UNIT - 4

4 DISCRETE DISTRIBUTION-2
(NEGATIVE BINOMIAL AND GEOMETRIC DISTRIBUTIONS)

1. Negative binomial distribution :

In first chapter we studied Poisson distribution and Hypergeometric distribution. We know that the values of mean and variance are equal in Poisson distribution and the value of variance is less than the value of mean in Binomial distribution. We study one more distribution of a discrete variable in which variance is greater than mean. Such a distribution is negative binomial distribution. We shall discuss one illustration to understand negative binomial distribution.

Suppose a person aims a target. Suppose the probability that he will hit the target in any attempt is $p = 0.4$. So the probability that he will not hit the target in any attempt is $q = 0.6$. Here $p + q = 1$.

Suppose we are interested in finding the probability that he will hit the target for the fourth time at the tenth attempt. For this he must have success at the tenth attempt and must have 3 successes in the first 9 attempts.

Hence the probability that he will hit the target fourth time at the tenth attempt = The probability that he will hit the target three times in the first nine attempts × Probability that he will hit the target in the tenth attempt;

$$= {}^{9}C_3\, p^3 q^6 \times p$$
$$= {}^{9}C_3\, p^4 q^6$$
$$= 84\,(0.4)^4\,(0.6)^6$$
$$= 84\,(0.0256)\,(.046656)$$
$$= 0.1003$$

We shall take one more illustration to understand this distribution.

Suppose during an epidemic the probability that a particular insect will bite a person is p; and the probability that it will not bite him is q. Here $p + q = 1$. Suppose the person dies when the insect bites him at the n^{th} time. Suppose we want to find out the probability that the insect will bite the person

for the n^{th} time at $(n + x)^{th}$ trial. The probability that the insect will bite him for n^{th} time at $(n + x)^{th}$ trial = Probability that insect will bite $(n - 1)$ times in $(n + x - 1)$ trials × Probability that it will bite him at $(n + x)^{th}$ trial.

$$= {}^{(n+x-1)}C_{(n-1)}\, p^{n-1} q^x \times p$$
$$= {}^{(n+x-1)}C_{(n-1)}\, p^n \cdot q^x$$

Hence, the probability that the insect will not bite for x times before it bites for the n^{th} time at $(n + x)^{th}$ trial is

$$\therefore P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n \cdot q^x;\ x = 0, 1, 2, \ldots\ldots\ldots$$

This probability distribution is known as negative binomial distribution.

Definition of negative binomial distribution :

If the probability mass function of a discrete random variable x is $P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n q^x$, $x = 0, 1, 2, \ldots\ldots$ then the distribution of x is said to be negative binomial distribution.

Here, $$\sum P(x) = \sum_{x=0}^{\infty} {}^{(n+x-1)}C_{n-1}\, p^n q^x$$

$$= p^n \sum_{x=0}^{\infty} {}^{(n+x-1)}C_{n-1}\, q^x$$
$$= p^n\,(1 - q)^{-n}$$
$$= p^n \cdot p^{-n}$$
$$= p^o$$
$$= 1$$

Thus the total probability = 1

Here the probabilities of different values of the variable x are the different terms of the binomial expansion of $p^n(1 - q)^{-n}$. Hence this distribution is known as negative binomial probability distribution.

The negative binomial distribution originated from the studies of deaths of persons during a particular epidemic. If a person dies only when an insect bites him for the n^{th} time, then the probability that the insect does not bite him for x times before it bites for the n^{th} time can be found out by this distribution.

In negative binomial distribution the trials also follow Bernoulli's conditions. In it (i) Each trial is independent (ii) the probability p of success remains constant in each trial.

2. Properties of negative binomial distribution :

1. This is a distribution of a discrete variable.
2. n and p are the parameters of the distribution.
3. The mean of negative binomial distribution is $\frac{nq}{p}$ which gives the average number of failures before the n^{th} success.
4. The variance of negative binomial distribution $= \frac{nq}{p^2}$
5. In this distribution variance is greater than mean.
6. When $n \to \infty$ and $\frac{q}{p} \to 0$, negative binomial distribution tends to Poisson distribution.
7. When $n = 1$, negative binomial distribution becomes geometric distribution.

