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Factor Momentum and the Momentum Factor

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ABSTRACT

Momentum in individual stock returns relates to momentum in factor returns. Most factors are positively autocorrelated: the average factor earns a monthly return of six basis points following a year of losses and 51 basis points following a positive year. We find that factor momentum concentrates in factors that explain more of the cross section of returns and that it is not incidental to individual stock momentum: momentum-neutral factors display more momentum. Momentum found in high-eigenvalue principal component factors subsumes most forms of individual stock momentum. Our results suggest that momentum is not a distinct risk factor—it times other factors.

Momentum appears to violate the efficient market hypothesis in its weakest form. Past returns should not predict future returns if asset prices respond to new information immediately and to the right extent unless past returns correlate with changes in systematic risk. Researchers have sought to explain momentum with time-varying risk, behavioral biases, and trading frictions.[1] At the same time, the pervasiveness of momentum over time and

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[1] See, for example, Conrad and Kaul (1998), Berk, Green, and Naik (1999), Johnson (2002), and Sagi and Seasholes (2007) for risk-based explanations; Daniel, Hirshleifer, and Subrahmanyam

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across asset classes has given momentum the status of an independent factor—models without momentum cannot explain it and those with momentum cannot explain anything more than just momentum (Fama and French (2016)).[2] In this paper, we show that momentum is a dynamic portfolio that times other factors. Rather than being unrelated to the other factors, momentum relates to *all* of them.

We first show that factors' prior returns are informative about their future returns. Small stocks, for example, are likely to outperform big stocks when they have done so over the prior year. This effect is economically and statistically large among the 20 factors we initially study: the average factor earns 51 bps per month following a year of gains but just 6 bps following a year of losses, with the difference significant with a t-value of 4.22. Moreover, this result is not specific to the use of obscure asset pricing factors, as we use off-the-shelf factors that are regularly updated and published by academics and a hedge fund.

Why are factors autocorrelated? We show that Kozak, Nagel, and Santosh's (2018) model of sentiment investors leads to factor reversal or momentum depending on the persistence of sentiment. If sentiment is sufficiently persistent, this persistence carries over to factor returns. Although arbitrageurs know that factor premiums are predictable, they do not trade sufficiently aggressively to neutralize this effect because, in doing so, they would expose themselves to factor risk. This model predicts that momentum should concentrate in more systematic factors—much as in Kozak, Nagel, and Santosh (2018) [KNS hereafter], it is the sentiment-driven demand component that aligns with covariances that distorts asset prices.

We extract principal components (PCs) from 47 factors from Kozak, Nagel, and Santosh (2020). We find that factor momentum concentrates in the high-eigenvalue PCs, that is, in factors that explain more of the cross section of returns. A strategy that trades the first 10 high-eigenvalue PCs has a five-factor model alpha that is significant with a t-value of 6.51. Momentum in this set of PCs either greatly reduces momentum in the other subsets of PCs (the first half of the sample) or fully subsumes it (the second half). The finding that momentum concentrates in high-eigenvalue factors is consistent with the absence of near-arbitrage opportunities—if low-eigenvalue factors exhibited

(1998), Hong and Stein (1999), Frazzini, Israel, and Moskowitz (2012), Cooper, Gutierrez, and Hameed (2004), Griffin, Ji, and Martin (2003), and Asness, Moskowitz, and Pedersen (2013) for behavioral explanations; and Korajczyk and Sadka (2004), Lesmond, Schill, and Zhou (2004), and Avramov et al. (2013) for trading friction-based explanations.

[2] Jegadeesh (1990) and Jegadeesh and Titman (1993) document momentum in the cross section of stocks, Jostova et al. (2013) in corporate bonds, Beyhaghi and Ehsani (2017) in corporate loans, Hendricks, Patel, and Zeckhauser (1993), Brown and Goetzmann (1995), Grinblatt, Titman, and Wermers (1995), and Carhart (1997) in mutual funds, Baquero, Ter Horst, and Verbeek (2005), Boyson (2008), and Jagannathan, Malakhov, and Novikov (2010) in hedge funds, Bhojraj and Swaminathan (2006), Asness, Moskowitz, and Pedersen (2013), and Moskowitz, Ooi, and Pedersen (2012) in major futures contracts, Miffre and Rallis (2007) and Szakmary, Shen, and Sharma (2010) in commodity futures, Menkhoff et al. (2012) in currencies, and Lee, Naranjo, and Sirmans (2021) in credit default swaps.

momentum, arbitrageurs could profit from this effect without assuming much factor risk. Haddad, Kozak, and Santosh (2020) find that predictability based on factors' valuation ratios also concentrates in this way.

Momentum in factor returns transmits into the cross section of security returns. The amount that transmits depends on the dispersion in factor loadings. The more these loadings differ across assets, the more factor momentum shows up as *cross-sectional* momentum in individual security returns. This transmission mechanism motivates the main hypothesis that we test in this paper: Do individual stock returns display momentum beyond that due to factor returns? Our empirical strategy in testing this hypothesis is to confront various strategies that trade individual stock momentum with factor momentum.

We begin by pricing portfolios sorted by prior one-year returns. We find that, if anything, factor momentum prices portfolios sorted by prior one-year returns better than Carhart (1997) up minus down (UMD) factor, which *directly* targets momentum in stock returns. When we augment the five-factor model with a factor that trades momentum in the high-eigenvalue PCs, mean absolute alphas are negligible and the Gibbons, Ross, and Shanken (1989) (GRS) test does not reject the null that these alphas are jointly zero.

Factor momentum also explains other forms of stock momentum: industry momentum, industry-adjusted momentum, intermediate momentum, and Sharpe ratio momentum. The left-hand side of Figure 1 shows two t-values for each version of individual stock momentum: the first corresponds to the strategy's five-factor model alpha while the second corresponds to a model that also captures momentum found in the first 10 high-eigenvalue PCs. Factor momentum renders these types of individual stock momentum strategies statistically insignificant. The right-hand side of the same figure shows that a five-factor model augmented with the aforementioned five types of individual stock momentum leaves factor momentum with an alpha that is significant with a t-value of 5.32.

Residual momentum strategies are of independent interest. A strategy that selects stocks based on their Capital Asset Pricing Model (CAPM) residuals is more profitable than a strategy that selects stocks based on their total past returns. However, as we remove additional factors from stock returns—such as value and size—residual momentum strategies weaken. This pattern also appears to relate to factor momentum. If an investor works with a misspecified asset pricing model, residual momentum strategies profit from "omitted-factor momentum" even when firm-specific innovations are independent and identically distributed (IID). If the factors in the investor's model are less autocorrelated than those it omits, residuals display *more* momentum than total stock returns. The finding that residuals exhibit more momentum than raw stock returns should therefore not be construed as evidence that *firm-specific* returns display momentum. Consistent with this omitted-factor argument, no residual momentum strategy is significant net of factor momentum—Figure 1 shows $t(\hat{\alpha})$'s for one such strategy.

If factors are linear combinations of individual stocks, is factor momentum ultimately a reflection of individual stock momentum? Our result that the

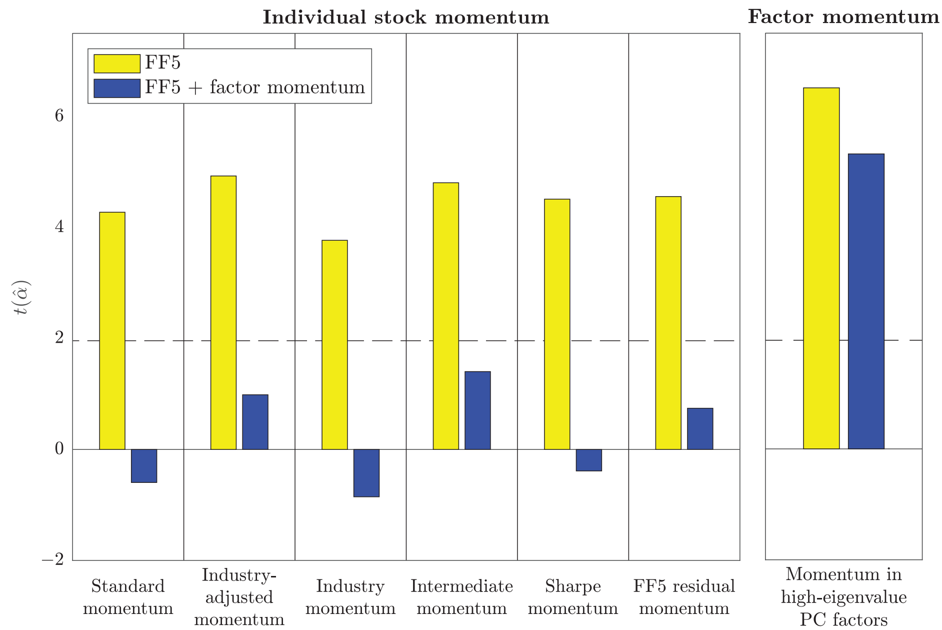

Figure 1. Individual stock momentum versus factor momentum. This figure shows t-values associated with alphas for six momentum strategies that trade individual stocks and a factor momentum strategy that trades the first 10 high-eigenvalue PCs extracted from 47 factors. For individual stock momentum strategies, we report t-values from the five-factor model (yellow bars) and this model augmented with the factor momentum strategy (blue bars). The blue residual momentum regression also includes betting-against-beta factors. For factor momentum, we report t-values from the five-factor model (yellow bar) and this model augmented with the first five individual stock momentum strategies (blue bar). The dashed line denotes a t-value of 1.96.

nature of factors matters—more systematic factors display more momentum—suggests that factors are distinct from individual stocks. Going a step further, we construct *momentum-neutral factors*, that is, factors whose weights are as close as possible to the original factors but orthogonal to past stock returns. An investor investing in a momentum-neutral size factor, for example, would buy and sell small and large stocks that are identical in terms of their past returns. We show that momentum-neutral factors exhibit *more* momentum than standard factors and that factor momentum in momentum-neutral factors subsumes standard factor momentum. Thus, factor momentum is not merely incidental to individual stock momentum. Of all the factor momentum strategies we consider, the one with the highest Sharpe ratio is the one that trades momentum in the high-eigenvalue PC factors extracted from momentum-neutral factors. This strategy's five-factor model alpha is significant with a t-value of 7.53.

Our results suggest that momentum is largely about timing other factors. This characterization of momentum resolves the perennial question about covariances and momentum (Cochrane (2011, p. 1075)): "...why should all the momentum stocks then rise and fall together the next month, just as if they are exposed to a pervasive, systematic risk?" Momentum stocks comove because they are exposed to the same systematic risks; winners, for example, load positively on factors that have done well and negatively on those that have done poorly. Because momentum's loadings change over time, we are easily left with the impression that momentum is distinct from other risk factors.

