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TEACHINGOLOGY

It's Off the Screen: Unearthing Megagons Through Technology

Mathematical action technology can foster equitable student discourse. Students engage in cycles of proof to create, test, and revise conjectures through dynamic exploration of the Pythagorean theorem.

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Technology will not inherently make students in any classroom more mathematically inquisitive. However, the ways we use technology can foster student-centered inquiry. Let's explore the impact technology has on mathematics learning when it emphasizes the ways it supports student-centered pedagogies. Through this lens, the authors focus on implementing tasks that promote reasoning and problem solving (National Council of Teachers of Mathematics [NCTM], 2000, 2014) through the mathematical action technology of GeoGebra and its opportunity for dynamic manipulation of shapes to support the lesson's goals, including mathematical reasoning and conversations, as students explore the Pythagorean theorem. Because the technology alleviates area calculations, the tasks promote equal opportunity by leveling the field for students to access the standard. Embedded into the task's design, GeoGebra supports promoting cycles of proof: creating, testing, revising, and proving mathematical conjectures

(Cullen et al., 2020) because students use the technology to test and revise their initial conjectures on the validity of a Pythagorean theorem extension for similar figures.

The GeoGebra activity starts with an image (see Figure 1) representing how the sum of the areas of two smaller squares equals the area of the larger square formed from the sides of a right triangle. The two tasks extend the Pythagorean theorem to regular polygons and then similar irregular figures, thus inviting mathematical discourse that encourages students to explore and prove their conjectures through reasoning and problem solving. The following questions guide the two tasks:

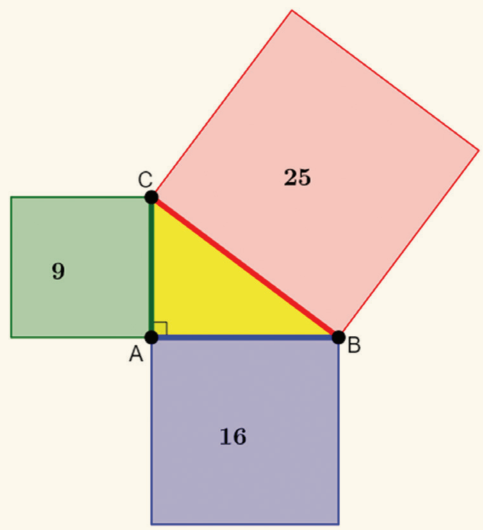
1. What if the shapes are not squares? Would the Pythagorean theorem still hold?
2. What if the shapes are not regular? Would the Pythagorean theorem still hold?

The teachers explicitly ask these two questions to frame students' discourse, and the remainder of teacher questions are assessing or advancing. The majority of the class time is given to students to discuss the mathematics in small groups. The questions promote a low-floor/high-ceiling atmosphere that elicits informal mathematical thinking and reasoning that can be linked to formalized mathematics.

The primary mathematical conveyance technology is a student recording sheet (Google Document and Google Form examples are available in the supplemental materials online), allowing the teacher to monitor students' progress. As an example, the guiding questions for Task 1 are as follows:

a. What if the shapes are *not* squares? Would the Pythagorean theorem still hold?
b. When the shapes are *not* squares, do the areas still follow the rule Area(a) + Area(b) = Area(c)?
c. What are your conclusions?
d. Are you convinced? Why or why not?
e. Insert a screenshot of a design that convinced you here:
f. For what types of polygons does the Pythagorean theorem appear to hold?

Figure 1 Geometrical Representation of the Pythagorean Theorem



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In this way, *all* students have a voice in making their own conjectures, playing with the technology to test and revise their conjectures, and providing mathematical evidence to support their claim. The teacher circulates around the room during small-group discussions to listen, monitor student progress, and respond to students' thinking to scaffold or extend, elevate their discussions to the whole group when appropriate, and use the written data as artifacts for assessments.

Technology can elicit breakthroughs for students, but the true beauty of technology is when it fosters conversations that would not have occurred without it. Picture a classroom in which students are successful only if they can perform previously learned calculations and procedures quickly. If the calculations are not the goal in a lesson, why have students do them? Now imagine a classroom in which prerequisite skills unrelated to the goals of the lesson are removed as barriers. The authors chose the GeoGebra technology *not* on the basis of the engagement with the technology itself, but instead based on how GeoGebra, along with the intentional facilitation of student discourse, affords students equality in conversations about the mathematics (NCTM, 2014) and the opportunity to explore conjectures and develop a positive mathematical identity while fostering agency through cycles of proof.