Uses of negative binomial distribution :

This distribution is useful in finding out the probability of x failures before n^{th} success in $(n + x)$ trials.

This distribution is useful in studying the probability of a death of a person due to an insect bites during an epidemic.

The distribution is also useful in the study of certain biological problems.

Illustration 1 : The probability that a person can hit a target in any trial is $\frac{2}{3}$. Find the probability that he will hit the target fourth time at the ninth trial.

Ans. : Here, $p = \frac{2}{3}$, $\therefore\ q = \frac{1}{3}$.

$$n + x = 9,\ n = 4,\ x = 5$$

Now $P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n \cdot q^x$

$$\therefore \quad = {}^{8}C_3 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^5$$

$$= 56 \times \frac{16}{81} \times \frac{1}{243}$$

$$= 0.0455.$$

Illustration 2 : 80 per cent of mangoes are sweet in a lot. The quality of the mango can be judged only by its taste. A person needs 5 sweet mangoes. Find the probability that he will get the 5^{th} sweet mango when he tastes 8^{th} mango.

Ans. : Here, $p = \frac{80}{100} = \frac{4}{5};\ q = \frac{1}{5}$,

$$n + x = 8;\ n = 5;\ x = 3$$

Now, $P(x) = {}^{n+x-1}C_{(n-1)}\, p^n\, q^x$

$$= {}^{7}C_4 \left(\frac{4}{5}\right)^5 \left(\frac{1}{5}\right)^3$$

$$= 35 \times \frac{1024}{3125} \times \frac{1}{125}$$

$$= 0.0918$$

Illustration 3 : The probability of getting head when a coin is tossed is $\frac{1}{2}$. A person tosses a coin continuously. Find the probability of getting 6^{th} head at the 10^{th} trial. Also find the mean and variance of number of tails before getting the 6^{th} head.

Ans. : Here, $p = \frac{1}{2},\ q = \frac{1}{2}$,

$$n + x = 10;\ n = 6;\ x = 4$$

$$P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n q^x$$

$$\therefore \quad P(4) = {}^{9}C_5 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$$

$$= \frac{126}{1024}$$

$$= 0.123$$

Now the average number of tails before getting the 6^{th} head

$$\text{Mean} = \frac{nq}{p} = \frac{6 \times \frac{1}{2}}{\frac{1}{2}} = 6$$

$$\text{Variance} = \frac{nq}{p^2} = \frac{6 \times \frac{1}{2}}{\frac{1}{4}} = 12$$

Illustration 4 : A die is thrown and obtaining 5 on the die is regarded as success. Find the probability of getting second success at the 6th trial.

Ans. : Here, $p = \frac{1}{6}; q = \frac{5}{6}$

$n + x = 6, n = 2, x = 4$

$$P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n q^x$$

$$\therefore \quad P(4) = {}^5C_1 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4$$

$$= 5 \times \frac{1}{36} \times \frac{625}{1296}$$

$$= 0.067$$

Illustration 5 : The probability that a person can hit a target is 0.6. He is to be given a prize when he hits the target for the 4th time. Find the probability that he will require more than 8 trials to obtain the prize.

Ans. : Here, $p = 0.6; q = 0.4; n = 4, n + x > 8$

$$\therefore \quad n + x \geq 9$$

$$\therefore \quad 4 + x \geq 9$$

$$\therefore \quad x \geq 5$$

Here $P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n q^x$

$$= {}^{(4+x-1)}C_{4-1}\, p^4 q^x$$

$$= {}^{(3+x)}C_3 (0.6)^4 (0.4)^x$$

$\therefore$ $P(x \geq 5)$

$= P(5) + P(6) + P(7) + ...$

$= 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$= 1 - \left[{}^3C_3 (0.6)^4 (0.4)^0 + {}^4C_3 (0.6)^4 (0.4)^1 + {}^5C_3 (0.6)^4 (0.4)^2 + {}^6C_3 (0.6)^4 (0.4)^3 + {}^7C_3 (0.6)^4 (0.4)^4\right]$