A clarifying note about momentum's status as a distinct risk factor is in order. Momentum is distinct from, for example, the five factors of the Fama-French model in the sense that a static combination of these factors does not span momentum. Our contribution is to show that we can capture all of momentum profits by timing other factors. Alternatively, we could redefine existing factors and push momentum back into them. Ehsani and Linnainmaa (2020) show that UMD is spanned in an unconditional regression against what they call a "time-series efficient" Fama-French five-factor model. There is no need to construct a separate momentum factor from security-level data—the factors that we already have will do. However, even if one accepts our conclusion that momentum is not a distinct factor, this does not mean that investors can ignore momentum; to capture it, investors still need to time the other factors, redefine these other factors or, if they so insist, trade individual stock momentum as if it were distinct from the other factors.

Our empirical tests indicate that momentum found in the first 10 PC factors subsumes all versions of individual stock momentum. This evidence, however, should not be construed as suggesting that there is no momentum in stock returns beyond that emanating from factor momentum. A more balanced and measured interpretation of our results is that factor momentum explains a significant portion of stock momentum profits. Our findings do not preclude the possibility that momentum exists in stock returns net of factor momentum. We discuss the econometric difficulties in distinguishing between factor and pure firm-specific momentum in the conclusion.

Our results relate to McLean and Pontiff (2016), Avramov et al. (2017), and Zaremba and Shemer (2017) who show that anomaly returns predict the cross section of anomaly returns at the one-month and one-year lags. Arnott et al. (2021) show that short-term cross-sectional factor momentum explains short-term industry momentum. However, that alternative form of factor momentum explains none of individual stock momentum, consistent with the finding of Grundy and Martin (2001) that industry momentum is largely unrelated to stock momentum.

The paper proceeds as follows. Section I measures autocorrelations in the returns of well-known equity factors. Section II shows that factors in the KNS model are autocorrelated when sentiment is sufficiently persistent and that factor momentum concentrates in high-eigenvalue factors. It then shows that the actual factors also display this property. Section III shows that factor momentum explains other forms of momentum. Section IV shows that

Table I
Descriptive Statistics

This table reports the start date, original study, and average annualized returns, standard deviations, and t-values for 15 U.S. and seven global factors. The universe of stocks for the global factors is the developed markets excluding the United States. The end date for all factors is December 2019.

Factor	Original Study	Start Date	Annual Return Mean	Annual Return *SD*	Annual Return t-Value
U.S. Factors					
Size	Banz (1981)	Jul 1963	2.7%	10.4%	1.97
Value	Rosenberg, Reid, and Lanstein (1985)	Jul 1963	3.7%	9.7%	2.82
Profitability	Novy-Marx (2013)	Jul 1963	3.1%	7.5%	3.13
Investment	Titman, Wei, and Xie (2004)	Jul 1963	3.3%	6.9%	3.59
Momentum	Jegadeesh and Titman (1993)	Jul 1963	7.8%	14.5%	4.02
Accruals	Sloan (1996)	Jul 1963	2.8%	6.6%	3.19
Betting against beta	Frazzini and Pedersen (2014)	Jul 1963	9.8%	11.2%	6.55
Cash flow to price	Rosenberg, Reid, and Lanstein (1985)	Jul 1963	3.4%	8.6%	2.94
Earnings to price	Basu (1983)	Jul 1963	3.5%	8.9%	2.95
Liquidity	Pástor and Stambaugh (2003)	Jan 1968	4.4%	11.6%	2.77
Long-term reversals	Bondt and Thaler (1985)	Jul 1963	2.5%	8.7%	2.16
Net share issues	Loughran and Ritter (1995)	Jul 1963	2.8%	8.2%	2.52
Quality minus junk	Asness, Frazzini, and Pedersen (2019)	Jul 1963	4.6%	7.7%	4.47
Residual variance	Ang et al. (2006)	Jul 1963	1.6%	17.3%	0.68
Short-term reversals	Jegadeesh (1990)	Jul 1963	6.0%	10.6%	4.21
Global Factors					
Size	Banz (1981)	Jul 1990	1.1%	7.1%	0.83
Value	Rosenberg, Reid, and Lanstein (1985)	Jul 1990	4.0%	7.4%	2.92
Profitability	Novy-Marx (2013)	Jul 1990	4.3%	4.7%	4.91
Investment	Titman, Wei, and Xie (2004)	Jul 1990	1.9%	6.1%	1.74
Momentum	Jegadeesh and Titman (1993)	Nov 1990	7.9%	12.1%	3.54
Betting against beta	Frazzini and Pedersen (2014)	Jul 1990	9.6%	9.7%	5.70
Quality minus junk	Asness, Frazzini, and Pedersen (2019)	Jul 1990	6.3%	6.8%	5.06

factor momentum is not incidental to individual stock momentum. Section V concludes.

I. Autocorrelation in Off-the-Shelf Factors

A. Data

We take monthly factor data from three sources, namely, the data libraries of Kenneth French, AQR, and Robert Stambaugh.[3] Table I lists the factors, start dates, average annualized returns, standard deviations of returns, and t-values associated with the average returns. If the factor return data are not provided, we compute factor return as the average return on the three

[3] These data sets are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, https://www.aqr.com/insights/datasets, and http://finance.wharton.upenn.edu/~stambaug/.

top deciles minus that on the three bottom deciles, where the top and bottom deciles are defined in the same way as in the original study.

The 15 anomalies that use U.S. data are size, value, profitability, investment, momentum, accruals, betting against beta, cash flow to price, earnings to price, liquidity, long-term reversals, net share issues, quality minus junk, residual variance, and short-term reversals. Except for the liquidity factor of Pástor and Stambaugh (2003), the return data for these factors begin in July 1963; those for the liquidity factor begin in January 1968. The seven global factors are size, value, profitability, investment, momentum, betting against beta, and quality minus junk. Except for the momentum factor, the return data for these factors begin in July 1990; those for the momentum factor begin in November 1990. We refer to these 22 factors as the "off-the-shelf" factors. We later study a broader set of 47 U.S. factors.

Table I shows significant variation in average annualized returns. The global size factor, for example, earns 1.1%, while both the U.S. and global betting-against-beta factors earn almost 10%. Factors' volatilities also vary significantly. The global profitability factor, for example, has an annualized standard deviation of returns of just 4.7%; at the other extreme, the volatility of the residual variance factor is 17.3%.

B. Factor Returns Conditional on Past Returns

Table II shows that factors' prior returns significantly predict their future returns. We estimate time-series regressions in which the dependent variable is a factor's month t return and the explanatory variable is an indicator variable for the factor's performance over the prior year from month $t-12$ to $t-1$. This indicator variable takes the value of one if the factor's return is positive, and zero otherwise.[4]

The intercepts in Table II measure the average factor returns earned following a year of underperformance. The slope coefficient represents the average return difference between the up and down years. In these regressions all slope coefficients, except that for the U.S. momentum factor, are positive. Six of the estimates are significant at the 5% level and an additional four at the 10% level. Although all factors' unconditional means are positive (Table I), the intercepts show that six anomalies earn a negative average return following a year of underperformance. The first row shows that the amount of predictability in factor premiums is economically and statistically large. We estimate this

[4] Table A.I shows estimates from regressions of factor returns on prior one-year factor returns. We present the indicator-variable specification of Table II as the main specification because it is analogous to a strategy that signs the positions in factors based on their prior returns. Christoffersen and Diebold (2006) show that the *signs* of returns may display serial dependence even if means are unpredictable. Sign autocorrelation and the lack of autocorrelation in means can coexist if means are positive and volatility is serially dependent. The regressions in Table II are of the "return-on-sign" rather than "sign-on-sign" variety and therefore not subject to this mechanism. The estimates show that signs predict differences in conditional means. The pooled estimate of 0.25 (t-value = 2.59) in Table A.I's "return-on-return" regression also indicates that mean returns are autocorrelated.

Table II

Average Factor Returns Conditional on Their Own Past Returns

This table reports estimates from regressions in which the dependent variable is a factor's monthly return and the independent variable takes the value of one if the factor's average return over the prior year is positive and zero otherwise. We estimate these regressions using pooled data (first row) and separately for each anomaly (remaining rows). The pooled data exclude the two momentum factors. In the pooled regression, we cluster standard errors by month. Table I reports the factor start dates. The sample ends in December 2019.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.06	0.72	0.45	4.22
		U.S. Factors		
Size	−0.10	−0.62	0.58	2.51
Value	0.04	0.20	0.41	1.78
Profitability	0.04	0.22	0.34	1.67
Investment	0.12	0.97	0.24	1.55
Momentum	0.72	2.70	−0.09	−0.29
Accruals	0.15	1.18	0.10	0.65
Betting against beta	−0.22	−0.63	1.32	3.53
Cash flow to price	0.13	0.78	0.24	1.16
Earnings to price	0.10	0.62	0.30	1.46
Liquidity	0.16	0.74	0.36	1.29
Long-term reversals	−0.25	−1.66	0.76	3.85
Net share issues	0.17	1.32	0.09	0.49
Quality minus junk	0.09	0.65	0.43	2.51
Residual variance	−0.46	−1.64	1.06	2.74
Short-term reversals	0.49	1.43	0.01	0.04
		Global Factors		
Size	−0.06	−0.39	0.28	1.33
Value	0.04	0.15	0.47	1.77
Profitability	0.14	1.03	0.26	1.62
Investment	−0.06	−0.41	0.38	1.94
Momentum	0.67	1.77	0.02	0.04
Betting against beta	0.19	0.58	0.84	2.30
Quality minus junk	0.39	1.76	0.12	0.49

pooled regression using data on the 20 nonmomentum factors. The average anomaly earns a monthly return of just 6 bps (t-value = 0.72) following a year of underperformance. When the anomaly's return over the prior year is positive, this return increases by 45 bps (t-value = 4.22) to 51 bps.

In Section I of the Internet Appendix, we construct and decompose factor momentum strategies that trade the factors listed in Table II.[5] A time-series momentum strategy, which is long factors with positive returns and short those with negative returns, earns an annualized return of 3.9% (t-value = 7.01). A cross-sectional strategy, which is long factors with above-median returns and short those with below-median returns, earns an annualized return of 2.4%

[5] The Internet Appendix may be found in the online version of this article.