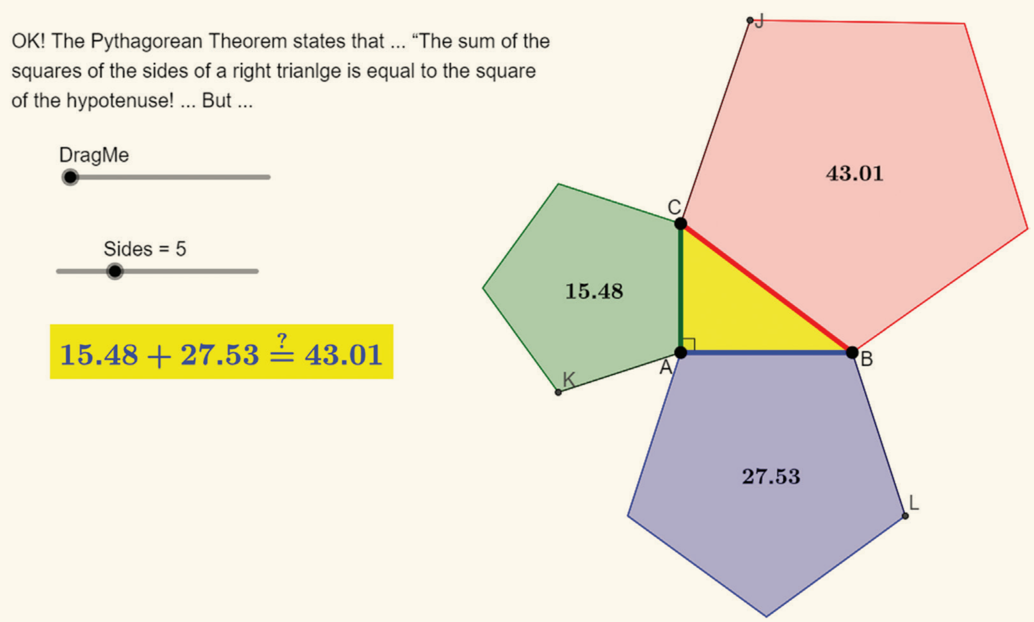
OVERVIEW OF THE PYTHAGOREAN THEOREM LESSON

The lesson (available in the online supplemental materials) begins with an introduction of the legends and cult of the Pythagoreans in a Vi Hart (2022) video. The subsequent whole-class discussion helps students reconstruct the Pythagorean theorem, concluding with a check for student understanding using the visual from Figure 1.

In Task 1, students use the GeoGebra activity Pythagorean Polygons (link online) to explore what happens if the shapes are not squares, which allows students to move a slider and examine different regular polygons (see Figure 2). In groups of three or four, students make conjectures to see if the Pythagorean theorem still holds, clarifying for students that they are investigating whether the sum of the areas of the two smaller shapes equals the area of the largest shape. The technology allows students to play and even peek over the edge of a standard Pythagorean theorem lesson to see deeper, more advanced mathematical connections.

By the end of Task 1, students recognize that the Pythagorean theorem seems to hold for regular polygons other than just squares. They verify a previously learned concept that $a^2 + b^2 = c^2$ is an algebraic way to relate the lengths of the sides of the triangle, rooted in the geometric discoveries through GeoGebra, thus expanding

Figure 2 Pythagorean Polygons



the reasoning and proof past squares. The students also realize that *any* regular polygon works, which embeds a review of the definition of a *regular polygon*. Their exploration expands the notion of why this works past squares, thus exploring the rigidity of a single proof or conjecture.

In Task 2, the teacher asks students, “What if it is *not* a regular polygon? What if it is a shape like this?” Then the teacher displays the shape seen in Figure 3 as students access the Organic Pythagorean Theorem task (link online). Students interact with the technology and one another in the same groups they used in Task 1. By the end of Task 2, students are making conjectures on what they think is true about the three figures to make the Pythagorean theorem still hold, invoking the definition of similarity and criteria for similar figures.

