3 DISCRETE DISTRIBUTION-1

(POISSON DISTRIBUTION AND HYPER-GEOMETRIC DISTRIBUTION)

(A) Poisson Distribution

1. Introduction :

As we know that arrangement which shows how the total probability one is distributed over the different values of random variable is called probability distribution. A rule according to which the total probability one is distributed over the different values of random variable is called probability distribution function. It satisfies the following two conditions.

(i) All probability values must be non-negative

(ii) Total probability must be one

When the basic random variable is discrete, then its distribution function is called probability mass function (p.m.f.). In this chapter, we will learn two discrete probability distributions : (i) Poisson distribution, and (ii) Hyper-geometric distribution.

2. Poisson Distribution :

Poisson distribution is a distribution of a discrete variable. It was first given by a French Mathematician **Simeon De Poisson** in 1837, as a limiting case of binomial distribution. In binomial distribution, when n is very large, p is very small and np is finite, then it tends to Poisson distribution. Mathematical form of Poisson distribution is given by the following probability function :

$$\text{Probability of } x \text{ successes } P(x) = \frac{e^{-m} m^{x}}{x\,!}, \; x \leq 0, 1, 2 \; ......$$

Where, $e = 2.7183$

m = mean = np

x = number of successes.

3. Properties of Poisson Distribution :

(1) This is a distribution of a discrete variable.
(2) This is a distribution of rare occurrences.
(3) m is parameter of the distribution.
(4) The mean of this distribution is m.
(5) The variance of this distribution is also m. Hence, its S.D. = $\sqrt{m}$.
(6) For Poisson distribution mean = Variance.
(7) Sum of two independent Poisson variates is also a Poisson variate.
(8) This is a distribution with positive skewness.

4. Uses of Poisson Distribution :

The following are some of the instances, where Poisson distribution can be applied :

(1) Number of accidents on a road.
(2) Number of misprints per page of a book.
(3) Number of defects in a radio set.
(4) Number of air bubbles in a glass bottle.
(5) Number of suicides committed per day.
(6) Number of goals scored in a football match.
(7) Number of telephone calls received during a given interval of time.
(8) Poisson distribution is used in C chart of statistical quality control.
(9) This distribution is used in acceptance sampling.

We shall now use Poisson distribution, in few illustrations.

5. Illustrations :

Illustration 1 : A factory having large number of employees find that, over a period of time, the average absentee rate is three employees per day. Calculate the probability that, on a given day.

(i) exactly one employee will be absent.

(ii) more than two employee will be absent. **($e^{-3} = 0.0498$)**

Ans : If the number of emplyees absent per day is denoted by x, then x will follow Poission distribution.

i.e. $p(x) = \dfrac{e^{-m} . m^{x}}{x!}$

Here, m = average number of employees absent

$\therefore m = 3$

(i) Probability that exactly one employee will be absent.

$$P(1) = \frac{e^{-3}\ 3^{1}}{1!}$$

$$= \frac{0.0498 \times 3}{1}$$

$$= 0.1494$$

(ii) Probability that more than two employees will be absent

$= P(3) + P(4) + P(5) + \ldots\ldots$

$= 1 - [P(0) + P(1) + P(2)]$

$$= 1 - \left[\frac{e^{-3}3^{0}}{0!} + \frac{e^{-3}3^{1}}{1!} + \frac{e^{-3}3^{2}}{2!}\right]$$

$$= 1 - e^{-3}\left[\frac{1}{1} + \frac{3}{1} + \frac{9}{2}\right]$$

$= 1 - (0.0498)(8.5)$

$= 1 - 0.4233$

$= 0.5767$

Illustration 2 : Between the hours of 2 and 4 p.m. the average number of phone calls per minute coming into the switch board of a company is 2.5. Find the probabilities that during one particular minute there will be (i) no phone call at all (ii) exactly 3 calls.

($e^{-2.5} = 0.0821$)

Ans : Let x be the number of telephone calls received per minute. x will follow Poisson distribution.

$\therefore \quad P(x) = \dfrac{e^{-m} m^{x}}{x\,!}$ Where, $m = 2.5$

(i) Probability that there will be no call during one particular minute

$$= P(0) = e^{-m} = e^{-2.5} = 0.0821$$

(ii) Probability that there will be exactly 3 calls during a particular minute.

$$P(3) = \frac{e^{-2.5}(2.5)^3}{3!}$$

$$= \frac{0.0821 \times 15.625}{6}$$

$$= 0.2135 \text{ (approximately)}$$

Illustration 3 : There are 100 misprints in a book of 100 pages. If a page is selected at random, find the probabilities that (i) there will be no misprint in the page (ii) there will be 1 misprint (iii) there will be at the most 2 misprints.