(t-value = 5.04).[6] The time-series strategy outperforms the cross-sectional strategy because it is a pure bet on autocorrelations in factor returns. A cross-sectional strategy, by contrast, also bets that a high return on a factor predicts low returns on the other factors (Lo and MacKinlay (1990)); in the data, however, a high return on one factor typically predicts high returns on other factors as well.

A noteworthy feature of factor momentum strategies is that investors can capture the momentum premium without prespecifying which leg of the factor *on average* earns a higher return. Consider, for example, the discovery of a new A-minus-B factor, AMB. An investor who believes that this factor is associated with an unconditional return premium needs to determine, ex ante, which leg outperforms the other. She would rely on historical data, an economic model, or both to make this determination. An investor who seeks to profit from the autocorrelation in factor returns, by contrast, does not need an estimate of the factor's unconditional mean. An investor who believes that AMB displays momentum would invest in AMB after a year of gains and in its reverse, "BMA," after a year of losses. Table II shows that an investor with perfect foresight about the signs of the factors' unconditional premiums would have earned 51 bps, approximately the same return as the momentum investor (45 bps).

II. Factor Momentum and the Covariance Structure of Returns

A. Factor Momentum in Economies with Sentiment Investors

Why are factors autocorrelated? In this section, we build on KNS to derive the conditions under which factors exhibit momentum and characterize the properties of the factors that exhibit the most momentum. We first describe the key elements of the KNS model. The economy has two types of risk-averse investors, namely, fully rational arbitrageurs and sentiment investors with distorted beliefs about asset returns' true distributions. Asset cash flows are IID and the covariance matrix of these cash flows has a few dominant factors. Sentiment investors' demand has an additional sentiment-driven demand component. Sentiment investors cannot take substantial leverage or short extensively. By market clearing, rational arbitrageurs trade against sentiment investors. KNS study the extent to which, and under what conditions, sentiment distorts asset prices.

The key finding of KNS is that arbitrageurs almost fully subsume any sentiment-driven demand not aligned with common factor covariances. The intuition is that arbitrageurs can make these profitable trades without assuming any factor risk, therefore neutralizing these components of sentiment investors' demand. Conversely, arbitrageurs are reluctant to take the other side of those sentiment-driven trades that align with common factor covariances; such trades would expose them to factor risk. This dichotomy implies that

[6] In Section II of the Internet Appendix, we compute time-series and cross-sectional factor momentum strategies with different formation and holding periods.

even if sentiment-driven demand has nothing to do with the covariances of cash flows, those mispricings that align with covariances remain. KNS conclusion is that the absence of near-arbitrage opportunities together with the substantial commonality in asset returns ensures that the stochastic discount factor (SDF) can be represented as a function of a few dominant factors. The ability to do so provides no clues as to whether pricing is rational or subject to behavioral distortions.

We now derive the condition under which asset returns and the factors in this model are autocorrelated. In what follows, we assume that the reader is familiar with Sections III and IV and Appendix C of the original paper. Kozak, Nagel, and Santosh (2018, equation (C5)) gives the realized returns as

$$R_{t+1} = D_{t+1} + a_1(\xi_{t+1} - \xi_t) - R_f(a_0 + a_1\xi_t), \tag{1}$$

where R_{t+1} is an $N \times 1$ vector of asset returns, D_{t+1} are the dividends, R_f is the risk-free rate, a_0 and a_1 are vectors of constants, and ξ_t is sentiment-investor demand. This demand follows an AR(1) process, $\xi_{t+1} = \mu + \phi\xi_t + \nu_{t+1}$, with $\text{var}(\nu_{t+1}) = \omega^2$. Sentiment investors' demand is distorted in direction δ by the amount ξ_t. From equation (1), the return autocovariance matrix is

$$\begin{aligned} \text{cov}(R_t, R_{t+1}) &= a_1a_1'\text{cov}(\xi_t - R_f\,\xi_{t-1}, \xi_{t+1} - R_f\,\xi_t) \\ &= a_1a_1'\sigma^2\Big[(1+R_f^2)\phi - R_f - R_f\,\phi^2\Big], \end{aligned} \tag{2}$$

where the second row uses the properties of the AR(1) process, $\sigma^2 \equiv \text{var}(\xi_t) = \frac{\omega^2}{1-\phi^2}$ and $\text{cov}(\xi_t, \xi_{t+h}) = \phi^{|h|}\sigma^2$.

KNS note that a_1 can be solved from the arbitrageurs' first-order condition (equation (C10)) combined with the market-clearing condition (equation (31)) using the method of undetermined coefficients. Specifically, b_2 appears in the term multiplying ξ_t in the first-order condition and, because market clearing has to hold for any value of ξ_t, this slope must be zero. Collecting terms, a_1 can be written as

$$a_1 = \frac{\gamma\theta\Gamma\delta}{R_f + \frac{1}{1+2b_2\omega^2}\left(\frac{\gamma\theta\delta' a_1}{2b_2} - \phi\right) - \frac{\gamma\theta\delta' a_1}{2b_2}}, \tag{3}$$

and therefore[7]

$$a_1a_1' = \frac{\gamma^2\theta^2\Gamma\delta\delta'\Gamma}{\left[R_f + \frac{1}{1+2b_2\omega^2}\left(\frac{\gamma\theta\delta' a_1}{2b_2} - \phi\right) - \frac{\gamma\theta\delta' a_1}{2b_2}\right]^2} = \Gamma\delta\delta'\Gamma c_0. \tag{4}$$

[7] Constant $c_0 > 0$ has (scalar) $\delta' a_1$ in the denominator. It could be eliminated by premultiplying both sides of equation (3) by δ', solving for $\delta' a_1$, and plugging it back into this expression. For our purposes, the value of the denominator does not matter, but it has to be positive for the solution to a_1 to exist.

The factors in KNS are the eigenvectors of the covariance matrix of asset cash flows, $\Gamma = Q\Lambda Q$, where Q is the matrix of eigenvectors and Λ is a diagonal matrix with the eigenvalues. Following KNS, we consider factor q_k, which is the k^{th} PC. The autocovariance of this factor is

$$\begin{aligned}\text{cov}(PC_t^k, PC_{t+1}^k) &= \text{cov}(q_k' R_t, q_k' R_{t+1}) = q_k' \text{cov}(R_t, R_{t+1}) q_k \\ &= q_k' a_1 a_1' q_k \sigma^2 \Big[(1+R_f^2)\phi - R_f - R_f \phi^2\Big] \\ &= q_k' \Gamma \delta \delta' \Gamma q_k c_0 \sigma^2 \Big[(1+R_f^2)\phi - R_f - R_f \phi^2\Big]. \end{aligned} \tag{5}$$

Kozak, Nagel, and Santosh (2018, equation (16)) characterize the association between the PCs and δ by expressing δ as a linear combination of the PCs, $\delta = Q\beta$. With this mapping together with the eigenvalue decomposition of the covariance matrix, the term $q_k' \Gamma \delta \delta' \Gamma q_k$ in equation (5) becomes

$$q_k' \Gamma \delta \delta' \Gamma q_k = q_k' Q \Lambda \beta \beta' \Lambda Q' q_k = \iota_k' \Lambda \beta \beta' \Lambda \iota_k = \lambda_k^2 \beta_k^2, \tag{6}$$

where ι_k is a vector of zeros with one as the k^{th} element. The autocovariance of the k^{th} PC is therefore

$$\text{cov}(PC_t^k, PC_{t+1}^k) = \lambda_k^2 \beta_k^2 c_0 \sigma^2 \Big[(1+R_f^2)\phi - R_f - R_f \phi^2\Big]. \tag{7}$$

When are factors serially correlated? The bracketed expression in equation (7) determines the sign of the autocovariance. This expression is quadratic and concave in ϕ with two roots: $\phi = \frac{1}{R_f}$ and $\phi = R_f$. Factors therefore positively correlate when sentiment is sufficiently persistent, $\phi \in (\frac{1}{R_f}, 1]$. The persistence in sentiment drives the momentum in factors for the same reason that factor premiums align with covariances in KNS: although arbitrageurs are aware that factors exhibit reversals (when $\phi < \frac{1}{R_f}$) or momentum (when $\phi > \frac{1}{R_f}$), they are reluctant to trade so aggressively that they would neutralize this pattern because, in doing so, they would assume factor risk. Autocorrelation in factor returns emerges from the connection between sentiment and prices. If sentiment is high today, so are prices. But mean reversion in sentiment would mean that both sentiment and prices are lower tomorrow. The extent to which sentiment autocorrelates therefore pins down the dynamics of factor returns.

In this model, sentiment has to be highly autocorrelated to generate factor *momentum*. With an average monthly risk-free rate of 0.39% between July 1965 and December 2018, the momentum threshold is $\phi > 0.996$. Is this, then, a reasonable mechanism for driving factor momentum? Perhaps. First, the first-order autocorrelation in the Baker and Wurgler (2006) sentiment index over the same 1965 to 2018 period is 0.986, and the Dickey and Fuller (1979) test does not reject the null hypothesis of a unit root at the 10% level.[8] By extension, we also cannot reject the null hypothesis that ϕ is above the critical threshold for factor momentum. Moreover, if the Baker and Wurgler (2006) index measures sentiment, it does so with noise; the latent sentiment index could be highly persistent. Second, the KNS model is a stylized model for tractability—the risk-free rate, the sentiment index, and the effect of the sentiment on stock returns, for example, are all exogenous, and cash flows are IID with a fixed covariance matrix. The model's qualitative prediction, namely, that persistence in sentiment can generate factor momentum, can be true even if it were to miss the mark on quantities. Factors *are* positively autocorrelated in the data, which implies that if a model in the spirit of KNS generates those data, sentiment must be sufficiently autocorrelated to clear the hurdle in such a generalized model.

What factors have more momentum in the KNS model? Equation (7) shows that those high-eigenvalue factors that line up with δ have more momentum. This result again parallels the distortion result in KNS: the sentiment-driven demand component δ has a large impact on SDF variance only when δ lines up "primarily with the high-eigenvalue (volatile) PCs of asset returns" (p. 1203). Our analysis suggests that the high-eigenvalue factors are also those that should display more factor momentum.

B. High-Variance PCs and Factor Momentum

The prediction that momentum should concentrate in high-eigenvalue factors transcends the specifics of the sentiment model. Both KNS and Haddad, Kozak, and Santosh (2020), for example, assume the absence of near-arbitrage opportunities to motivate their study of the extent to which low-order PCs explain unconditional differences in expected returns and generate time-series predictability.