Classroom data were collected from eight classrooms. The first author taught five classrooms of students in Grades 9–12 who were repeating algebra in a public California high school. The second author taught three classrooms of Grade 8 general education students in an Illinois public junior high school. In all eight classrooms, students’ ability to engage in mathematical discourse and wonderment are evident in the vignettes below, and the dynamic nature of the technology and the teachers’ facilitation showcased the students’ strengths of mathematical argumentation that otherwise would not have been observable had they been given just a static activity sheet. The audio recording is available as a link online. The transcript is available in the online supplemental materials.

IT’S OFF THE SCREEN: TESTING AND REVISING AS A MEANS OF PROOF

As students enter the California classroom for first period during the first week of the school year, the teacher welcomes them with the Vi Hart video. The teacher asks reflective questions about the Pythagorean theorem, and students comment, “Pythagorus was really afraid of beans?”

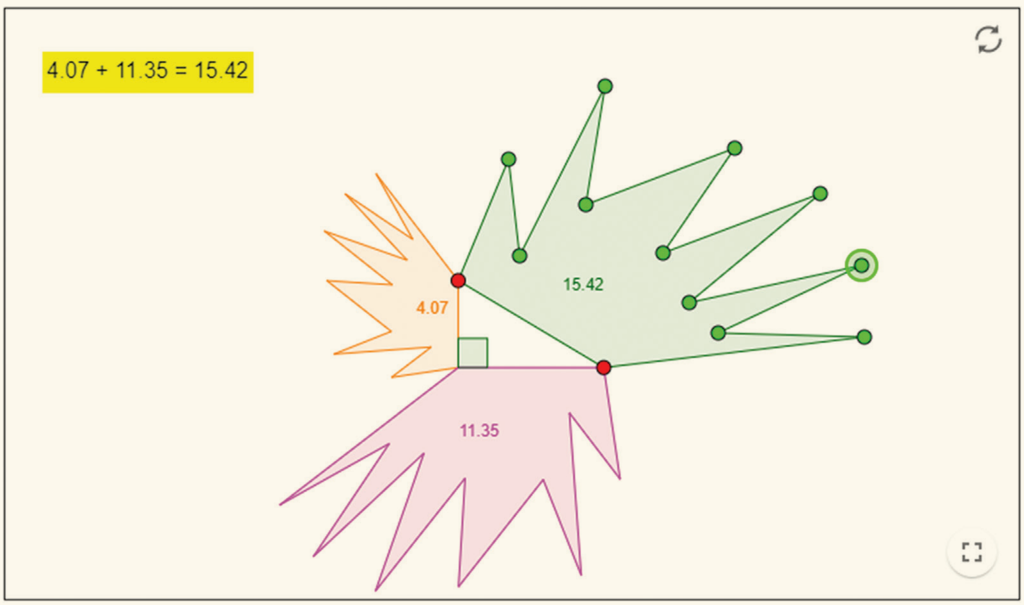
“Who did he kill?”

“Did he get caught?”

The students transition to group work, exploring Task 1, using inductive iterations to explore their conjectures that the Pythagorean theorem does not work for squares of all sizes and it definitely will not work for any shape other than squares.

One student, Maria, captures the class’s view, saying, “Pythagoras said it works with squares, and that’s what he used it for. So that’s the only thing it will work on.” Thinking about how the students engage in the proof cycle, their initial conjecture has now been created; so now they explore and must revise their conjecture on the basis of the data they generate with the technology.

Figure 3 Initial Figure for the Organic Pythagorean Theorem



As the teacher walks by a group of four students, he listens in on the conversation. Marcus, Maria, Tony, and Edgar are testing conjectures they made. They had already explored squares and triangles, but some in the group were uncertain that the pattern of the Pythagorean theorem would hold for other regular polygons such as octagons or pentagons. Some students in the group had accepted the original conjecture that the Pythagorean theorem would hold only with squares, so now the group stood divided. Maria thought the pattern would hold for any regular polygon, but Marcus and Edgar were not yet convinced.

During their exploration, one group was convinced the Pythagorean theorem proof would hold for any regular polygon. The teacher glanced at a Chromebook from the group of four students exploring nonagons, then walked away. Then he heard Maria say, "Whoa, wait, look at this, Nank; check it out. Look at this shape! Bigger works, smaller works; this sucker's off the screen, and it still works!"

Teacher: Wait, what? Show me.

Tony: OK, look here, this whatever you call it [nonagon], it has a lot of sides; it's off the screen, and it's still working!