$(e^{-1} = 0.3679)$

Ans : If the number of misprints per page is denoted by x, then x will follow Poisson distribution.

i.e, $$P(x) = \frac{e^{-m}m^x}{x!}$$

Here, m = mean of the distribution

= average number of misprints per page

$$= \frac{100}{100} = 1.$$

(i) Probability that there will be no misprint

$$P(0) = \frac{e^{-1}1^0}{0!}$$

$$= e^{-1}$$

$$= 0.3679$$

(ii) Probability that there will be 1 misprint

$$P(1) = \frac{e^{-1}1^1}{1!}$$

$$= 0.3679$$

(iii) Probability that there will be at the most 2 misprints.

$$= P(0) + P(1) + P(2)$$

Now, $$P(2) = \frac{e^{-1}1^2}{2!}$$

$$= \frac{0.3679}{2}$$

$$= 0.1840.$$

$\therefore$ Required probability $= 0.3679 + 0.3679 + 0.1840$

$= 0.9198.$

Illustration 4 : On an average 1.5 per cent of electric bulbs are found to be defective in a bulb manufacturing factory. Using Poisson distribution find the probability of 4 defective bulbs in a box of 200 bulbs.

$(e^{-3} = 0.0498)$

Ans. : If x denotes the number of defective bulbs in a box,

then $$P(x) = \frac{e^{-m}m^x}{x!}$$

Here, m = Average number of defective bulbs

$= np.$

$$p = \frac{1.5}{100} \text{ and } n = 200$$

$$\therefore \quad m = 200 \times \frac{1.5}{100} = 3$$

$\therefore$ Probability of 4 defective bulbs

$$P(4) = \frac{e^{-3}3^4}{4!}$$

$$= \frac{0.0498 \times 81}{24}$$

$$= 0.1681.$$

Illustration 5 : The probability that a blade manufactured by a factory is defective is $\frac{1}{500}$. Blades are packed in packets of 10 blades. Find the expected number of packets containing (i) no defective blade (ii) one defective blade (iii) 2 defective blades, in a consignment of 10,000 packets.

$(e^{-0.02} = 0.9802)$

Ans. : Here, $m = np$

$$= 10 \times \frac{1}{500}$$
$$= 0.02$$

The probability of x defective blades is given by

$$P(x) = \frac{e^{-m} m^x}{x!}$$
$$= \frac{e^{-0.02}(0.02)^x}{x!}$$

Now, (i) the probability that no blade is defective in a packet.

$$P(0) = \frac{e^{-0.02}(0.02)^0}{0!}$$
$$= \frac{e^{-0.02} \times 1}{1}$$
$$= e^{-0.02}$$
$$= 0.9802$$

∴ The expected number of packets with no defective blade $= 10000 \times 0.9802 = 9802$.

(ii) The probability that one blade is defective in a packet

$$P(1) = \frac{e^{-0.02}(0.02)^1}{1!}$$
$$= \frac{0.9802 \times 0.02}{1}$$
$$= 0.019604$$

∴ The expected number of packets with 1 defective blade $= 10000 \times 0.019604 = 196.04 = 196$ (approximately)

(iii) The probability that 2 blades are defective in a packet.

$$P(2) = \frac{e^{-0.02}(0.02)^2}{2!}$$
$$= \frac{0.9802 \times 0.0004}{2}$$
$$= 0.000196$$

∴ The expected number of packets with 2 defective blades

$$= 10{,}000 \times 0.000196$$
$$= 2 \text{ (approximately)}.$$

Illustration 6 : The probability that a patient will get reaction of a particular injection is 0.001. 2000 patients are given that injection. Find the probabilities that (i) 3 patients will get reaction (ii) more than 2 patients will get reaction.

($e^{-2} = 0.135$)

Ans. : Here, $m = np$

$$= 2000 \times 0.001$$
$$= 2.$$

$$P(x) = \frac{e^{-m} m^x}{x!}$$

(i) Probability that 3 patients will get reaction

$$P(3) = \frac{e^{-2} 2^3}{3!}$$
$$= \frac{0.135 \times 8}{6}$$
$$= 0.18$$

(ii) Probability that more than 2 patients will get reaction

$$= P(3) + P(4) + P(5) + \ldots\ldots\ldots\ldots$$
$$= 1 - \{P(0) + P(1) + P(2)\}$$

Now, $P(0) = e^{-2} = 0.135$

$$P(1) = \frac{e^{-2} 2^1}{1!}$$
$$= \frac{0.135 \times 2}{1}$$
$$= 0.270$$

$$P(2) = \frac{e^{-2} 2^2}{2!}$$
$$= \frac{0.135 \times 4}{2}$$
$$= 0.270$$

$\therefore$ The required probability

$= 1 - \{0.135 + 0.270 + 0.270\}$

$= 0.325.$

Illustration 7 : The following is a distribution of number of accidents occurred in a city during 100 days. Fit a Poisson distribution to the given data.

Number of accidents	0	1	2	3	4 or more
Number of days	37	36	19	6	2

$(e^{-1} = 0.368)$

Ans. : We shall first find out the mean of the given distribution.

Number of accidents x	Days f	fx	$P(x)$	Expected frequency $= N \cdot P(x)$
0	37	0	0.368	36.8
1	36	36	0.368	36.8
2	19	38	0.184	18.4
3	6	18	0.061	6.1
4 or more	2	8	0.019	1.9
Total	100 = N	100	1.00	100

For the above distribution, mean

$$m = \frac{\sum fx}{N} = \frac{100}{100} = 1$$

Now, $$P(x) = \frac{e^{-m} m^x}{x!}$$

$$P(0) = e^{-m} = e^{-1} = 0.368$$

$$P(1) = \frac{e^{-1}\, 1^1}{1!} = 0.368 \times 1 = 0.368$$

$$P(2) = \frac{e^{-1}\, 1^2}{2!} = \frac{0.368 \times 1}{2} = 0.184$$

$$P(3) = \frac{e^{-1}\, 1^3}{3!} = \frac{0.368 \times 1}{6} = 0.061$$

Probability of 4 or more accidents

$= 1 - \{ P(0) + P(1) + P(2) + P(3) \}$

$= 1 - \{ 0.368 + 0.368 + 0.184 + 0.061 \}$

$= 1 - 0.981$

$= 0.019.$

The expected frequencies are shown in the last column.