$= 1 - (0.6)^4 \left[1 \times 1 + 4 (0.4) + 10 (0.4)^2 + 20 (0.4)^3 + 35 (0.4)^4\right]$

$= 1 - (0.6)^4 \left[1 \times 1 + 4 \times 0.4 + 10 \times 0.16 + 20 \times 0.064 + 35 \times 0.0256\right]$

$= 1 - 0.1296 \left[1 + 1.6 + 1.6 + 1.28 + 0.896\right]$

$= 1 - 0.1296$ [6.376]

$= 1 - 0.8263$

$= 0.1737$

Illustration 6 : A person draws a card one after the other from a pack of 52 cards. Each time the card is replaced before the next draw. Find the probability that he will get 3rd spade at the 8th draw. Also find the mean and variance of the number of failures before the 3rd spade is drawn.

Ans. : Here, $p = \frac{1}{4}; q = \frac{3}{4}$

$n + x = 8; n = 3; x = 5$

$$P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n \cdot q^x$$

$$\therefore \quad P(5) = {}^7C_2 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^5$$

$$= 21 \times \frac{1}{64} \times \frac{243}{1024}$$

$$= 0.0779$$

The average number of failures before the 3rd spade is drawn $= \frac{nq}{p} = \frac{3 \times 3/4}{1/4} = 9$

Variance $= \frac{nq}{p^2} = \frac{3 \times 3/4}{\frac{1}{16}} = 36$

3. Geometric distribution :

We have seen that the probability mass function of negative binomial distribution is $P(x) = {}^{(n+x-1)}C_{(n-1)}\, p^n q^x$

If we put $n = 1$ in this distribution we get the following probability function

$$\therefore \quad P(x) = {}^xC_0\, p^1 q^x$$

$$= p \cdot q^x$$

$$= q^x \cdot p$$

This probability distribution gives the probability of number of failures x before the first success. Taking values of x as 0, 1, 2, ... the different probabilities $p, qp, q^2p, q^3p, ...$ can be obtained from the above distribution. These probabilities are the different terms of the geometric series and hence the distribution is known as Geometric distribution.

Definition : If the probability mass function of a discrete random variable x is $P(x) = q^x \cdot p$, $x = 0, 1, 2, ...$ then the distribution of x is said to be geometric distribution.

Here, $$\sum P(x) = \sum_{x=0}^{\infty} q^x p$$

$$= p + qp + q^2p + ...$$

$$= \frac{p}{1-q} = \frac{p}{p} = 1$$

Here the variable x denotes the number of failures before the first success.

4. Properties of geometric distribution :

1. This is a distribution of a discrete variable.
2. p is a parameter of the distribution.
3. The mean of the distribution $= \frac{q}{p}$ which shows the average number of failures before the first success.
4. Variance of this distribution $= \frac{q}{p^2}$
5. Variance is greater than mean in this distribution.

Uses of geometric distribution :

This distribution is useful for finding the probability of number of failures before getting first success in trials following Bernoulli's conditions.

Illustration 7 : The probability that a person can hit a target in any trial is 0.7. Find the probability that he will hit the target for the first time at the 4th trial.

Ans. : Here, $p = 0.7$, $q = 0.3$, $x = 3$

$$P(x) = q^x p$$

$$\therefore \quad P(3) = (0.3)^3 (0.7)$$

$$= 0.0189$$

Illustration 8 : The probability of getting head when a coin is tossed is 0.5. Find the probability of obtaining first head at the third trial.

Ans. : Here $p = 0.5$, $q = 0.5$, $x = 2$

$$\therefore \quad P(x) = q^x p$$

$$\therefore \quad P(2) = (0.5)^2 (0.5)$$

$$= 0.125$$

Illustration 9 : A die is thrown and getting 5 is regarded as success. Find the probability that more than 4 trials will be required before getting first success.