We use data on 54 factors from Kozak, Nagel, and Santosh (2020) to measure factor momentum's concentration in high-eigenvalue PCs.[9] We exclude the seven predictors that relate to momentum or that combine momentum with other characteristics.[10] Similar to Kozak, Nagel, and Santosh (2020), we exclude all-but-microcaps from the analysis to ensure that very small and illiquid stocks do not unduly influence the results.[11] The characteristics are expressed as weights on zero-investment long-short factors. Each firm characteristic $c_{i,t}$, where i indexes firms, is first transformed into a cross-sectional rank,

[8] The Dickey-Fuller test statistic with 641 months of data is −2.36. The 10% critical z-value to reject the null hypothesis of a unit root is −2.57.

[9] We thank Serhiy Kozak for making these data available at https://www.serhiykozak.com/data.

[10] The characteristics we exclude are (i) momentum (6 m), (ii) industry momentum, (iii) value momentum, (iv) value-momentum-profitability, (v) momentum (1 year), (vi) momentum-reversal, and (vii) industry momentum-reversal.

[11] Following Kozak, Nagel, and Santosh (2020), we compute the total market value of all common stocks traded on NYSE, Amex, and Nasdaq in month t and exclude stocks with market values less than 0.01% of the total market value.

$rc_{i,t} = \frac{\text{rank}(c_{i,t})}{n_t+1}$, where n_t is the number of stocks in month t. These ranks are then centered around zero and normalized by the sum of absolute deviations from the mean,

$$w_{i,t} = \frac{rc_{i,t} - \overline{rc}_{i,t}}{\sum_{i=1}^{n_t} |rc_{i,t} - \overline{rc}_{i,t}|}. \tag{8}$$

If a firm's characteristic $c_{i,t}$ is missing, we set the weight corresponding to this characteristic to zero (Kozak, Nagel, and Santosh (2020)). The month t return on a factor based on characteristic j is then $f_t = \sum_{i=1}^{n_{t-1}} w_{i,t-1} r_{i,t}$. Table A.II lists the 47 characteristics and the annualized CAPM alphas for long-short factors based on these characteristics. The factors are not re-signed based on the direction in which each characteristic predicts returns; the size factor, for example, is long large stocks and short small stocks and therefore earns a negative average return.

Table III reports results on the profitability of factor momentum strategies that trade PC factors extracted from these 47 factors. To avoid lookahead bias, we compute month $t+1$ returns on PC factors using only information that is available as of the end of month t. Our out-of-sample procedure consists of five steps:

1. Compute eigenvectors using daily returns on the 47 factors from July 1973 through the end of month t from the correlation matrix of factor returns.
2. Compute monthly returns for the PC factors up to month $t+1$ using these eigenvectors. PC factor f's return is $r^{pc}_{f,t} = \sum_{j=1}^{47} v^f_j r_{j,t}$, where v^f_j is the j^{th} element of the f^{th} eigenvector and $r_{j,t}$ is the return on individual factor j.
3. Compute individual factors' variances using data up to month t. Demean and lever the PC factors so that their variances up to month t are equal to the variance of the average individual factor and their average returns up to month t are zero.
4. Construct a factor momentum strategy that is long factors with positive average returns from month $t-11$ to t and short factors with negative average returns.
5. Compute the return on the resulting factor momentum strategy in month $t+1$.

This strategy's return in month $t+1$ is out of sample relative to the computation of the eigenvectors in the first step, which uses data only up to the end of month t. Similarly, the demeaning and leveraging in the third step use information only up to the end of month t.[12] When we construct time-series factor

[12] Goyal and Jegadeesh (2017) and Huang et al. (2020) note that time-series momentum strategies that trade individual assets (or futures contracts) are not as profitable as they might seem because they are net long assets with positive risk premiums. The argument is that, because average returns are positive, a time-series strategy buys more often than it sells. We compute the PC eigenvectors from the correlation matrix, which is equivalent to computing PCs from the

Table III

Factor Momentum in High- and Low-Eigenvalue Factors

This table reports estimates from time-series regressions in which the dependent variable is the return on factor momentum. We construct factor momentum strategies from the 47 factors listed in Table A.II using either the individual factors or the PC extracted from these factors. We compute the factor PC momentum strategy's month $t+1$ return in five steps: (i) compute eigenvectors from the correlation matrix of daily factor returns from July 1963 up to the end of month t; (ii) compute monthly returns for PC factors up to month $t+1$ using these eigenvectors; (iii) demean and lever up or down all PC factors so that their average returns up to month t are zero and their time-series variances match that of the average original factor up to month t; (iv) take long positions in the PC factors with positive average returns from month $t-11$ to t and short positions in factors with negative average returns; and (v) compute the return on the resulting strategy in month $t+1$. This strategy's returns are out of sample relative to the computation of the eigenvectors in step (i). We similarly lever individual factor returns so that when we compute the month $t+1$ return on the strategy that trades these factors, these factors' variances up to month t are all equal to the average factor's variance up to month t. Panel A reports monthly average returns and t-values for momentum strategies that trade subsets of PC factors ordered by eigenvalues. Panels B and C report estimates from regressions that explain the returns of momentum strategies with each other. The two intercepts correspond to the first and second halves of the sample. The sample begins in July 1973 and ends in December 2019. The first half runs from July 1973 through September 1996 and the second half from October 1996 through December 2019.

Panel A: Factor Momentum in Subsets of PC Factors Ordered by Eigenvalues

Set of PCs	Full Sample $\bar{r}$	Full Sample $t(\bar{r})$	First Half $\bar{r}$	First Half $t(\bar{r})$	Second Half $\bar{r}$	Second Half $t(\bar{r})$
1–10	0.19	7.07	0.27	8.49	0.11	2.60
11–20	0.13	5.23	0.20	6.13	0.05	1.50
21–30	0.10	5.02	0.18	7.93	0.02	0.63
31–40	0.10	4.05	0.16	5.07	0.04	1.08
41–47	0.07	2.51	0.09	2.71	0.06	1.17

Panel B: Explaining Factor Momentum in Low-Eigenvalue PC Factors

Explanatory Variable	Set of PCs 11–20	21–30	31–40	41–47
$\alpha_{\text{first half}}$	0.12	0.12	0.06	−0.01
	(3.50)	(4.27)	(1.86)	(−0.31)
$\alpha_{\text{second half}}$	0.02	0.00	0.00	0.03
	(0.56)	(−0.16)	(−0.10)	(0.78)
$\text{FMOM}_{\text{PC1–10}}$	0.34	0.28	0.34	0.43
	(9.78)	(9.50)	(9.50)	(10.64)
FF5	Y	Y	Y	Y
N	558	558	558	558
Adj. R^2	20.8%	21.7%	20.3%	22.4%

(*Continued*)

Table III—*Continued*

Panel C: Explaining Factor Momentum in High-Eigenvalue PC Factors

Explanatory Variable	Regression (1)	(2)	(3)	(4)	(5)
$\alpha_{\text{first half}}$	0.17	0.16	0.20	0.22	0.10
	(4.63)	(4.36)	(5.38)	(6.19)	(2.92)
$\alpha_{\text{second half}}$	0.08	0.09	0.09	0.08	0.06
	(2.30)	(2.59)	(2.57)	(2.16)	(1.94)
$\text{FMOM}_{\text{PC11–20}}$	0.43				0.26
	(9.78)				(6.26)
$\text{FMOM}_{\text{PC21–30}}$		0.51			0.29
		(9.50)			(5.83)
$\text{FMOM}_{\text{PC31–40}}$			0.41		0.20
			(9.50)		(4.65)
$\text{FMOM}_{\text{PC41–47}}$				0.39	0.21
				(10.64)	(5.66)
FF5	Y	Y	Y	Y	Y
N	558	558	558	558	558
Adj. R^2	24.6%	24.0%	24.0%	26.6%	40.5%

momentum strategies using the original factors, we similarly scale all factors to have the same volatility up to the end of month t so that they are comparable with the PC factors. Any instability in factor rotations does not affect the momentum signal. In the procedure above, we compute the month $t+1$ return and the average return from month $t-11$ to t using the same time t eigenvectors. That is, even if the rotation of the factors changes over, say, a six-month period, this instability does not matter because we fix the rotation each month before we look one year backward and one month forward in time. We use daily factor returns starting in July 1963 to compute the eigenvectors; we require at least 10 years of data to extract the PCs. The returns on the factor momentum strategies therefore begin in July 1973.[13]

Panel A of Table III shows average returns and t-values for momentum strategies that trade subsets of PC factors ordered by eigenvalues. Over the full

covariance matrix of demeaned factors. All of our PC factors earn zero mean returns *in-sample*. The momentum strategy that trades these PC factors is therefore a pure bet on autocorrelations and not subject to the Goyal and Jegadeesh (2017) bias.

[13] The PC factors are quite stable. A comparison between three alternative sets of PC factors illustrates this stability. We first compute returns on PC factors in month t from a covariance matrix estimated using daily data (i) up to the end of month $t-1$, (ii) up to the end of month $t-2$, or (iii) over the full-sample period. We then compute average returns for the first 10 PC factors for each of these alternative definitions. The correlation between the first two definitions (timely and not-so-timely PC factors) is 0.977 because the covariance matrix rarely changes that much from one month to the next. The correlation between the first and third definitions (timely and full-sample PC factors) is 0.810. That is, factor PC returns computed using the full-sample covariance matrix are in considerable agreement with those extracted using information available in real time.

sample, the strategy that trades the first 10 PC factors earns 19 bps per month (t-value = 7.07). Because the Kozak, Nagel, and Santosh (2020) factors have low volatilities, so do the PC factors and, by extension, these momentum strategies. The average returns and t-values are lower among the low-eigenvalue PC factors. A momentum strategy trading the last set of PC factors, for example, earns 7 bps per month (t-value = 2.61). Factor momentum strategies are less profitable in the second half of the sample; although the strategy that trades the first 10 PC factors is statistically significant at the 1% level in the second half, the others are not significant even at the 10% level.[14]

In Panels B and C, we measure the extent to which the momentum realized returns found in the five subsets of factors correlate with and subsume each other. In Panel B, we report estimates from regressions such as

$$\mathrm{FMOM}_t^{\mathrm{PC11–20}} = \alpha_{\text{first half}} \mathbb{1}_{t \in \text{first half}} + \alpha_{\text{second half}} \mathbb{1}_{t \in \text{second half}} + b\, \mathrm{FMOM}_t^{\mathrm{PC1–10}} + \mathrm{FF5} + \varepsilon_t^{\mathrm{PC11–20}}, \tag{9}$$

where the two alphas represent the $\mathrm{FMOM}^{\mathrm{PC11–20}}$ strategy's incremental returns over the $\mathrm{FMOM}^{\mathrm{PC1–10}}$ strategy in the first and second halves of the sample and FF5 denotes the five factors of the Fama-French model. In Panel C, we reverse this regression to explain the momentum present in the first 10 PC factors with that found in the other (or all) subsets of PC factors.