Illustration 8 : The mean of a Poisson distribution is 3. Find its standard deviation.

Ans : We know that for a Poisson distribution mean = variance.

Here, Mean = 3 $\quad\therefore$ Variance = 3

$\therefore$ S.D. $= \sqrt{3} = 1.7321.$

Illustration 9 : For a poisson variate P (1) = P (2), find the value of P (0) :

Ans. : For a poisson variate, the probability of x successes is given by

$$P(x) = \frac{e^{-m} m^x}{x!}$$

We have $P(1) = P(2)$

$$\therefore \quad \frac{e^{-m} m^1}{1!} = \frac{e^{-m} m^2}{2!}$$

$$\therefore \quad \frac{m}{1} = \frac{m^2}{2}$$

$$\therefore \quad m = 2$$

Now, $P(0) = \frac{e^{-2}2^0}{0!}$

$= e^{-2}$

Illustration 10 : The standard deviation of a Poisson variable is 0.8 Find its mean, P(0) and P(1). $(e^{-0.64} = 0.5273)$

Ans. : Here, S.D. = 0.8

$\therefore$ Variance $= (0.8)^2 = 0.64$

Now, Mean = Variance $\therefore$ Mean $m = 0.64$

Now, $P(0) = e^{-m}$

$= e^{-0.64}$

$= 0.5273$

$P(1) = \frac{m}{1} P(0)$

$= 0.64(0.5273)$

$= 0.3375.$

Illustration 11 : For a Poisson variate $3P(x = 2) = P(x = 4)$. Find mean and variance.

Ans. : Here, $3 \cdot P(x = 2) = P(x = 4)$

$\therefore \quad 3 \cdot \frac{e^{-m} \cdot m^2}{2!} = \frac{e^{-m} \cdot m^4}{4!}$

$\therefore \quad \frac{3m^2}{2} = \frac{m^4}{24}$

$\therefore \quad \frac{3 \times 24}{2} = \frac{m^4}{m^2}$

$\therefore \quad m^2 = 36$

$\therefore \quad m = 6$

$\therefore$ Mean = 6, Variance = m = 6.

EXERCISES

SECTION A

Answer in one line :

1. What is probability distribution ?
2. What is probability distribution function ?
3. Give properties of probability distribution function.
4. Give three names of discrete distribution function.
5. Give probability mass function of Poisson distribution.
6. What is the standard deviation of Poisson distribution.
7. Give the name of the distribution for which the value of mean an variance are equal.
8. State additive property of Poisson distribution.
9. What is the skewness of Poisson distribution ?
10. Give one application of Poisson distribution.
11. For a Poisson distribution if mean = 2 and $P(x = 0) = 0.1353$ then find $P(x = 1)$. *[Guj., Uni., Nov., 2017]* **[Ans. : 0.2706]**
12. In a Poisson distribution $P(x = 0) = 0.22$ then find the value of mean *[Guj., Uni., Nov., 2017]* **[Ans. : 1.5]**

SECTION B

1. State five instances where a Poisson distribution can be suitably applied.
2. Write down the probability mass function of a discrete variable in which mean and variance are equal. State its parameter.
3. Give the properties of Poisson distribution
4. A person has some cars, and the average demand of cars per day is 3, find the probability that on any day not more than 2 cars are in use. $(e^{-3} = 0.0498)$ **[Ans. : P(0) + P(1) + P(2) = 0.4233]**
5. The probability that a match stick is found without head is $\frac{1}{100}$. Each match box contains 50 sticks. Using Poisson distribution, find the percentage of number of boxes having 0, 1, 2, sticks without heads. $(e^{-0.5} = 0.61)$ **[Ans. : 61, 30.5, 7.63]**

6. In the production of electric fuses 2% are defective. Find the probability of getting (i) all non–defective fuses in a box containing 200 fuses. (ii) at the most 2 defective fuses (iii) 3 defective fuses. ($e^{-4} = 0.0183$)

[**Ans.** : (i) 0.0183, (ii) 0.2379, (iii) 0.1952]

7. In the manufacturing of cotter pins it is known that 5% of the pins are defective. The pins are sold in boxes of 100 and it is guaranted that not more than 4 pins will be defective in a box. What is the probability that a box will meet this guarantee ? ($e^{-5} = 0.0067$)

[**Ans.** : 0.4380]

8. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs, exactly 5 bulbs are defective. ($e^{-3} = 0.0498$)

[**Ans.** : 0.1008]

9. In one hospital 3 per cent of the patients demand special rooms. On a particular day 3 special rooms were vacant. If 50 patients were admitted in the hospital on that day, find the probabilities that (i) no patient demanded special room (ii) the demands for special room were not met. ($e^{-1.5} = 0.2231$)

[**Ans.** : (i) 0.2231 (ii) 0.0658]

