

# An Analytic Approach to Safe Withdrawal Rates

Brian Wiksell

Symplectic Capital

March 1, 2026

## **Abstract**

Retirement withdrawal rules are typically evaluated using historical simulations or Monte Carlo methods. These numerical approaches provide realism but obscure simple relationships among key variables.

This paper models retirement wealth as a stochastic process with drift and volatility. The probability of ruin over a fixed horizon may then be expressed analytically. The resulting expression links the safe withdrawal rate directly to expected return, volatility, time, and an explicitly chosen probability of failure.

The analysis clarifies what determines safe withdrawal rates and provides intuition to complement simulation-based methods.

## Introduction

The concept of a safe withdrawal rate traces to the Trinity Study (Cooley, Hubbard, and Walz 1998), a seminal empirical investigation of historical U.S. equity and fixed-income returns. The study sought to estimate the maximum constant real withdrawal rate that would have sustained a portfolio over a thirty-year retirement horizon without exhausting principal. The 4 percent rule derived from this analysis became foundational in both the academic literature and practitioner discourse.

The broader literature on retirement income intersects with classical results in portfolio theory and actuarial science. Merton (1969, 1971) derived the continuous-time consumption–investment problem under uncertainty, establishing that optimal spending depends jointly on expected return, volatility, and horizon. Earlier actuarial treatments of ruin probabilities (e.g., Cramér 1930; Feller 1951) formalized the mathematics of capital exhaustion under stochastic processes. More recent contributions extend the withdrawal framework to incorporate regime dependence (Blanchett, Finke, and Pfau 2013), valuation-aware return assumptions (Kitces 2008), and adaptive spending rules (Guyton and Klinger 2006). Collectively, this body of work reframes the withdrawal problem as one of risk management under uncertainty rather than the identification of a single universally safe percentage.

Among contemporary researchers, Jeske (2016–2026) has provided one of the most extensive modern treatments of the subject. His research critiques overly simplistic interpretations of the 4 percent rule and emphasizes sequence-of-returns risk as a central determinant of portfolio sustainability.

Most modern analyses of withdrawal rates rely on historical back-tests or Monte Carlo simulations. These numerical approaches allow for realistic modeling of return distributions and path dependence but often obscure the fundamental relationships among key variables.

For researchers and practitioners seeking both intuition and tractable structure, portfolio wealth may instead be modeled analytically as a stochastic process characterized by drift and diffusion. Within such a framework, the probability of portfolio exhaustion over a fixed horizon can be derived using standard results from continuous-time probability theory. This approach yields an implicit equation linking the safe withdrawal rate to expected return, volatility, horizon length, and a chosen ruin probability.

Retirement calculators and simulation-based tools remain valuable for applied planning. However, an explicit analytic relationship clarifies the underlying mechanics.

## Model

A retiree withdraws a constant inflation-adjusted fraction  $w$  of their initial wealth each year from a portfolio with expected real return  $\mu$  and real volatility  $\sigma$ .

Portfolio wealth  $W_t$  may be modeled in continuous time as:

$$W_t = 1 + (\mu - w)t + \sigma B_t,$$

where  $B_t$  is Brownian motion (a standard random walk), the probability  $p$  that wealth ever hits zero by time  $T$  may be written in closed form:

$$p = \Phi\left(\frac{-1 - (\mu - w)T}{\sigma\sqrt{T}}\right) + \exp\left(\frac{-2(\mu - w)}{\sigma^2}\right) \Phi\left(\frac{-1 + (\mu - w)T}{\sigma\sqrt{T}}\right).$$

Here  $\Phi$  is the standard normal cumulative distribution function. This expression gives the ruin probability as a function of the withdrawal rate and portfolio characteristics.

Rearranging the above to define withdrawal rate  $w$  implicitly in terms of a target ruin probability  $p$ , horizon  $T$ , expected return  $\mu$ , and volatility  $\sigma$  yields:

$$\text{Find } w \text{ such that } \Phi\left(\frac{-1 - (\mu - w)T}{\sigma\sqrt{T}}\right) + \exp\left(\frac{-2(\mu - w)}{\sigma^2}\right) \Phi\left(\frac{-1 + (\mu - w)T}{\sigma\sqrt{T}}\right) = p$$

This is exact under the continuous diffusion assumption. There is no closed-form expression for  $w$  in elementary functions, but this equation is the exact analytical mapping from  $(T, \mu, \sigma, p)$  to  $w$ .