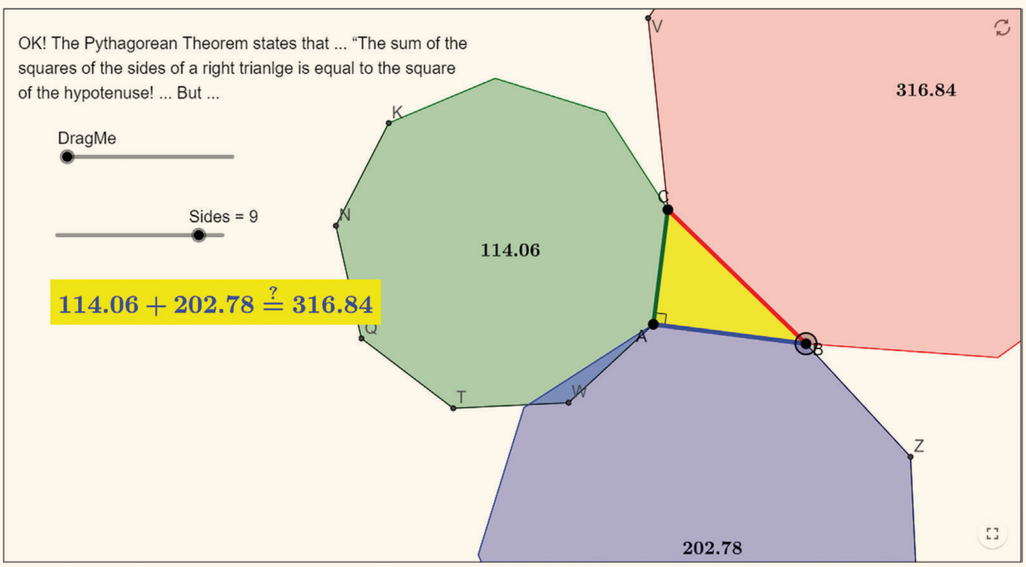
Teacher: Whoa, it is off the screen, you can barely see some of this one. So, what do you think? Does it work for other regular polygons?

After further interaction and although this group of students' repeated testing of different size regular polygons does not constitute a formal proof, they have come to a consensus that they have seen a sufficient number of cases to change their original conjecture, providing them the mathematical agency to determine what constitutes a convincing argument. Their upload of the nonagon is Figure 4.

The teacher synthesizes the learning with a whole-class discussion and then moves on to Task 2. Maria says, "Different shape, bigger, smaller, bigger; like, how many examples do we need? There's hundreds of them! It works!" She demonstrates to her friends that as long as the figures remained similar, the Pythagorean theorem still held true. Opportunities like these—when students can feel confident in their conjectures as they convince others—allow for positive mathematical identities to be cultivated. The original conjecture from the students is that the Pythagorean theorem will not hold for irregular shapes, but through explorations similar to Task 1, most students are convinced otherwise, using representations such as that in Figure 5.

According to Dick and Hollebrands (2011), "Technology-based learning scenarios should allow students to take *deliberate, purposeful,* and *mathematically meaningful actions* and provide *immediate, perceptible* (usually visual) and *mathematically meaningful*

Figure 4 It's Off the Screen



consequences to those actions" (p. xiv, emphasis in original). The activities students embarked on above gave them the immediate visuals that informed the meaning they made of mathematics while influencing their discussions, conjectures, and conclusions. Even though formally proving all cases was beyond the scope of this activity, the mathematical discourse that the technology elicited afforded students like Maria the opportunity to present a valid argument to her classmates, increasing her mathematical agency.

THE BEAUTY OF TEACHERS TRUSTING STUDENTS: AN EPILOGUE TO IT'S OFF THE SCREEN

The California day concludes with students in fifth period exploring the tasks. Because the classes included integration of students with disabilities and English learners, Ms. Garcia is in the classroom to support students. Ms. Garcia sits with a group of students to work with them. She runs excitedly to the teacher and says, "So, I'm working with students, and they made a pentagon! And wait, it worked. But it's only supposed to work with squares. I've only seen teachers do this with squares over the years."

The teacher replies, "This is cool! I mean, I can't tell you how many times I've looked at stuff I learned before and I thought to myself, 'I wish I had seen this before!'"