10. In a book on an average there are 3 misprints in 5 pages. Using Poisson distribution, find the number of pages having more than 2 misprints in that book of 100 pages. ($e^{-0.6} = 0.5488$)

[**Ans.** : 2.31]

11. A factory produces 0.5% defective articles. If a sample of 100 articles is taken from the production, find the probability of getting 2 or more defective articles. ($e^{-0.5} = 0.6065$)

[**Ans.** : 0.0902]

12. A random variable x follows Poisson distribution with mean 2, find $P(x \geq 1)$. ($e^{-2} = 0.1353$)

[**Ans.** : 0.8647]

13. The mean of a Poisson variate is 0.81, find its S.D. Find the probabilities for $x = 0$ and $x = 2$. ($e^{-0.81} = 0.449$)

[**Ans.** : S.D. = 0.9; P(0) = 0.449; P(2) = 0.1473]

14. x is a Poisson variate such that $P(x = 3) = P(x = 4)$, Prove that $P(x = 2) = 8e^{-4}$.

15. For a Poisson variate x.

$P(x = 1) = P(x = 2)$. Prove that $P(x = 4) = \frac{2}{3} e^{-2}$

16. If for a Poisson variate x, $P(x = 0) = P(x = 1) = k$, prove that $k = \frac{1}{e}$.

17. x is a Poisson variate and $P(x = 1) = P(x = 2)$, find $P(x = 0)$.

[**Ans.** : e^{-2}]

18. x is a Poisson variate and $P(x = 2) = 9.P(x = 4) + 90.P(x = 6)$, find its mean and variance.

[**Ans.** : 1 ; 1]

19. If x is a Poisson variate and $P(x = 0) = 0.05$, prove that $P(x > 2) = 0.575$. ($e^{-3} = 0.05$, $e^{-0.05} = 0.9512$)

20. The number of mistakes committed by a typist in typing 100 pages are as follows. Using Poisson distribution, find the expected frequencies.

Number of mistakes per page	0	1	2	3	4	5
Number of pages	42	33	14	6	4	1

($e^{-1} = 0.368$)

[**Ans.** : 36.8, 36.8; 18.4; 6.1; 1.5; 0.4]

21. Fit a Poisson distribution to the following data :

x :	0	1	2	3	4
f :	110	65	21	3	1

($e^{-0.6} = 0.5488$)

[**Ans.** : 110, 66, 20, 4, 0]

22. Using Poisson distribution, find the expected frequencies for the following data :

Number of accidents	0	1	2	3	4
Days	123	59	14	3	1

($e^{-0.5} = 0.6065$)

[**Ans.** : 121, 61, 15, 3, 0]

23. The distribution of number of mistakes committed by a typist is given below. Assuming Poisson distribution, find the expected frequencies.

Number of mistakes	0	1	2	3	4	5
Number of pages	142	156	69	27	5	1

($e^{-1} = 0.368$)

[Ans. : 147, 147, 74, 25, 6, 1]

24. Answer the following :

(i) How does a Poisson distribution differ from a Binomial distribution?

(ii) For a Poisson variate x, $P(x = 4) = P(x = 5)$, find $P(x = 2)$.

(iii) For a Poisson variate x $P(x = 2) = P(x = 3)$, find its mean.

(iv) If mean of a Poisson distribution is 2, find its S.D.

[Ans. : (ii) $\frac{25}{2}e^{-5}$ (iii) 3 (iv) $\sqrt{2}$]

25. State with reasons whether the following statements are true or false:

(i) The mean of a Poisson variate is 4, hence its S.D. is also 4.

(ii) The mean of a Poisson variate is 2, hence co–efficent of variation is also 2.

(iii) The number of boys in families follow Poisson distribution.

(iv) Mean and variance for Poisson distribution are equal.

[Ans. : (i) False (ii) False (iii) False (Iv) True]

26. Fill up the blanks :

(i) The mean of Poisson distribution is 1.44. its S.D = ...

(ii) In a Poisson distribution P(0) = ...

(iii) In Binomial distribution when $n \to$ ....., $p \to$ ....., and np = ... it tends to Poisson distribution.

(iv) The number of misprints in pages of a book follows a .... distribution.

[Ans. : (i) 1.2 (ii) e^{-m}, (iii) ∞, 0, constant (iv) Poisson]

27. (a) State probability mass function of Poisson distribution. State its properties.

(b) The probability of a printing mistake in a book of 200 pages is 0.01. Find the probability that –

(1) there is no mistake

(2) there are at the most 2 printing mistakes in the page.

[$e^{-2} = 0.135$]

(c) For a Poisson variate X, if $3P(X = 2) = P(X = 4)$, obtain mean and variance. Also find the $P(X \le 2)$. [$e^{-6} = 0.0025$]

(d) When does Binomial distribution tend to Poisson distribution?

[Guj. Uni., Dec., 2013]

[Ans. : (b) (i) 0.135, (ii) 0.675

(c) Mean = 6, Variance = 6, 0.0625 **(d)** $n \to \infty$ and $p \to 0$]

28. (a) In which situation, Binomial distribution tends to Poisson distribution ? State properties of Poisson distribution.

(b) The probability that a person develops an allergy to a particular injection is 0.2%. Find the probability that (i) exactly 3 persons (ii) more than 3 persons develop allergy from 1000 persons given injection.