The momentum strategies significantly correlate with each other: the t-values for the slope estimates $\hat{b}$ are close to 10. This correlation is noteworthy because the individual PC factors are, by definition, orthogonal to each other in-sample (and approximately orthogonal out-of-sample). These positive correlations indicate that all sets of PC factors display momentum in a synchronized way: they all tend to be profitable or unprofitable at the same time. The $\alpha_{\text{first half}}$ intercepts in Panel B show that, during the first half of the sample, the strategy that trades the first 10 PC factors spans the last two strategies but not the strategies that trade PC factors 11 to 20 and 21 to 30. This slow decay of alphas stands in contrast to KNS who find that a model with a small number of low-order PC factors does well in explaining the expected returns on anomaly portfolios. The significant alphas in Panel B suggest that, during the first half of the sample, a large number of PCs is required to capture all momentum profits.

One possible explanation for this slow alpha decay relates to arbitrage activity. Arbitrageurs might have learned more about momentum (and how to harvest it) over time. The result that a small number of low-order PCs suffice to characterize the SDF relies on the assumption of the absence of near-arbitrage opportunities. If such opportunities were more plentiful in the past (because arbitrageurs did not know about them) but have grown scarce, we would expect alphas to decay faster later in the sample. Consistent with this argument,

[14] This deterioration in the momentum profits parallels the individual stock momentum. UMD's average return in the first half is 81 bps per month (t-value = 4.00); in the second half, it is 38 bps (t-value = 1.21).

the momentum in the first 10 PC factors subsumes the momentum found in all other sets of PC factors during the second half of the sample. Panel C shows that these spanning results do not work both ways. Momentum in the first 10 PC factors is informative about the cross section of returns in both the first and second halves when we control for any or all other momentum strategies.[15]

The result that PC factors—and, in particular, the high-eigenvalue factors—exhibit more momentum suggests that momentum is interconnected with the covariance structure of returns. In addition to being consistent with KNS model of sentiment investors and, more generally, the absence of near-arbitrage opportunities, it is also consistent with Haddad, Kozak, and Santosh (2020) empirical finding that the high-eigenvalue PC factors are predictable using the value spreads of Cohen, Polk, and Vuolteenaho (2003).

The finding that more systematic factors are more autocorrelated is specific neither to the Kozak, Nagel, and Santosh (2020) factors or the use of the PC methodology. Section IV of the Internet Appendix shows the same result for the 14 U.S. factors from Table I. Factors based on characteristics that explain more of the cross-sectional variation are also the ones more predictable by their own past returns. Size, market beta, idiosyncratic volatility, and quality-minus-junk, for example, are among the most predictable; at the same time, the characteristics underneath these factors explain more of the cross-sectional variation in returns.

III. Factor Momentum and Individual Stock Momentum

A. Transmission of Factor Momentum into the Cross Section of Stock Returns

If stock returns obey a factor structure, factor momentum transmits into the cross section of stock returns in the form of *cross-sectional* stock momentum of Jegadeesh and Titman (1993). In multifactor models of asset returns, such as the Intertemporal CAPM of Merton (1973) and the Arbitrage Pricing Theory

[15] The estimates in Panel B suggest that, at least in the first half of the sample, an investor could have earned a higher Sharpe ratio by combining the high-eigenvalue portfolio with the lower-eigenvalue portfolios. Over the entire sample period, the in-sample mean-variance efficient (MVE) portfolio of the five-factor momentum strategies invests 50% in FMOM $^{\text{PC1–10}}$, −13% in FMOM $^{\text{PC41–47}}$, and spreads the remaining weight across the middle strategies. This ex post optimal portfolio's Sharpe ratio is 15% higher than that of the highest-eigenvalue momentum portfolio. A real-time trader, however, would need to estimate the weights of this portfolio from historical data. We use a bootstrap procedure similar to that in Fama and French (2018) to measure the extent to which an investor can earn a higher Sharpe ratio, *out of sample*, by trading all PC factor momentum strategies instead of just the highest-eigenvalue strategy. We run 5,000 simulations in which we select half of the sample months at random, compute the weights of the MVE portfolio, and then compute the return for the resulting portfolio for the other half of the sample. The MVE and FMOM $^{\text{PC1–10}}$ strategies are statistically indistinguishable in this out-of-sample test: the FMOM $^{\text{PC1–10}}$ trader earns an average Sharpe ratio of 1.068, the MVE trader earns a Sharpe ratio of 1.089, and the difference of 0.022 is not statistically significantly different from zero. The bootstrapped 95% confidence interval for this 0.022 difference is $[-0.26, 0.17]$, indicating that the MVE trader is more likely to underperform the FMOM $^{\text{PC1–10}}$ trader by a large amount than the other way around.

of Ross (1976), multiple sources of risk determine expected returns. Consider a factor model in which asset excess returns obey an F-factor structure,

$$R_{i,t} = \sum_{f=1}^{F} \beta_i^f r_t^f + \varepsilon_{i,t}, \tag{10}$$

where $R_{i,t}$ is stock i's excess return, r_t^f is the return on factor f, β_i^f is stock i's beta on factor f, and $\varepsilon_{i,t}$ is the stock-specific return component. We assume that the factors do not exhibit any lead-lag relationships with the stock-specific return components, that is, $\mathrm{E}[r_{t'}^f \varepsilon_{i,t}] = 0$.

We now assume that asset prices evolve according to equation (10) and examine the payoffs to a cross-sectional momentum strategy; this strategy, as before, chooses weights that are proportional to stocks' performance relative to the cross-sectional average. The expected payoff to the position in stock i is

$$\mathrm{E}[\pi_{i,t}^{\mathrm{mom}}] = \mathrm{E}\big[(R_{i,-t} - \bar{R}_{-t})(R_{i,t} - \bar{R}_t)\big], \tag{11}$$

where $\bar{R}$ is the return on an equal-weighted index. Under the return process of equation (10), this expected profit becomes

$$\begin{aligned} \mathrm{E}[\pi_{i,t}^{\mathrm{mom}}] = &\sum_{f=1}^{F} \Big[\mathrm{cov}(r_{-t}^f, r_t^f)\,(\beta_i^f - \bar{\beta}^f)^2\Big] \\ &+ \sum_{f=1}^{F}\sum_{g\neq f}^{F} \Big[\mathrm{cov}(r_{-t}^f, r_t^g)\,(\beta_i^g - \bar{\beta}^g)\,(\beta_i^f - \bar{\beta}^f)\Big] \\ &+ \mathrm{cov}(\varepsilon_{i,-t}, \varepsilon_{i,t}) + (\eta_i - \bar{\eta})^2, \end{aligned} \tag{12}$$

where η_i is stock i's unconditional expected return. The expectation of equation (12) over the cross section of N stocks gives the expected return on the cross-sectional momentum strategy,

$$\begin{aligned} \mathrm{E}[\pi_t^{\mathrm{mom}}] = &\underbrace{\sum_{f=1}^{F} \Big[\mathrm{cov}(r_{-t}^f, r_t^f)\,\sigma_{\beta f}^2\Big]}_{\text{factor autocovariances}} + \underbrace{\sum_{f=1}^{F}\sum_{g\neq f}^{F} \Big[\mathrm{cov}(r_{-t}^f, r_t^g)\,\mathrm{cov}(\beta^f, \beta^g)\Big]}_{\text{factor cross-serial covariances}} \\ &+ \underbrace{\frac{1}{N}\sum_{i=1}^{N} \big[\mathrm{cov}(\varepsilon_{i,-t}, \varepsilon_{i,t})\big]}_{\substack{\text{autocovariances}\\ \text{in residuals}}} + \underbrace{\sigma_\eta^2}_{\substack{\text{variation in}\\ \text{mean returns}}}, \end{aligned} \tag{13}$$

where N is the number of stocks and $\sigma_{\beta f}^2$ and σ_η^2 are the cross-sectional variances of the portfolio loadings and stocks' unconditional expected returns.[16]

[16] Equation (13) does not assume that there are no arbitrage opportunities. If there are no arbitrage opportunities, then the firm-specific component $\varepsilon_{i,t}$ is mean zero and the last term in the

Equation (13) shows that the profits of the cross-sectional stock momentum strategy can come from four sources:

1. Positive autocorrelation in factor returns induces momentum profits through the first term. Cross-sectional variation in betas amplifies this effect. The amount of momentum in the cross section of stocks depends on the number of factors and how autocorrelated the typical factor is. Even if the typical factor is only weakly autocorrelated, the cross section of stocks will display a lot of momentum if the number of factors is large.[17]
2. The lead-lag return relationships between factors could also contribute to stock momentum profits. The strength of this effect depends on both the cross-serial covariance in factor returns and the covariances between factor loadings. This condition is restrictive: the cross-serial covariances of returns and the covariances of betas have to have the same signs. It would need to be, for example, that, first, SMB's return today positively predicts HML's return tomorrow and, second, stocks' SMB and HML loadings positively correlate.
3. Autocorrelation in firm-specific returns can also add to the profits of the cross-sectional momentum strategy.
4. The cross-sectional variation in mean returns of individual securities contributes to momentum profits through the Conrad and Kaul (1998) mechanism.

B. Pricing Momentum-Sorted Portfolios with Equity Momentum and Factor Momentum

Does factor momentum contribute to the returns of cross-sectional momentum strategies? In Table IV, we examine the connection between individual stock and factor momentum. In Panel A, we compare the performance of four asset pricing models in pricing portfolios sorted by prior one-year returns skipping a month. This sorting variable is the same as that used to construct Carhart (1997) UMD factor. The first model is the Fama-French five-factor model, the second model is this model augmented with the UMD factor, and the third and fourth models augment the five-factor model with factor momentum (FMOM) constructed from the 20 factors listed in Table I or the 10 high-eigenvalue PC factors from Table III. The factor momentum strategies are long factors with positive returns over the prior year and short those with negative returns.[18] We report alphas for the deciles and the factor loadings against UMD and FMOM.

decomposition, σ_η^2, represents variation in stocks' *risk premiums*. If there are arbitrage opportunities, this term also picks up cross-sectional variation in mispricings.