For practical purposes, especially for low ruin probabilities, a high-survival approximation may be used:

$$w \approx \mu + \frac{1}{T} + \frac{\sigma}{\sqrt{T}} z_p,$$

where  $z_p = \Phi^{-1}(p)$  is the standard normal *quantile* corresponding to ruin probability  $p$ . Here,  $|z_p|$  is how many standard deviations into the left tail that probability sits. As  $z_p$  is negative, the last term acts as a safety buffer beneath  $\mu$ . To make the safety buffer more explicit, the expression may be rewritten as:

$$w \approx \mu - \frac{\sigma}{\sqrt{T}} |z_p| + \frac{1}{T}.$$

From the above we see that the expression matches intuition: higher expected return,  $\mu$ , produces a higher withdrawal rate. Higher volatility,  $\sigma$ , longer horizon,  $T$ , and higher  $|z_p|$  or lower probability of ruin, each produce a lower withdrawal rate.

In the case of a risk-free investment,  $\sigma = 0$ , the expression reduces to:

$$w \approx \mu + \frac{1}{T}.$$

where the safe withdrawal rate is equal to the risk-free return  $\mu$ , plus an equal fraction of initial wealth over time horizon  $T$ . In this case, portfolio wealth is exhausted exactly at  $T$  with probability one.

### Comparison with Simulation

Using a thirty-year horizon and a 1 percent probability of ruin,  $z_{0.01} = -2.326$ , the analytical framework generates withdrawal rates that are broadly consistent with the estimates reported by Jeske (2016–2026), though modestly more conservative at higher equity allocations. For an all-equity portfolio ( $\mu = 6.92$  percent,  $\sigma = 16.57$  percent), the model implies a withdrawal rate of 3.22 percent, compared with 3.88 percent under historical simulation. At more balanced allocations, the estimates converge. A 60/40 portfolio yields 3.93 percent versus 3.60 percent, while an all-bond portfolio produces 3.86 percent relative to 3.41 percent. Overall, the analytical results fall within a narrow band of the simulation benchmarks, providing some level of validation.

Safe Withdrawal Rates:  $T = 30$ ; 1% probability of ruin:  $z_{0.01} = -2.326$

Stocks	Bonds	$\mu$	$\sigma$	Wiksell	Jeske
100.00	0.00	6.92	16.57	3.22	3.88
60.00	40.00	5.38	11.27	3.93	3.60
0.00	100.00	4.91	10.33	3.86	3.41

## References

- Bengen, William P. 1994. "Determining Withdrawal Rates Using Historical Data." *Journal of Financial Planning* 7 (4): 171–80.
- Blanchett, David M., Michael S. Finke, and Wade D. Pfau. 2013. "Low Bond Yields and Safe Portfolio Withdrawal Rates." *Journal of Wealth Management* 16 (4): 55–62.
- Cooley, Philip L., Carl M. Hubbard, and Daniel T. Walz. 1998. "Retirement Savings: Choosing a Withdrawal Rate That Is Sustainable." *Journal of the American Association of Individual Investors* 20 (1): 16–21.
- Cramér, Harald. 1930. "On the Mathematical Theory of Risk." *Skandia Jubilee Volume*, Stockholm.
- Feller, William. 1951. "Two Singular Diffusion Problems." *Annals of Mathematics* 54 (1): 173–82.
- Guyton, Jonathan T., and William J. Klinger. 2006. "Decision Rules and Portfolio Management for Retirees: Is the 'Safe' Initial Withdrawal Rate Too Safe?" *Journal of Financial Planning* 19 (10): 48–57.
- Jeske, Karsten. 2016–2026. "Safe Withdrawal Rate Series." *Early Retirement Now* (blog). Accessed March 1, 2026. <https://earlyretirementnow.com>
- Kitces, Michael E. 2008. "Resolving the Paradox: Is the Safe Withdrawal Rate Sometimes Too Safe?" *Journal of Financial Planning* 21 (3): 48–57.
- Merton, Robert C. 1969. "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case." *Review of Economics and Statistics* 51 (3): 247–57.
- Merton, Robert C. 1971. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory* 3 (4): 373–413.