(c) Find the expected frequencies from the following data by using Poisson distribution :

x :	0	1	2	3	4	5
f :	142	156	69	27	5	1

($e^{-1} = 0.3680$)

(d) For a Poisson variate $P(x = 0) = P(x = 1) = \frac{1}{k}$, then find the value of k. *[Guj. Uni. Nov., 2014]*

[Ans. : (b) (i) 0.18 (ii) 0.1454; **(c)** 147, 147, 74, 25, 6, 1 **(d)** $m = 1$, $k = e$]

29. (a) State probability mass function of Poisson distribution. State its properties and uses.

(b) Using Poisson distribution obtain the expected frequencies for the following data : [$e^{-0.61} = 0.5434$]

x :	0	1	2	3	4
f :	109	65	22	3	1

(c) For a Poisson variate X, if $2P(X = 1) = 3P(X = 3)$, then find its parameter and $P(X < 2)$ [$e^{-1} = 0.368$, $e^{-2} = 0.135$]

(d) For a Poisson distribution, if mean = 4, then find the $P(x \le 1)$ [$e^{-4} = 0.0183$]

[Guj. Uni., Dec., 2015]

[Ans. : (b) 109, 66, 20, 4, 1 **(c)** m = 2, $P(x < 2) = 0.405$

(d) $P(x \le 1) = 0.0183 + 0.0732 = 0.0915$]

30. (a) When Binomial distribution tends to Poisson distribution ? State properties of Poisson distribution.

(b) For a Poisson variate X, if P(X = 3) = 5 P (X = 5), then find its parameter and P(X ≥ 1).

($e^{-4} = 0.0183$, $e^{-2} = 0.1353$)

(c) The probability of a printing mistake in a book of 300 pages is 0.1 per cent. Find the probability that –

(i) there is no mistake.

(ii) there are at least 2 printing mistake in a page.

($e^{-0.1} = 0.9048$, $e^{-0.3} = 0.7408$)

[Guj. Uni. Nov., 2017]

[Ans. : (b) 0.8647 **(c)** (i) 0.7408 (ii) 0.037]

31. (a) Explain the meaning of Poisson distribution and describe its uses.

(b) There are 100 misprints in a book of 200 pages. Find the probability of : (i) 0 misprint (ii) 1 misprint, and (iii) at the most 2 misprints, if a page is selected at random.

[$e^{-0.5} = 0.6065$]

(c) State the formula to find mean and variance of Poisson distribution.

(d) In which cases does Bionmial distribution follows Possion distribution ?

(e) If for a Poisson distribution $P(x = 2) = P(x = 3)$, find its Mean.

[Guj. Uni., Nov., 2018]

[Ans. : (b) 0.6065, 0.30325, 0.9855625 **(e)** m = 3]

32. (a) State probability mass function of Poisson distribution. State its properties and describe its uses.

(b) The number of mistakes committed by a typist in typing 100 pages are as follows. Using Poisson distribution find the expected frequencies.

No. of mistakes per page	0	1	2	3	4 or more
Number of pages	42	36	14	6	2

[$e^{-1} = 0.368$, $e^{-0.4} = 0.4066$]

(c) Answer the following :

(1) For a Poisson distribution, if mean = 4, find its coefficient of variation.

(2) For a Poisson variable x, its prob. mass function $P(x) = \dfrac{e^{-2.25}(2.25)^x}{x!}$, find its standard deviation.

[Guj. Uni., Oct., 2019]

[Ans. : (b)

No. of Mistake per page	0	1	2	3	4 or more
$p(x)$	0.4066	0.3659	0.1646	0.0494	0.0135
Expected frequency	41	37	16	5	1

(c) (1) C.V. = 50 (2) S.D. = 1.5

33. (a) Give definition of Poisson distribution. Also state its properties.

(b) For a Poisson distribution, if mean = 3, then find $P(x \geq 2)$. ($e^{-3} = 0.049$)

(c) Answer the following :

(1) Mean of a Poisson distribution is 8, then its standard deviation = _______.

(a) 8

(b) $\sqrt{8}$

(c) 8^2

(2) Number of air bubbles in a glass bottle follows _______ distribution.

(a) Normal

(b) Poisson

(c) Binomial

[Guj. Uni., Jan., 2021]

[Ans. : (b) $P(x \geq 2) = 0.804$

(c) (1) (b) $\sqrt{8}$ (2) (b) Poisson]

(B) Hyper-geometric Distribution

1. Introduction :

If an experiment results only in two ways success and failure and if the probability of success remains the same in each trial, then for finding out the probability of x successes out of n independent trials, we use Binomial distribution.

A situation in which the experiment can result in two ways, success and failure, but **the probability of success changes from trial to trial**, hyper- geometric distribution is used for finding the probabilities of different number of sucesses.

2. Hyper-geometric distribution :

If the units of a population can be divided into two categories with respect to some characteristic, i.e. the units possessing that characteristic and the units not possessing that characeristic; and if the units are taken one by one without replacement from that population, then for finding the probabilities of different number of units possessing that characteristic hyper-geometric distribution is used. The following are the conditions for using Hyper-geometric distribution.

(1) The result of each trial should be divided in two categories : success and failure.

(2) The results of different trials should not be independent, i.e., the probability of success changes from trial to trial.