[17] In Section III.E.1, we set up simulations in which all momentum emanates from factor autocorrelations. We build on these simulations in Section V of the Internet Appendix to demonstrate how the cross section of stocks aggregates the autocorrelations found across multiple factors.

[18] The first term in equation (13), which links cross-sectional momentum to factor momentum, multiplies factor autocovariances with cross-sectional dispersion in betas. If there is no dispersion in betas, factor autocorrelation cannot transmit into the cross section. In the data, the differences

Table IV

Pricing Momentum-Sorted Portfolios with Momentum and Factor Momentum

Panel A compares the performance of four asset pricing models in explaining the monthly excess returns on 10 portfolios sorted by prior one-year returns skipping a month, $r_{t-12,t-2}$, (i) the Fama-French five-factor model, (ii) the five-factor model augmented with Carhart (1997) UMD factor, and (iii) and (iv) the five-factor model augmented with factor momentum. Factor momentum is long factors with positive prior one-year returns and short factors with negative returns. FMOM $_{\text{ind.}}$ uses the 20 factors listed in Table I; FMOM $_{\text{PC1–10}}$ uses the first 10 PC factors from Table III. We report alphas for these models and the loadings against the UMD and FMOM factors. Panel B reports estimates from regressions of Carhart (1997) UMD factor against the five-factor model (first row) and this model augmented with a momentum strategy that trades different subsets of PC factors (other rows). The sample in Panel A, except for the last column, begins in July 1964 and ends in December 2019. The sample in Panel A's last column and Panel B begins in July 1973.

Panel A: Pricing Decile Portfolios Sorted on Past Returns

	Asset Pricing Model						
		FF5 + UMD		FF5 + FMOM $_{\text{ind.}}$		FF5 + FMOM $_{\text{PC1–10}}$	
Decile	FF5 $\hat{\alpha}$	$\hat{\alpha}$	$\hat{b}_{\text{umd}}$	$\hat{\alpha}$	$\hat{b}_{\text{fmom}}$	$\hat{\alpha}$	$\hat{b}_{\text{fmom}}$
Losers	−0.75	−0.10	−0.93	−0.04	−2.46	−0.02	−3.65
	(−4.05)	(−0.94)	(−36.59)	(−0.28)	(−20.06)	(−0.09)	(−12.95)
2	−0.35	0.13	−0.70	0.16	−1.78	0.18	−2.66
	(−2.74)	(2.08)	(−46.76)	(1.54)	(−21.26)	(1.36)	(−14.05)
3	−0.20	0.18	−0.54	0.17	−1.30	0.20	−2.06
	(−1.90)	(2.92)	(−38.35)	(1.93)	(−17.78)	(1.83)	(−13.16)
4	−0.16	0.07	−0.33	0.12	−0.95	0.16	−1.43
	(−1.93)	(1.20)	(−22.77)	(1.69)	(−16.70)	(1.93)	(−11.67)
5	−0.16	−0.04	−0.17	−0.02	−0.47	0.02	−0.78
	(−2.45)	(−0.65)	(−12.30)	(−0.39)	(−9.07)	(0.22)	(−7.49)
6	−0.13	−0.09	−0.05	−0.07	−0.22	−0.05	−0.41
	(−2.05)	(−1.46)	(−3.52)	(−1.02)	(−4.26)	(−0.65)	(−3.90)
7	−0.12	−0.16	0.07	−0.14	0.09	−0.12	0.06
	(−1.94)	(−2.72)	(4.73)	(−2.32)	(1.83)	(−1.70)	(0.58)
8	0.04	−0.11	0.22	−0.09	0.44	−0.10	0.69
	(0.62)	(−2.05)	(16.96)	(−1.34)	(8.42)	(−1.44)	(6.68)
9	0.08	−0.14	0.33	−0.11	0.66	−0.14	0.94
	(1.08)	(−2.46)	(23.85)	(−1.45)	(11.04)	(−1.61)	(7.56)
Winners	0.57	0.17	0.57	0.16	1.42	0.01	2.38
	(4.82)	(2.32)	(32.93)	(1.60)	(17.21)	(0.05)	(14.26)
Winners	1.33	0.27	1.51	0.20	3.88	0.02	6.03
− Losers	(4.91)	(2.43)	(56.81)	(0.99)	(23.13)	(0.09)	(15.84)
N	666	666		666		558	
Avg. $\lvert\hat{\alpha}\rvert$	0.26	0.12		0.11		0.10	
GRS F-value	4.24	3.10		2.33		1.30	
GRS p-value	0.00%	0.04%		1.06%		20.29%	

(Continued)

Table IV—*Continued*

Panel B: Pricing UMD with Momentum in Subsets of PC Factors

Subset of PCs	Alpha		Factor Momentum		FF5	R^2
	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{b}_{\text{fmom}}$	$t(\hat{b}_{\text{fmom}})$		
None	0.62	3.36			Y	10.7%
1–10	−0.09	−0.59	3.90	17.50	Y	42.5%
11–20	0.35	1.95	2.05	6.89	Y	17.6%
21–30	0.28	1.60	3.14	9.07	Y	22.1%
31–40	0.34	2.01	3.03	10.91	Y	26.4%
41–47	0.40	2.33	2.56	10.54	Y	25.5%

Stock momentum is evident in the alphas of the Fama-French five-factor model. The alphas for the loser and winner portfolios are −0.75% and 0.57% per month (t-values = −4.05 and 4.82). The average absolute alpha across deciles is 26 bps. We significantly improve the model's ability to price these portfolios by adding UMD. The average absolute monthly alpha falls to 12 bps, and the return on the long-short portfolio falls from 1.33% to 0.27%. Yet, the alpha associated with the long-short portfolio is statistically significant with a t-value of 2.43.

The model augmented with factor momentum constructed from the 20 individual factors performs just as well as—or even better than—the Carhart (1997) six-factor model. The average absolute alpha falls to 11 bps per month, the GRS test statistic falls from 3.10 to 2.33, and the alpha of the high-minus-low portfolio falls from 0.27% to 0.20% (t-value = 0.99). Similar to the Carhart (1997) model, the estimated slopes against factor momentum increase monotonically from the bottom decile's −2.46 to the top decile's 1.42.

A momentum strategy that trades the first 10 PC factors performs just as well in pricing the momentum-sorted portfolios. The absolute pricing error is 9 bps per month and the GRS test does not reject the null that the alphas across the 10 test portfolios are zero.[19] The fact that the five-factor model augmented with factor momentum performs as well as (or better than) the Carhart six-factor model is surprising. The Carhart model sets a high bar because both the

in beta dispersions are not large enough for this effect to matter, perhaps because each factor is defined using the cross-sectional spread in characteristics or, in the case of the liquidity factor, cross-sectional variation in estimated betas. A factor momentum strategy that gives factors weights proportional to the cross-sectional variances of their betas earns an average return of 0.31% (t-value = 7.03) from July 1965 through December 2019; the unweighted strategy earns an average return of 0.34% (t-value = 6.86) over this period. In this computation, we estimate betas for individual stocks from univariate regressions using five years of monthly data up to month t, requiring a minimum of two years of data, and compute month $t + 1$ returns using this information. The correlation between the weighted and unweighted strategies is 0.95.

[19] The sample period in the last column starts in July 1973 because it uses the PC factors. Over this sample period, the winners-minus-losers portfolio's five-factor model alpha is 1.14% (t-value = 3.72).

factor and the test assets sort on the same variable, that is, UMD targets momentum as directly as, say, HML targets portfolios sorted by book-to-market.

In Panel B, we report estimates from time-series regressions in which Carhart (1997) momentum factor, UMD, is the dependent variable. The model on the first row is the Fama-French five-factor model and the other rows augment this model with strategies that trade momentum in different subsets of the PC factors. Momentum found in the high-eigenvalue factors explains all of UMD's profits; the alpha is −6 bps. Strategies based on the other subsets perform worse, leaving UMD with substantial alphas that are, except for the 21 to 30 set, statistically significant at the 5% level. Across the five models, momentum in the high-eigenvalue factors also explains the most of the time-series variation in UMD's return at $R^2 = 43\%$.

C. Alternative Momentum Factors: Spanning Tests

In Table V, we show that in addition to the "standard" individual stock momentum of Jegadeesh and Titman (1993), factor momentum also subsumes other cross-sectional momentum strategies. We construct three other momentum factors using the UMD methodology: industry-adjusted momentum of Cohen and Polk (1998) sorts stocks' by their industry-adjusted returns, intermediate momentum of Novy-Marx (2012) sorts stocks by their returns from month $t-12$ to $t-7$, and the Sharpe ratio momentum of Rachev et al. (2007) sorts stocks by the returns scaled by the volatility of returns. We also construct the industry momentum strategy of Moskowitz and Grinblatt (1999). This strategy sorts 20 industries based on their prior six-month returns and takes long and short positions in the top and bottom three industries.

Panel A of Table V introduces these alternative momentum factors alongside the two factor momentum strategies: momentum in the 20 individual factors and that in the first 10 PC factors. All momentum factors earn statistically significant average returns and Fama-French five-factor model alphas. Although the average returns associated with the two factor momentum strategies are the lowest, they are also the least volatile. Their Sharpe and information ratios, which are proportional to the t-values associated with the average returns and five-factor model alphas, are the highest among all of the momentum strategies.

Panel B shows estimates from spanning regressions in which the dependent variable is one of the individual stock momentum factors. The model is the Fama-French five-factor model augmented with one of the factor momentum strategies. These regressions have two interpretations. From an investment perspective, a statistically significant alpha implies that an investor would have earned a higher Sharpe ratio by having traded the left-hand-side factor in addition to the right-hand-side factors (Huberman and Kandel (1987)). From an asset pricing perspective, a statistically significant alpha implies that the asset pricing model that only contains the right-hand-side factors is dominated by a model that also contains the left-hand-side factor (Barillas and Shanken (2017)). Although all definitions of momentum earn statistically

Table V

Alternative Definitions of Momentum: Spanning Tests

Panel A reports monthly average returns and Fama-French five-factor model alphas for alternative momentum factors. Every factor, except for industry momentum, is similar to the UMD factor of Jegadeesh and Titman (1993) ("standard momentum"). We sort stocks into six portfolios by market values of equity and prior performance. A momentum factor's return is the average return on the two high portfolios minus that on the two low portfolios. Industry momentum uses the Moskowitz and Grinblatt (1999, table I) methodology; it is long the top three industries based on their prior six-month returns and short the bottom three industries, with each stock classified into one of 20 industries. Panel A also reports references for the original studies that use these alternative definitions. Panel B reports estimates from regressions in which the dependent variable is the monthly return on one of the individual stock momentum strategies. These regressions augment the five-factor model with momentum found in either the 20 individual factors or the first 10 PC factors. Panel C reports estimates from regressions in which the dependent variable is one of the factor momentum strategies. These regressions augment the five-factor model with one of the individual stock momentum factors (UMD*) or, in the last row, with all five individual stock momentum factors on the right-hand side at the same time. The sample begins in July 1964 and ends in December 2019 except for the regressions that use the PC-based factor momentum, in which the sample begins in July 1973.