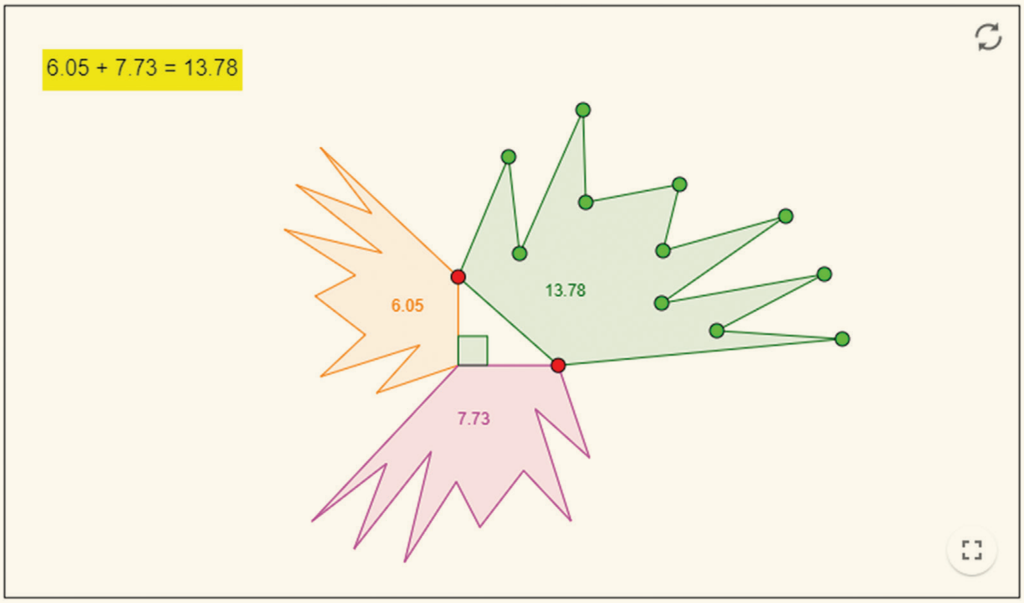
Expressing the beauty and excitement both Ms. Garcia and the teacher felt at this moment is important. Later, the two conversed about how the best lessons are those in which we as adults learn to think of mathematics in a way we never had thought about before. If you trust and believe in the students, then they will show you things you never imagined because we were taught mathematics without an inquiry-based approach.

MEGAGONS AND ZEROS: ITERATIONS APPROACHING INFINITY

In the Illinois junior high school eighth-grade class, the teacher begins with the introduction of the Pythagorean legends, and then the students work in small groups to investigate if the Pythagorean theorem still holds for shapes other than squares by adjusting the GeoGebra figures to test and revise their conjectures in their cycles of proof. In the whole-group synthesis, the teacher asks what types of polygons the Pythagorean theorem holds for. Students respond:

Demitri: A megagon.
Teacher: What's that?
Demitri: A megagon.
Teacher: A megagon?

Figure 5 Irregular Explorations of the Pythagorean Theorem



Demitri: One, one trillion sides!
Andrea: What does that mean?
James: What's a megagon?
Teacher: Ooh, I'm liking where you're going. I think you mean it would work even if there were a trillion sides?
Demitri: I got the biggest amount of sides!

Although this GeoGebra investigation was coded to have a limit on the number of sides, the students are visualizing how the number of sides could potentially increase infinitely. The teacher then leads the whole-class synthesis of Task 1, also eliciting the definition of *regular polygons*.

The teacher continues with the lesson as students explore Task 2, moving from testing regular polygons to irregular figures. She exclaims, "Whoa, that is the craziest one I've ever seen! . . . Whoa, this one looks like a star. Whoa, that looks like a bird. . . . What do you see?" The teacher stops for a moment when she catches a glimpse of one student's work who has collapsed her irregular figures into three degenerate figures with areas of zero. She asks Niveen, "Do you think that's a true statement, 0 + 0 = 0?"

Niveen: Yes.
Teacher: What do you think? Does this mean that [the Pythagorean theorem] still works?
Niveen: I guess. I mean, it is zero.
Teacher: If you're not convinced, make it like a little more than zero and see if the math works. [*The teacher walks away from Niveen and her partner to allow them time and space to explore their conjecture with the technology*.]

This unexpected zero equation was interesting because the students were unsure if this degenerate case supported or refuted their initial conjecture. Furthermore, because the technology did not easily allow the user to zoom in and create infinitely smaller and smaller irregular figures because of the size of the pixels, this clearly showed the limitations of this technology, something that the teacher could explore further in subsequent class meetings.