(3) The trials should be repeated for a fixed number of times.

The probability mass function of hyper geometric distribution can be obtained in the following way :

Suppose, there are m white and n black balls in an urn. From these $(m+n)$ balls r balls are taken at random and we want to find the probability of x white balls and $(r-x)$ black balls.

The total number of ways of taking r balls from $(m+n)$ balls $= {}^{(m+n)}C_r$.

Favourable ways of getting x white balls from m white balls and $(r-x)$ blacks balls from n black balls

$$= {}^mC_x \times {}^nC_{(r-x)}$$

$\therefore$ The probability of getting x white balls is

$$P(x) = \frac{\text{Number of favourable ways}}{\text{Total number of ways}}$$

$$\therefore \quad P(x) = \frac{{}^mC_x \times {}^nC_{(r-x)}}{{}^{(m+n)}C_r}, \quad \text{where, } x = 0, 1, 2, ..., r.$$

For finding probabilities of different number of successes i.e. for different values of x the above formula can be used. This probability distribution is called hyper geometric distribution.

3. Properties and uses of hyper-geometric distribution :

Properties :

(1) This is a probability distribution of a discrete variable.

(2) m, n and r are the parameters of the distribution.

(3) Its mean $= \dfrac{mr}{m+n}$.

(4) Its variance $= \dfrac{mnr\,[m+n-r]}{(m+n)^2\,(m+n-1)}$

(5) When $(m+n)$ is very large, hyper geometric distribution tends to Binomial distribution.

Uses :

(1) To find the probability of number of successes, when the probability of success in each trial changes.

(2) This distribution has a wide application in acceptance sampling.

Illustrations :

Illustration 1 : There are 7 red and 4 white balls in an urn. 3 balls are taken from it one after the other. Find the probabilities that (i) 2 balls are red and 1 ball is white (ii) all balls are Red (iii) all balls are of the same colour.

Ans. : In the urn there are 7 red and 4 white balls i.e. 11 balls in all.

(i) The probability of getting 2 red and 1 white ball

$$= \frac{{}^7C_2 \times {}^4C_1}{{}^{11}C_3}$$

$$= \frac{21 \times 4}{165}$$

$$= \frac{28}{55}$$

$$= 0.5091$$

(ii) The probability that all the balls are red

$$= \frac{{}^7C_3}{{}^{11}C_3}$$

$$= \frac{35}{165}$$

$$= 0.2121$$

(iii) All the balls are of the same colour i.e., all the balls are either red or all the balls are white.

∴ Required probability

$$= \frac{{}^7C_3 + {}^4C_3}{{}^{11}C_3}$$

$$= \frac{35 + 4}{165}$$

$$= \frac{39}{165}$$

$$= 0.2364$$

Illustration 2 : A transport company has 8 maruti cars and 6 fiat cars. If five cars are on hire, find the probabilities that of them (1) 3 are marutis and 2 are fiats (ii) at least 3 are marutis (iii) all are marutis.

Ans. : There are 8 marutis + 6 fiats = 14 cars in all.

(i) The probability that 3 marutis and 2 fiats are on hire

$$= \frac{{}^8C_3 \times {}^6C_2}{{}^{14}C_5} = \frac{60}{143}$$

(ii) At least 3 are marutis

i.e, 3 marutis and 2 fiats

OR

4 marutis and 1 fiat

OR

5 marutis and 0 fiat

Required probability

$$= \frac{\left({}^8C_3 \times {}^6C_2\right) + \left({}^8C_4 \times {}^6C_1\right) + \left({}^8C_5 \times {}^6C_0\right)}{{}^{14}C_5}$$

$$= \frac{840 + 420 + 56}{2002}$$

$$= \frac{1316}{2002}$$

$$= 0.6573$$

(iii) The probability that all are maruti cars

$$= \frac{{}^8C_5}{{}^{14}C_5}$$

$$= \frac{56}{2002}$$

$$= 0.02797$$

Illustration 3 : There are 50 screws in a lot; and 2% of them are defective. If a random sample of 20 screws is taken from the lot, find the probability that none among them is defective. Also find the mean and variance of number of defective screws in the sample.

Ans. : There are 50 screws in the lot and 2% of them are defective, i.e, there is only 1 defective screw and 49 non–defective screws in the lot. If a sample of 20 screws is taken from the lot, the probability that none of the screws is defective

$$P(0) = \frac{{}^1C_0 \times {}^{49}C_{20}}{{}^{50}C_{20}}$$

$$= \frac{49\,!}{20!\,29!} \times \frac{20!\,30!}{50!}$$

$$= \frac{30}{50}$$

$$= 0.6$$

The average number of defective screws in the sample = mean

$$= \frac{mr}{m+n}, \quad m = 1, n = 49; r = 20$$

$$= \frac{1 \times 20}{50}$$

$$= 0.4$$

The variance of number of defective screws in the sample

$$= \frac{mnr\,(m+n-r)}{(m+n)^2\,(m+n-1)}$$

$$= \frac{1 \times 49 \times 20 \times (1+49-20)}{(1+49)^2\,(1+49-1)}$$

$$= \frac{980 \times (30)}{(50)^2 \times (49)}$$

$$= \frac{29,400}{1,22,500}$$

$$= 0.24$$

Illustration 4 : There are 10 defective bulbs in a lot of 80 bulbs. From it 8 bulbs are taken one by one. Find the probabilities that (i) 2 bulbs are defective (ii) at the most 2 bulbs are defective.