Ans : Here, $p = \frac{1}{6}, q = \frac{5}{6}$;

$$n + x \geq 5$$

$$n = 1$$

$$\therefore x \geq 4$$

$$\therefore \quad P(x \geq 4) = \sum_{x=4}^{\infty} P(x)$$

$$= 1 - \left[\sum_{x=0}^{3} P(x)\right]$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$\therefore \quad P(x \geq 4) = 1 - [p + qp + q^2p + q^3p]$$

$$= 1 - \left[\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{25}{36} \times \frac{1}{6} + \frac{125}{216} \times \frac{1}{6}\right]$$

$$= 1 - \left[\frac{216 + 180 + 150 + 125}{1296}\right]$$

$$= 1 - \left[\frac{671}{1296}\right]$$

$$= 1 - 0.5177$$

$$= 0.4823$$

Illustration 10 : Assuming that the probability that a male child is born to a woman is $\frac{1}{2}$, find the probability that the 4th child born to a woman is a first male child. Also find the mean and variance of number of girls born before the first male child is born.

Ans. : Here, $p = \frac{1}{2}; q = \frac{1}{2}, x = 3$

$$P(x) = q^x p$$

$$\therefore \quad P(3) = \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)$$

$$= \frac{1}{16}$$

The average number of girls before the first male child is born

$$= \frac{q}{p} = \frac{1/2}{1/2} = 1$$

$$\text{Variance} = \frac{q}{p^2} = \frac{1/2}{1/4} = 2$$

EXERCISE

SECTION - A

Answer in one line :

1. Give probability mass function of negative binomial distribution.
2. Give the relation between mean and variane between negative binomial distribution.
3. What are the parameters of negative binomial distribution ?
4. What is the mean and variance of negative binomial distribution ?
5. When negative binomial distribution follows poisson distribution ?
6. Give one use of negative binomial distribution.
7. What is the relation between negative binomial and geometric distribution.
8. Give probability mass function of geometric distribution.
9. What is the parameter of geometric distribution ?
10. Give mean and variance of geometric distribution.
11. Give use of geometric distribution.

SECTION - B

1. Give the probability mass function of negative binomial distribution. State its properties and uses.

2. When does negative binomial distribution become geometric distribution ? State the probability mass function, properties and uses of geometric distribution .
3. A person aims a target. The probability that he will hit the target in any trial is 0.2. Find the probability that he will hit the target third time at the eighth trial. **[Ans. : 0.0551]**
4. The probability of getting head when a coin is tossed is 1/2. Find the probability that a person will get 5^{th} head at the 12^{th} trial. **[Ans. : 0.0806]**
5. Ninety percent of mangoes are sweet in a lot. The quality of a mango can be known only by its taste. A person requires 6 sweet mangoes. Find the probability of getting 6^{th} sweet mango when 8^{th} mango is tasted. **[Ans. : 0.1116]**
6. The probability that a potato is stale in a lot of potatoes is 1/4. A person requires 5 good potatoes. Find the probability that he will get 5^{th} good potato when he inspects 9^{th} potato. **[Ans. : 0.0649]**
7. A die is thrown and an even number is regarded as success. Find the probability that 6^{th} success is obtained at the 10^{th} throw. **[Ans. : 0.1230]**
8. In an objective test 3 alternatives are given in each question and of them only one is correct. A student does not know correct answer of any question. Hence he ticks any one of the answers at random. Find the probability that his 3^{rd} answer will be correct at the 8^{th} trial. **[Ans. : 0.1024]**
9. The probability that any bulb is defective in a lot is 0.1. Whether a bulb is defective or not can be known only by testing it. A person requires 7 good bulbs. Find the probability that he will get 7^{th} good bulb on testing 9^{th} bulb. **[Ans. : 0.1339]**
10. The probability that a cricketer can hit a six on any ball is 1/3. Find the probability that he will hit 3^{rd} six on the 6^{th} ball. **[Ans. : 0.1097]**
11. The probability that a person can hit a target is 0.8. He gets a prize when he hits the target 4^{th} time. Find the probability that he will require more than 7 trials to get the prize. **[Ans. : 0.0333]**
12. A person draws cards one after the other with replacement. Find the probability that he will get 3^{rd} spade at the 7^{th} draw. Also find mean and variance of number of failures before getting 3^{rd} spade. **[Ans. : 0.0742, mean = 9, Variance = 36]**