Panel A: Factor Means and Fama-French Five-Factor Model Alphas

		Monthly Returns			FF5 Model	
Momentum Definition	Reference	$\bar{r}$	*SD*	$t(\bar{r})$	$\hat{\alpha}$	$t(\hat{\alpha})$
	Individual Stock Momentum					
Standard momentum	Jegadeesh and Titman (1993)	0.64	4.22	3.93	0.70	4.28
Ind.-adjusted momentum	Cohen and Polk (1998)	0.41	2.64	3.96	0.50	4.93
Industry momentum	Moskowitz and Grinblatt (1999)	0.63	4.60	3.54	0.69	3.77
Intermediate momentum	Novy-Marx (2012)	0.48	3.02	4.12	0.56	4.81
Sharpe ratio momentum	Rachev et al. (2007)	0.55	3.59	3.94	0.63	4.51
	Factor Momentum					
Momentum in individual factors		0.33	1.20	7.01	0.29	6.21
Momentum in PC factors 1–10		0.19	0.64	7.07	0.18	6.51

Panel B: Regressions of Individual Stock Momentum Strategies on Factor Momentum

Individual Stock	Factor Momentum				
	Individual Factors		PC Factors 1–10		
Momentum, UMD*	$\hat{\alpha}$	$FMOM_{ind.}$	$\hat{\alpha}$	$FMOM_{PC1-10}$	FF5
Standard momentum	0.00 (−0.04)	2.43 (24.72)	−0.09 (−0.60)	3.90 (17.52)	Y
Industry-adjusted momentum	0.14 (1.67)	1.23 (17.63)	0.10 (0.99)	1.90 (12.68)	Y
Industry momentum	0.02 (0.12)	2.32 (18.83)	−0.16 (−0.85)	4.10 (15.51)	Y
Intermediate momentum	0.15 (1.51)	1.41 (17.72)	0.17 (1.40)	2.20 (12.64)	Y
Sharpe ratio momentum	0.02 (0.19)	2.12 (25.45)	−0.05 (−0.39)	3.63 (19.74)	Y

(*Continued*)

Table V—*Continued*

Panel C: Regressions of Factor Momentum on Individual Stock Momentum Strategies

Individual Stock Momentum, UMD*	Dependent Variable: Momentum in Individual Factors $\hat{\alpha}$	Momentum in Individual Factors UMD*	Momentum in PC Factors 1–10 $\hat{\alpha}$	Momentum in PC Factors 1–10 UMD*	FF5
Standard momentum	0.15 (4.44)	0.20 (24.72)	0.13 (5.53)	0.09 (17.52)	Y
Industry-adjusted momentum	0.16 (4.07)	0.26 (17.63)	0.13 (5.17)	0.12 (12.68)	Y
Industry momentum	0.19 (4.88)	0.15 (18.83)	0.14 (5.88)	0.07 (15.51)	Y
Intermediate momentum	0.16 (4.15)	0.23 (17.72)	0.13 (4.95)	0.10 (12.64)	Y
Sharpe ratio momentum	0.14 (4.20)	0.23 (25.45)	0.11 (5.17)	0.11 (19.74)	Y
All of above	0.14 (4.30)	-	0.12 (5.32)	-	Y

significant average returns and five-factor model alphas, both factor momentums span all of them. The maximum t-value across the 10 specifications is industry-adjusted momentum's 1.67 when explained with the momentum in individual factors.[20]

Panel C shows that none of the alternative definitions of individual stock momentum spans either version of factor momentum. Across the 12 specifications, the lowest t-value for factor momentum's alpha is 4.07. The last row augments the Fama-French five-factor model with all five individual stock momentum factors. In these specifications, factor momentums' alphas are significant with t-values of 4.30 (individual factors) and 5.45 (PC factors 1 to 10).

Table V indicates that factor momentum contains information not present in any other forms of momentum and yet, at the same time, no other form of momentum is at all informative about the cross section of stock returns when controlling for factor momentum. These spanning results suggest that individual stock momentum is, at least in large part, a manifestation of factor momentum. An investor who trades individual stock momentum indirectly times factors; she would do better by timing the factors directly.

[20] We exclude the market factor from the set of individual factors when constructing the factor momentum strategy that trades individual factors. The results here and elsewhere are not sensitive to the decision to exclude or include the market factor. Panel B's alphas, for example, range from −1 bps (t-value = −0.10) to 13 bps (t-value = 1.60) when the momentum strategy $\text{FMOM}_{\text{ind.}}$ trades the market factor as well.

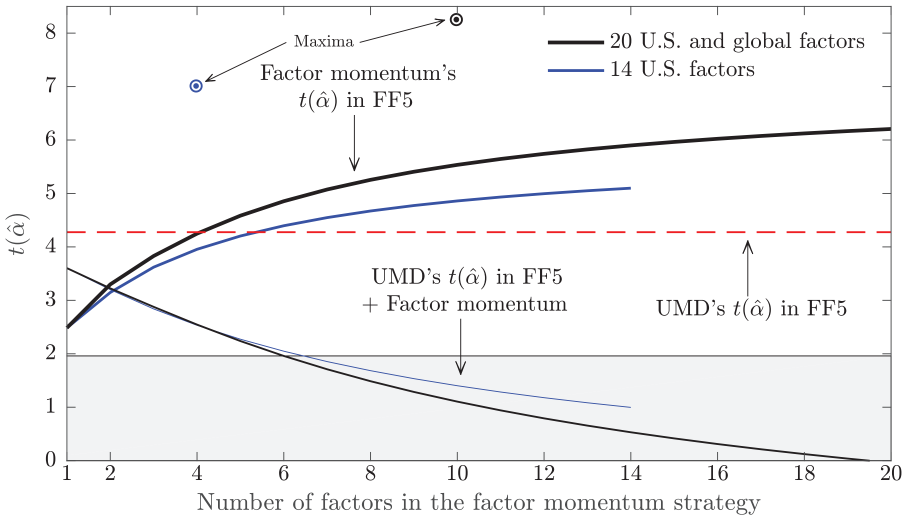

Figure 2. Individual stock momentum versus factor momentum as a function of the number of factors. We form all subsets of the 14 U.S. factors (blue lines) or 20 U.S. and global factors (black lines) listed in Table I and form time-series factor momentum strategies that trade these factors. A time-series factor momentum strategy is long factors with positive returns over the prior year and short those with negative returns. The thick line represents the factor momentum strategy's average $t(\hat{\alpha})$ from the Fama-French five-factor model regression, the thin line represents UMD's average $t(\hat{\alpha})$ from a regression that augments the five-factor model with the factor momentum strategy, and the dashed line denotes UMD's $t(\hat{\alpha})$ from the Fama-French five-factor model regression. The circles denote the combinations with the highest t-values in the two universes of factors. The shaded region indicates t-values below 1.96.

D. Individual Stock Momentum versus Factor Momentum with Alternative Sets of Factors

The factor momentum strategy that trades individual factors takes positions in up to 20 factors. Tables IV and V show that this factor momentum explains individual stock momentum. In Figure 2, we measure the extent to which this result is sensitive to the number and identity of the factors included in the version of factor momentum that trades the 20 individual factors.

We first construct all possible combinations of factors, ranging from one factor to the full set of 20 factors. We then construct a factor momentum strategy from each set of factors and estimate two regressions. The first regression is the Fama-French five-factor model with factor momentum as the dependent variable. The dependent variable in the second regression is UMD and the model is the Fama-French five-factor model augmented with factor momentum. We record the t-values associated with the alphas from all possible models, and plot averages of these t-values as a function of the number of factors.[21] The black lines in Figure 2 depict these combinations drawn from the full set of 20 factors. We also construct all possible factor momentum strategies that trade only the 14 U.S. factors. The blue lines in Figure 2 depict these combinations. We also plot, for reference, the t-value associated with UMD's alpha in the five-factor model.

Figure 2 shows that the t-value associated with factor momentum's five-factor model alpha monotonically increases in the number of factors. Consider first strategies drawn from the full set of 20 factors. When factor momentum alternates between long and short positions in just one factor, the average t-value is 2.49. When it trades 10 factors, it is 5.54. When we reach 20 factors, it is 6.21. At the same time, factor momentum's ability to span UMD improves. The typical one-factor factor momentum strategy leaves UMD with an alpha that is statistically significant with a t-value of 3.60. However, when the number of factors increases to 10, this average t-value decreases to 1.10, and with all 20 factors, the t-value is −0.04. The patterns are the same when we limit the analysis to the 14 U.S. factors. For example, the average t-value associated with UMD's alpha is 1.40 when we construct factor momentum from 10 U.S. factors.[22]

These estimates suggest that factor momentum's ability to span UMD is not specific to the set of factors used; as the number of factors increases, the autocorrelations found within most sets of factors aggregate to explain individual stock momentum. Figure 2 supports our prediction that individual stock momentum is an aggregation of the autocorrelations found in factor returns—the more factors we identify, the better we capture UMD's return.

E. Do Firm-Specific Returns Display Momentum?

E.1. Simulation Evidence

If factor momentum drives *all* momentum in the cross section of stock returns, firm-specific returns should not display any continuation. A natural test would therefore be to measure momentum in firm-specific returns. However, when these returns have to be estimated as residuals from factor models, we

[21] The sample begins in July 1964 and ends in December 2019. Because some factors have later start dates, we exclude those factor combinations that would result in a sample that does not span the full 1964 to 2019 period. There are, for example, $\frac{20!}{(20-6)!6!} = 38{,}760$ six-factor combinations. We exclude seven combinations that would result in start dates later than July 1964. The total number of one- to 20-factor combinations is 1,048,575 of which 1,048,448 span the full sample period.