Ans. : Here, there are 10 defective + 70 non defective = 80 bulbs in all.

(i) The probability that 2 bulbs are defective

$$= \frac{{}^{10}C_2 \times {}^{70}C_6}{{}^{80}C_8}$$

(ii) The probability that at the most 2 bulbs are defective

$$= P(0) + P(1) + P(2)$$

$$= \frac{{}^{10}C_0 \times {}^{70}C_8}{{}^{80}C_8} + \frac{{}^{10}C_1 \times {}^{70}C_7}{{}^{80}C_8} + \frac{{}^{10}C_2 \times {}^{70}C_6}{{}^{80}C_8}$$

Illustration 5 : From a pack of 52 cards 4 cards are drawn one by one. Find the probability of getting at least one ace in them.

Ans. : The probability of getting atleast one ace in 4 cards

$$= P(1) + P(2) + P(3) + P(4)$$

$$= 1 - P(0)$$

$$= 1 - \left[\frac{{}^{4}C_0 \times {}^{48}C_4}{{}^{52}C_4}\right] = 1 - 0.72 = 0.28$$

Illustration 6 : In a lot of 50 items, 4% of the items are defective. A sample of 5 items is taken from it. The lot will be accepted if the number of defective item in the sample is 1 or less. Find the probability that the lot will be accepted.

Ans. : In the given lot of 50 items 4% are defective i.e., 2 items are defective and 48 are non–defective. The lot is accepted if at the most one item is defective in the sample of 5 items.

∴ The probability that the lot is accepted

$$= P(0) + P(1)$$

$$= \frac{{}^{2}C_0 \times {}^{48}C_5}{{}^{50}C_5} + \frac{{}^{2}C_1 \times {}^{48}C_4}{{}^{50}C_5}$$

$$= \frac{396}{490} + \frac{90}{490}$$

$$= \frac{486}{490}$$

$$= 0.9918$$

EXERCISES

SECTION A

❑ **Answer in one line :**

1. Write probability mass function of Hypergeometric distribution.
2. What is the difference between Binomial and Hyper-geometric distribution.
3. State the parameters of Hyper-geometric distribution.
4. State the mean and variance of Hyper-geometric distribution.
5. When Hyper-geometic distribution follows Binomial distribution.
6. Give one use of Hyper-geomtric distribution.
7. In usual notations m = 5, n = 95, r = 5. then find mean of Hyper-geometric distribution.
8. In usual notations m = 10, n = 40, r = 5, then find variance of Hyper-geometric distribution.

SECTION B

1. State the probability mass function of hyper geometric distribution and give its properties and uses.
2. There are 5 red and 7 white balls in an urn. 4 balls are taken one after the other from it. Find the probabilitlies of getting-
   (i) 2 white and 2 red balls,
   (ii) all balls of the same colour.
   [Ans. : (i) 0.4242 (ii) 0.0808]
3. There are 100 mangoes in a basket of them 40 are bad. From the basket 20 mangoes are taken one by one at random. Find the probabilities that of them (i) 2 mangoes are bad (ii) at the most 2 mangoes are bad (iii) at least 2 mangoes are bad.

   [Ans. : (i) $\frac{{}^{40}C_2 \times {}^{60}C_{18}}{{}^{100}C_{20}}$

   (ii) $\frac{{}^{40}C_0 \times {}^{60}C_{20}}{{}^{100}C_{20}} + \frac{{}^{40}C_1 \times {}^{60}C_{19}}{{}^{100}C_{20}} + \frac{{}^{40}C_2 \times {}^{60}C_{18}}{{}^{100}C_{20}}$

   (iii) $1 - \left[\frac{{}^{40}C_0 \times {}^{60}C_{20}}{{}^{100}C_{20}} + \frac{{}^{40}C_1 \times {}^{60}C_{19}}{{}^{100}C_{20}}\right]$]
4. In a modern hotel, there are 28 rooms of which 8 are airconditioned. Assuming that the demands of both the types of rooms are equal and on a particular day 12 rooms are rented. Find the probabilities that of them (i) all the airconditioned rooms are rented (ii) none of the airconditioned room is rented.

   [Ans. : (i) $\frac{{}^{20}C_4}{{}^{28}C_{12}}$ (ii) $\frac{{}^{20}C_{12}}{{}^{28}C_{12}}$]
5. There are 40 screws in a packet of which 5 are defective. If 10 screws are taken at random from the packet, find the probability that none of them is defective. Also find mean and variance of defective screws.
   [Ans. : 0.2166, mean = 1.25, variance = 0.84]
6. There are 50 bulbs in a lot and 10% of them are defective bulbs. 5 bulbs are taken one after the other from it. Find the probabilities that (i) none of the bulbs is defective (ii) at the most 2 bulbs are defective in them. **[Ans. :** (i) 0.5766 (ii) 0.9952]