In summary, the beauty of the conversations is that the explorations and rich moments would not have taken place without the technology bringing students to a place they could not otherwise have gone, shedding calculations for conjectures, and replacing definitions and theorems with investigations. This allows all students to have an equitable voice within their small groups using the mathematical action technology of GeoGebra and when they each record their thinking using the conveyance technology.

We should not use "technology for the sake of technology" (Nank, 2017) or for classroom management, assuming software predetermines engagement. This example of mathematical thought, and in fact, investigation of mathematical relationships, would not have occurred without the dynamic technology. Although formal mathematical language was not always used, these students were conceptualizing and pondering polygons whose number of sides were increasing and approaching infinity.

MEGAGONS, INFINITY, AND CONVINCING AGAIN: AN EPILOGUE TO MEGAGONS AND ZEROS

Interestingly, students in California and Illinois, with different teachers, four months apart, had strangely similar conversations. Most notably, students had generated the same word—*megagon*—with no prompting. For example, analogous to the Illinois classroom, a similar conversation ensued in the California classroom during small-group work during Task 1 when a student said, "It's a what-gon? It's a deceptagon, optimus primagons. Yo, no wait, this will work even if it's a *megagon*!"

Another group next to the megagon group talked among themselves. One student said, "See, when I drag this point [vertex], then all three of these squares are getting bigger. So, like, I'm not making this biggest square bigger; I'm making all of them bigger. Now look: I'm making all of them smaller at the same time, not just one smaller."

A student in the megagon group replied, "Oh, I see, like our megagons! Super big shapes!" This conversation had students and an adult alike wondering what would happen if the shapes were not squares but instead were, as two classes put it, "*megagons*!"

Furthermore, the students in both California and Illinois were able to articulate how regular polygons could grow an infinite number of sides, and they could see the proportional relationship between the irregular figures as they adjusted their vertices, noticing how they grow proportionately to each other.

Finally, just as students in California had a conversation about how many tests and revisions in the cycles of proof need to be done before being convinced of a mathematical truth (though this was not a formal proof), a strangely similar conversation popped up in the Illinois classroom. Near the end of the class, the Illinois teacher overheard a conversation between two students who were trying to decide how many examples were necessary. One student, Isaac, said to Ben, “Are *you* convinced?”

Ben: I don’t know what convinced means.
Teacher: So, how much do *you* need to be convinced?
Ben: I don’t know.
Teacher: Yeah, what do mathematicians need to be convinced? That’s a good question!

These two students pondered this, and decided to explore more figures until they were sufficiently convinced. Noticing that the students were still wrestling with the notion of convincing and proving, the teacher elevated the conversation to the whole-group synthesis.

All eight classes started with the same conjecture, that the Pythagorean theorem will hold for only squares. The major rationale for the students is that Pythagoras was concerned only about squares, so it should not work with anything else. But through technological exploration that replaced the arduous task of using formulas to calculate the areas of each regular polygon, conversations ensued in which students tested their conjectures, modified their conclusions, and developed a deeper understanding of the how and the why behind the Pythagorean theorem.

CONCLUSION

Technology is not inherently good or bad, but how we use it can be. The main strength of the Pythagorean tasks is that they challenge assumptions, stretching student understanding using mathematical reasoning while including all students, especially those whose strength is not in calculating area, thus using technology to remove barriers.

The technology provides equity of access to students who are not as computationally strong. There is a movement to embrace conceptual understanding instead of speed as an indicator of mathematical intelligence. Students do not have to be able to quickly calculate area; it is not inherent in the goal of the lesson, so students can elevate and equalize their experience.

Consider how many students we unknowingly dis-invite to a lesson because they do not have the computational skills or do not enjoy computation but do have the reasoning needed to partake. Just because a student might not be able to calculate quickly should not determine their worthiness in classrooms. In this sense, technology provides an equalizing invitation to discourse. In this way, the technology allows the teacher to break from the traditional pattern of lecture and presenting definitions and theorems—often viewed as grand undeniable truths that famous mathematicians have handed down to us to memorize and not to question—and transition to exploration and wonderment through the dynamic discovery of relationships inherent in the Pythagorean theorem.

The overarching question persists: When do we (or should we) allow previous standards to hinder mastery of current standards? Let’s use technology to invite *all* students into deeper conversations, thus increasing their mathematical confidence, shifting their mathematical identity, and affording them the courage and hope to do and be better mathematically with past, current, and future standards.

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