13. The probability that a person can hit a target is 0.6. Find the probability that the will hit the target first time at the fifth trial. [**Ans. :** 0.01536]

14. Find the probability of getting number 6 for the first time at the fourth attempt when a die is thrown. Also find mean and variance of number of failures before getting number 6.

[**Ans. :** 0.0965, mean 5, Variance = 30]

15. Fifty percent of mangoes are sweet in a lot. Find the probability that the first sweet mango will be obtained when the 3rd mango is tasted.

[**Ans. :** $\frac{1}{8}$]

16. Cards are drawn one after the other with replacement from a pack of 52 cards. Find the probability of getting a king for the first time at the 7^{th} attempt. [**Ans. :** $\frac{(12)^6}{(13)^7}$]

17. 80 per cent of the bulbs are defective in a big lot of bulbs. Bulbs are inspected one after the other from the lot. Find the probability that the first non-defective bulb will be obtained on testing the tenth bulb. Also find mean and variance of number of defective bulbs drawn before getting the first non-defective bulb.

[**Ans. :** 0.0268, mean = 4, variance = 20]

18. How a geometric distribution can be obtained from a negative binomial distribution ? Give probability mass function, properties and uses of geometric distribution.

19. In a big lot of mangoes 90% of mangoes are good. The quality of mango can be judged only by its taste. Find the probability that 6 or more mangoes are required to be tasted for getting 4 good mangoes.

[**Ans.:** 0.08146]

20. A die is thrown. Find the probability of getting 1 for the first time at the fourth attempt. Also find the mean and variance of number of failures before obtaining 1 for the first time.

[**Ans.:** $\frac{125}{1296}$, Mean = 5, Variance = 30]

21. In an objective test, there are three alternatives answers of each question and one of them is correct. A student does not know correct answer of any of the questions and hence he write the answers by guess. Find the probability that he will be required to answer more than 7 questions to get third correct answer. [**Ans. :** 0.5706]

22. The probability that a person can hit a target many trial is 0.8. Find the probability that he will hit the target second time at the fourth trial.

[**Ans.:** 0.0768]

23. For a negative binomial distribution mean is 9 and variance is 36. Find its parameters. [**Ans. :** $n = 3$, $p = \frac{1}{4}$, $q = \frac{3}{4}$]

24. For a geometric distribution mean is 5 and variance is 30. Find its parameters. [**Ans. :** $p = \frac{1}{6}$, $q = \frac{5}{6}$]

25. (a) Write properties of Negative Binomial Distribution.
    (b) A person draws a card one by one with replacement from a pack of 52 cards. Find the probability of getting third spade card when he draws eight card. Also find mean and variance of number of failures before it..
    (c) The probability that any person hit a target in any trial is 0.8. Find the probability that in eighth trial he hit the target first time ?

[**Ans.:** (b) 0.0779, Mean = 9, Variance = 36 (c) 0.00001024]

26. **(a)** State the probability mass function of negative binomial distribution. State its properties and uses.

(b) The probability that a person can hit a target is $\frac{1}{2}$. Find the probability that he will hit the target 6th time at the tenth trial. Also find the mean and variance of number of failures before 6th trial.

(c) In an objective test 4 alternatives are given in each question and of them only one is correct. A student does not know correct answer of any question. Hence he ticks any one of the answers at random. Find the probability that his 3rd answer will be correct at the 6th trial.

(d) Mean and variance of negative binomial distribution are 8 and 24 respectively. Find its parameters.