7. 3 cards are drawn at random from a pack of 52 cards. Find the probabilities that (i) all the cards are of spade (ii) all are aces.
   [Ans. : (i) 0.01294 (ii) 0.0001810]
8. There are 12 Ambassador and 8 Fiat cars with a company. From them 5 cars are in repair in a workshop. Find the probabilities that of these cars (i) there are 3 Ambassadors and 2 Fiats (ii) at least 3 Fiat cars. (iii) all the cars are of the same type
   [Ans. : (i) 0.397 (ii) 0.296 (iii) 0.0547]
9. In the dispensary of Dr. Urvish Shah, 10% patients out of 50 patients are suffering from cold. If 5 patients are inquired randomly, find the probability that not more than 2 of them are not suffering from cold. Also find the average number of patients suffering from cold. **[Ans. :** 0.9952; 0.5]
10. From a pack of 52 cards, 3 cards are selected at radom. Find the probabilities of :
    (i) all 3 cards are of diamond (ii) all 3 cards are of kings.
    [Ans. : (i) 0.01294 (ii) 0.0001810]
11. 4 cards are selected at random from a pack of 52 cards. Find the probability of selecting at most one heart card.
    [Ans. : 0.7426]
12. (a) State probability mass function of Hyper geometric distribution. State its properties.
    (b) There are 5 boys and 7 girls in a group. 4 persons are selected one after the other from it. Find the probability of selecting
    (i) 2 boys and 2 girls (ii) All boys or girls.
    (c) There are 5 defective bulbs in a lot of 50 bulbs. 2 bulbs are taken one after the other from it. Find the probability that none of them is defective. Also find mean and variance of defective bulbs.
    (d) When the probability of success changes from trial to trial which distribution is used ?
    (e) Give two examples of discrete probability distribution.
    [Guj. Uni., Dec., 2013]

[Ans.:

(b) (i) $\frac{42}{99} = 0.4242$ (ii) $\frac{8}{9} = 0.808$

(c) $\frac{198}{245} = 0.8082$; Mean $= \frac{1}{5} = 0.2$, Variance $= 0.1763$

(d) Hyper-geometric distribution

(e) Poission distribution, Hyper-geometric distribution, Negative Binomial distribution and Geometric distribution.]

13. (a) State its properties and uses of Hyper-geometric distribution.

(b) 2% of screws in a lot of 50 screws are defective. If a random sample of 10 screws is taken from the lot, find the probability that all selected screws are non-defective. Also find mean of number of defective screws in the sample.

(c) From a pack of 52 cards, two cards are randomly selected. Find the probability that (i) both are picture card (ii) both are queen.

(d) In a Hypar-geometric distribution, m = 5, n = 45 and r = 15, then find variance.

(Guj. Uni. Nov., 2014)

[Ans.: (b) (i) 0.8 (ii) Mean = 0.2 **(c)** (i) 0.0498 (ii) 0.0045 **(d)** 0.9643]

14. (a) State probability mass function of Hyper-geometric distribution. State its properties.

(b) A basket contains 2 dozen mangoes of which 3 mangoes are rotten. If 4 mangoes are selected from the basket, find the probability of having at least one rotten mango.

(c) There are 40 students in a class out of them 10% students are taking interest in sports. If 3 students are selected one after another from this class, then find the probability that 2 students of them are taking interest in the sports. Also find the mean of students who has interest in the sports.

(d) For Hyper-geometric distribution $m = 4$, $n = 6$ and $r = 2$, then find its mean and variance.

[Guj. Uni., Dec., 2015]

[Ans.:

(b) 0.4368

(c) 0.0219, Mean = 0.3

(d) Mean = 0.8, Variance = 0.4267]

15. (a) State probability mass function of Hypergeometric distribution. State its properties.

(b) A person invites 20 guests for a party which includes 12 males. He slects 4 guests at random to play a certain game.

Find the probability that (i) at least one male is selected, (ii) at the most 2 females are selected.

(c) From a pack of 52 cards, 4 cards are drawn one by one. Find the probability of getting at least one king in them. Also find the mean of king cards.

[Guj. Uni., Nov., 2017]

[Ans. : (b) (i) 0.9856 (ii) 0.8469 **(c)** 0.2813, 0.3077]

16. (a) Explain the meaning of hyper-geometric distribution and describe its characteristics.

(b) There are 50 screws in a packet in which 4% are defective. A sample of 20 screws is taken. Find the probability that not a single screw is defective. Also find mean and variance of defective screws.

[Guj. Uni., Nov., 2018]

[Ans. : (b) 0.1758, 0.8, 0.4702]

17. (a) State probability mass function of Hyper-geometric distribution. State its properties and describe its uses.

(b) From a pack of 52 cards, 3 cards are selected at random. Find the probabilities of (i) all three cards are of clubs (ii) all three cards are of queens (iii) all three cards are of red colour.

(c) For Hyper-geometric distribution $n = 20$, $r = 15$ and $m + n = 30$, then find its mean and variance.

[Guj. Uni., Oct., 2019]

[Ans. : (b) (i) 0.013 (ii) 0.0002 (iii) 0.1176

(c) Mean = 5, Variance = 1.72]

18. (a) Three cards are drawn at random from a pack of 52 cards. Find the probabilities that –

(a) all the cards are of queens

(b) all are heart

(b) State any two properties of hyper-geometric distribution.

[Guj. Uni., Jan., 2021]

[Ans. : (a) (i) $\frac{1}{5525}$ (ii) $\frac{11}{850} = 0.01294$]

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