P(B \cup C)$.

(e) A number is taken at random from the numbers 1 to 150. Find the probabilities that the number is divisible by (i) 4 (ii) 4 or 9.

(f) Answer the following :

(i) Give the example of mutually exclusive events.

(ii) Write any one definition of probability.

(iii) What is the range of probability ?

(iv) State Bayes theorem.

(Guj. Uni. Nov., 2019)

[Ans. : (b) $\frac{4}{11}$ **(d)** $P(A \cup B) = \frac{10}{13}$, $P(B \cup C) = \frac{7}{13}$,

(e) (i) Prob. that number is divisible by 4 = $\frac{37}{150}$,

(ii) Prob. that number is divisible by 4 or 9 = $\frac{49}{150}$

(f) (iii) $0 \le P(A) \le 1$]

60. (a) Define the following terms :

(i) Intersection of two events.

(ii) Independent Events

(iii) Exhaustive Events

(iv) Difference of two events

(b) Answer the following :

(i) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{6}$, find the value of $P(A \cup B)$, $P(A' \cap B')$ and $P(A'/B')$

(ii) Three persons P, Q and R hit a target. The probability of hitting the target respectively $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{4}$. Find the probability that

(1) At least one person will hit the target

(2) The target will not be hit

(c) Answer the following. (any five) :

(i) State addition theorem for two events and three events when events are not mutually exclusive.

(ii) If $2P(A) = 3P(B) = 5P(A \cap B) = \frac{1}{3}$, then find $P(A \cup B)$

(Guj. Uni. Jan., 2021)

[Ans. : (b) (i) $P(A \cup B) = 5/12$, $P(A' \cap B') = 7/12$, $P(A' / B') = 7/9$

(ii) (1) Prob. that at least one person will hit the target = 11/12

(2) Prob. that the target will not be hit = 1/12

(c) (ii) $P(A \cup B) = 19/90$]

61. (a) Define the following :

(1) Union event

(2) Mutually exclusive events

(3) Conditional probability

(4) Exhaustive events

(5) Complimentary events

(b) If $P(A) = 0.6$; $P(B) = 0.5$; $P(A \cap B) = 0.4$ then find $P(A \cup B)$, $P(A - B)$, $P(A/B)$, $P(B'/A)$ and $P(A' \cap B')$.

(c) Answer the following :

(1) What is the probability of having 5 Saturday in a February of a leap year ?

(2) In usual notations, P(U) = ... and P(ϕ) = ...

(a) (0.5, 0.5) (b) (1, 0)

(c) (5, –1) (b) (0.8, 0.2)

(Guj. Uni. Dec., 2021)

[**Ans. : (b)** $P(A \cup B) = 0.7$, $P(A - B) = 0.2$, $P(A/B) = 0.8$, $P(B'/A) = 0.33$, $P(A' \cap B') = 0.3$

(c) (1) 1/7 (2) (b) (1, 0)]

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