[Guj. Uni., Dec., 2013]

[**Ans.:** **(b)** $\frac{126}{1024} = 0.123$ Mean = 6 and Variance = 12

(c) $\frac{270}{4096} = 0.0659$ **(d)** $p = \frac{1}{3}$ and $n = 4$]

27. **(a)** State the probability mass function of geometric distribution. State its properties and uses.

(b) The probability that an apple is sour in a lot is $\frac{3}{4}$. Find the probability that a person gets first sweet apple when fifth apple is tested.

(c) The probability that a cricketer can hit a six on any ball is $\frac{1}{3}$. Find the probability that he will hit first six on the sixth ball.

[Guj. Uni., Dec., 2013]

[Ans. : (b) P(4) = 0.0791 **(c)** P(5) = 0.04389**]**

28. **(a)** State the probability mass function of geometric distribution. State its properties and uses.

(b) State the probability mass function of negative binomial distribution. State its properties and uses.

(c) The probability that a person can hit a target in any trial is 0.7. Find the probability that a person can hit the target third time at the eighth trial.

(d) In a lot of apples there are 20% are sour. Find the probability that a person gets third sweet apple when fifth apple is tested.

(e) In an objective test 3 alternatives are given in each question and of them only one is correct. A student ticks any one of the answers at random. Find the probability that his first answer will be correct at fourth trial.

(f) A person tosses an unbiased coin continuously. Find the probability of getting 1st head at the 9th trial. Also find the mean and variance of number of failure before getting 1st head.

(g) In a Negative Binomial distribution if mean & variance are 24 & 96 respectively, then find its parameters.

(h) For geometric distribution if probability of success is 1/4, then find its mean & variance.

[Guj. Uni., Dec., 2015]

[Ans. : (c) 0.0175 **(d)** 0.1229 **(e)** 0.0988 **(f)** 0.0020, Mean = 1, Variance = 2 **(g)** P = $\frac{1}{4}$ n = 8, **(h)** Mean = 3, Variance = 12**]**

29. **(a)** Explain probability function of a Negative Binomial distribution and describe its properties.

(b) Explain probability function of Geometric distribution and give its principles.

(c) Ten percent of potatoes are stale in a lot of potatoes. A person require 6 good potatoes. Find probability of getting 6th good potato when he inspect 8th potato. Also find mean and variance of number of failures before getting 6th good potato.

(d) 20 percent of bulbs are non-defective in a lot of bulbs. They are inspected one after the other. Find the probability that the first non-defective bulb will be obtained on testing the tenth bulb. Also find mean and variance of number of defective bulbs drawn before getting the first non-defective bulb.

(e) The mean and variance of a negative binomial distribution are 6 and 12. Find its parameters.

(f) The probability that a person can hit a target in any trial is 0.6. Find the probability that a person can hit the target 4th time at the eight trial.

(Guj. Uni. Dec., 2016)

[Ans. : (c) 0.1116, 0.67, 0.79 **(d)** 0.2684, 4, 20 **(e)** $\frac{1}{2}, \frac{1}{2}$, 6 **(f)** 0.1161**]**

30. **(a)** State probability mass function of Negative Binomial distribution. Write its properties.

(b) Under which condition Negative Binomial distribution is Geometric distribution ? Write its probability mass function and properties.

(c) In a lot of mango probability that mango is rotten is 0.3 A person requires 5 good mango. Find the probability that a person gets fifth good mango when he inspect ninth mango. Also find mean and variance of defective mango.

(d) In an objective test, 4 alternatives are given in each question and of them only one is correct. A student does not know correct answer of any questions. Hence he ticks any one of the answers at random. Find the probability that he will attempt more than 7 questions for getting third correct answer.

(e) For negative binomial distribution mean is 6 and variance is 24. Find its parameters.

(f) A die is thrown and getting 4 is regarded as success. Find the probability that more than 4 trials will be required before getting first success. *(Guj. Uni. Dec., 2017)*

[Ans. : (c) 0.0953, 2.14, 3.06, **(d)** 0.7563 **(e)** $\frac{1}{4}$, 2 **(f)** 0.4823]

31. (a) Give the probability mass function of negative binomial distribution. State its properties and uses.

(b) The probability that a person can hit a target is 2/3. Find the probaility that he will hit the target 5th time at ninth trial. Also find the mean and variance of numbers of failures before 5th trial.

(c) Under which condition negative binomial distribution tends to geometric distribution ? Give the probability mass function of geometric distribution. State its properties and uses.

(d) A die is thrown and getting '3' is regarded as success. Find the probabilty that more than 4 trials will be required before getting first success.

(e) For a negative binomial distribution mean is 6 and variance is 12. Find probability of success.

(f) When negative binomial distribution follows Poission distribution ?

(g) What is the difference between binomial distribution and negative binomial distribution ?

(h) For Geometric distribution mean is 3 and variance is 12, find probability of success.

(i) Negative binomial distribution is a distribution of which type of random variable ? *(Guj. Uni. Nov., 2018)*

[Ans. : (b) 0.1138 **(d)** 0.4823 **(e)** $\frac{1}{2}$ **(h)** $\frac{1}{4}$ **(i)** Discrete]

32. (a) State properties and uses of negative binomial distribution.

(b) The probability that a person can hit a target in any trial is 0.8. When he hits the target 4th time, he gets a prize. Find the probability that he will required 7th or 8th trial to win the prize.

(c) Write probability function of geometric distribution and also give its properties.

(d) The probability that cricketer Virat Kohli can hit a six on any ball is 3/4. In the match against South Africa, find the probability that he will required more than '5' balls to hit the first six.

(e) Answer the following :

(i) Under which condition negative binomial distribution follows geometric distribution.

(ii) If mean and S.D. of a geometric distribution are 20 and 10 respectively, find parameter of geometric distribution.

(iii) If 3rd success is obtained at the 8th trial in negative binomial distribution, find the number of failures. *(Guj. Uni. Nov., 2019)*

[Ans. : (b) Required Probability = P(3) + P(4)
= 0.0655 + 0.023 = 0.0885

(d) 0.001 **(e)** (i) When $n = 1$ negative binomial distribution becomes geometric distribution

(ii) P = 1/5, q = 4/5 (iii) $n + x = 8$, $x = 5$]

33. (a) State probability mass function of negative binomial distribution. Write its properties.

(b) A person draw a card one after the other from a pack of 52 cards and placed it back in the pack every time. Find the probability that he will get 3rd club at the 8th draw. Also find the mean and variance of the number of failures before the 3rd club is drawn.

(c) A die is thrown and getting '4' is regarded as success. Find the probability that more than 4 trials will be required before getting first success.

(d) Answer the following :

(i) State any two properties of geometric distribution.

(ii) In negative binomial distribution if mean = 24 and variance = 96, find its parameters. *(Guj. Uni. Jan., 2021)*

[**Ans. : (b)** Required Prob. = 0.0779, Mean = 9, Variance = 36
(c) Required Prob. = 0.4823 **(d)** (ii) $n = 8$, $p = 1/4$, $q = 3/4$]

34. (a) Write the properties of negative binomial distribution.
The probability that a person can hit a target in any trial is 0.7. Find the probability that he will hit the target fourth time when he tries for ninth time. Also find the mean and standard deviation.

(b) Write the properties of Geometric distribution.
From a pack of 52 cards, one card is selected at random one by one using with replacement. What is the probability that it needs more than 3. Find the variance if the probability of success in any trial is 0.4.

(c) Answer the following :

(1) The mean of a negative binomial distribution is 3. Find the variance if the probability of success in any trial is 0.4.

(2) For a negative binomial distribution, mean ..... variance and for a geometric distribution, mean ..... variance.

(a) (>, <) (b) (<, <)
(c) (>, >) (d) None of these

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[**Ans. : (a)** Required Probability = 0.0327, Mean = 1.71, S.D. = 1.57
(b) Required Probability = 1 – [P(0) + P(1) + P(2)] = 0.7865
(c) (1) S.D. = 2.74 (2) (b) (<, <)]

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