[22] The t-values we report in Figure 2 are averages of various combinations. We could indulge in some data dredging and ask which combinations of factors display the most momentum. Among the 14 U.S. factors, a combination of four factors produces a strategy with a $t(\hat{\alpha})$ of 6.99; in the set of all 20 factors, the highest t-value of 8.24 belongs to a 10-factor strategy. The blue and black circles in Figure 2 denote these maxima. More "powerful" factor momentum strategies than the 20-factor version therefore lurk within this set of factors. We use the all-factor strategy to err on the side of caution. Any strategy that uses a subset of all available factors would need to be justified on an ex ante basis or subjected to tests that address the multiple hypothesis testing problem.

encounter three problems: (i) we do not know the identities of all factors, (ii) we do not observe true factor returns, and (iii) we can only estimate stocks' factor loadings with noise. It is therefore not possible—absent a natural experiment that would allow us to identify true firm-specific returns—to attribute conclusively cross-sectional momentum to "factor momentum" and "residual momentum."

To illustrate the issue arising from omitted factors, suppose that two systematic factors drive excess stock returns,

$$R_{i,t} = \beta_{i,1}F_{1,t} + \beta_{i,2}F_{2,t} + \varepsilon_{i,t}. \tag{14}$$

A researcher who knows only about the first factor then estimates the residual as

$$\hat{\varepsilon}_{i,t} = \left[r_{i,t} - \beta_{i,1}F_{1,t}\right] + \beta_{i,2}F_{2,t}, \tag{15}$$

where we assume that the researcher observes the true factor F_1 and stock i's beta against it. If the researcher does not have the full set of factors, *estimated* residuals can display momentum even if firm-specific returns are IID. In terms of equation (15), $\hat{\varepsilon}_{i,t}$ would display momentum if the omitted factor, F_2, displays momentum. What the researcher views as the firm-specific residual is only the residual net of known factors. This problem of *omitted-factor momentum* grows worse if we do not observe true factors or betas.

Table VI demonstrates the difficulty of disentangling factor momentum from residual momentum. We simulate returns from an economy in which only factor returns are positively autocorrelated. Ten systematic factors and IID firm-specific innovations drive stock returns. Factors' risk premiums follow the AR(1) process, and a factor's return is the sum of its risk premium and an IID innovation. We simulate data under two assumptions about factors. In the "Symmetric factors" specification all 10 factors have the same variance and all factors' risk premiums are equally persistent. In the "Uncorrelated market factor" specification, the first factor explains five times as much of the cross section of returns as the other nine factors and its risk premium is uncorrelated. Section V of the Internet Appendix details these simulations. In the simulations, the average nonmarket factor's autocorrelation is similar to that in the data: the correlation between month t returns and the average returns from month $t-12$ to $t-1$ is 0.25 in the data and 0.20 in the simulations.

We construct three momentum strategies using the simulated returns. Individual stock momentum is long the top decile of stocks with the highest returns over the prior year and short the bottom decile. The residual momentum strategy is long and short the top and bottom deciles of stocks based on their firm-specific residuals over the prior year. We estimate firm-specific residuals from a factor model with $1, 2, \ldots$ or 10 factors from month $t-72$ to $t-13$ and use these beta estimates to compute residuals from month $t-12$ to $t-1$. The factor momentum strategy is long factors with positive returns over the prior year and short factors with negative returns. The column labeled "Number of known factors" in Table VI indicates the number of factors used to compute

Table VI

Residual Momentum versus Factor Momentum: Simulations

This table reports average t-values from simulations that assess the strengths of individual stock momentum, residual momentum, and factor momentum in an economy in which only factor returns are positively autocorrelated. We simulate 672 months of returns from a market with 2,000 stocks. Ten systematic factors and IID firm-specific innovations drive stock returns. In the "Symmetric factors" specification, the 10 factors have the same variance, their risk premiums are equally persistent, and stocks' betas against these factors are mean zero. In "Uncorrelated market factor" specification, the first factor explains five times as much of the cross section of the stock returns as each of the other nine factors, the first factor's risk premium is uncorrelated, and stocks' betas against the first factor have a mean of one; the betas against the other nine factors are mean zero. Section V of the Internet Appendix details these simulations. The individual stock momentum strategy is long the top decile of stocks with the highest average returns over the prior year and short the bottom decile. The residual momentum strategy is long the top decile and short the bottom decile formed by firm-specific residuals over the prior year. We estimate firm-specific residuals from a factor model with $1, 2, \ldots$ or 10 factors from month $t-72$ to $t-13$ and use the resulting betas to compute residuals for months $t-12$ to $t-1$. The factor momentum strategy is long factors with positive returns over the prior year and short factors with negative returns. The column labeled "Number of known factors" indicates the number of factors used to compute firm-specific residuals and for trading factor momentum. We report average t-values from 10,000 simulations.

	Symmetric Factors		Uncorrelated Market Factor	
Number of Known Factors	Residual Momentum	Factor Momentum	Residual Momentum	Factor Momentum
1	5.65	1.55	5.62	−0.02
2	5.28	2.23	5.21	0.84
3	4.86	2.72	4.82	1.46
4	4.45	3.14	4.38	2.00
5	4.01	3.49	3.94	2.46
6	3.52	3.85	3.42	2.88
7	2.97	4.16	2.91	3.23
8	2.33	4.44	2.26	3.56
9	1.51	4.69	1.42	3.88
10	0.00	4.95	−0.01	4.16
Individual stock momentum	6.01		4.68	

firm-specific residuals and for trading factor momentum. We report average t-values from 10,000 simulations.

Stock returns display momentum in these simulations. In the "Symmetric factors" specification, the individual stock momentum strategy is significant with an average t-value of 6.01. Although there is no momentum in stock returns per se, this strategy is profitable because it indirectly bets on the persistence in factor risk premiums: stocks with the best performance over the prior year, on average, load positively on factors with high past returns and negatively on factors with low past returns; stocks with the worst performance have, on average, the opposite loadings. Because there is no momentum in firm-specific residuals, residual momentum strategies are on average less profitable than the individual stock momentum strategy. Residual momentum strategies, however, remain statistically significantly profitable as long as the investor's asset pricing model omits two factors; at this point, the residual momentum's strategy average t-value is 2.33. When the investor has the full factor model, the estimated residuals on average equal firm-specific innovations and the residual momentum strategy earns no profits.

Factor momentum strategies are profitable because factors positively autocorrelate. The greater the number of factors the investor knows, the more profitable these strategies are. If the investor knows just two factors, the factor momentum strategy's average t-value is 2.23; if the investor knows all 10 factors, its t-value is 4.95.[23]

Residual momentum is stronger than individual stock momentum if the total amount of momentum in the known factors is less than that in the omitted factors. The "Uncorrelated market factor" specification illustrates this possibility by assuming that the first factor is more systematic than the other factors and serially uncorrelated. If an investor computes firm-specific residuals from a one-factor model with only the uncorrelated factor, residuals become better measures of the remaining nine autocorrelated factors. Although individual stock momentum strategy's average t-value in these simulations is 4.68, the residual momentum strategy under a one-factor model has a t-value of 5.62.

The residual momentum strategy becomes less profitable as we begin removing the autocorrelated factors. It remains more profitable than the individual stock momentum strategy up to three factors. Although the factor momentum strategy becomes more profitable as the investor adds more factors, this strategy continues to be hurt by including the market factor. If the investor traded only the nine autocorrelated factors, then the factor momentum strategy would be profitable with an average t-value of 4.69—this case corresponds to the "Symmetric factors" specification with nine known factors.

Table VI shows that attempts to disentangle factor momentum from residual momentum run into an omitted-variables problem. If the asset pricing model is incomplete, a residual momentum strategy may be profitable by virtue of trading omitted-factor momentum.

E.2. Actual data

In Table VII, we examine the profitability of three residual momentum strategies. We compute residuals from the CAPM and the Fama-French three- and five-factor models. We estimate stocks' factor loadings using data from

[23] Although all momentum resides in factors, factor momentum is not as profitable as individual stock momentum because it takes long and short positions in all 10 factors. This strategy therefore also invests in factors whose past returns (and therefore estimated risk premiums) are close to zero. The individual stock momentum and residual momentum strategies, by virtue of taking positions only in the top and bottom deciles of stocks, implicitly bet more on the factors with large positive or negative past returns and therefore large positive or negative risk premiums.

Table VII

Residual Momentum versus Factor Momentum: Actual Data

This table reports average returns and alphas for UMD-style individual stock momentum strategies. The strategy in the first row sorts stocks by their raw returns from month $t-12$ to $t-2$. The strategies in the other rows sort stocks by their estimated residuals based on the CAPM or the Fama-French three- or five-factor model. The independent variable is a factor momentum strategy that trades either the 20 off-the-shelf factors or the first then high-eigenvalue PC factors. The strategy in the first row, which is the same as the standard UMD, trades the same stocks as in the residual momentum strategies. t-Values are in parentheses. Except for the last column, the sample begins in July 1967 and ends in December 2019. In the last two columns, the sample begins in July 1973.

		Control for Factor Momentum			
		Individual Factors		PC Factors 1–10	
Sorting Variable	Average Return	$\hat{\alpha}$	$\hat{b}_{\text{fmom}}$	$\hat{\alpha}$	$\hat{b}_{\text{fmom}}$
Raw returns	0.45	−0.19	1.96	−0.29	3.69
	(2.88)	(−1.45)	(19.16)	(−2.04)	(17.18)
CAPM residuals	0.58	0.08	1.53	−0.05	3.08
	(4.29)	(0.67)	(16.68)	(−0.38)	(16.58)
FF3 residuals	0.44	0.15	0.90	0.00	2.09
	(3.83)	(1.35)	(10.27)	(0.04)	(11.92)
FF5 residuals	0.37	0.17	0.63	0.00	1.72
	(3.39)	(1.52)	(7.32)	(−0.03)	(10.13)

month $t-72$ to $t-13$, requiring a minimum of three years of data, and compute average residuals from month $t-12$ to $t-2$. We then construct UMD-like residual momentum strategies by sorting stocks into six portfolios by size and past returns and taking long and short positions in the winner and loser portfolios. For the sake of comparability, we recompute UMD in this table so that it is based on the same universe of stocks as that used for the residual momentum strategies.

The individual stock momentum strategy earns an average return of 45 bps per month (t-value = 2.88) in this sample. This strategy's alpha turns negative when we control for factor momentum strategy that trades either the 20 individual factors or the first 10 high-eigenvalue PC factors.

A momentum strategy based on the CAPM residuals is more profitable than the strategy based on stocks' raw returns. This strategy earns an average return of 58 bps (t-value = 4.29). This increase is consistent with the view that the market factor is weakly serially correlated relative to its importance in explaining cross-sectional variation in realized returns; removing this factor renders the residuals *more* informative about the other factors. This residual momentum strategy also correlates significantly with the two-factor momentum strategies. Its alpha net of factor momentum in the individual factors is 8 bps (t-value = 0.67) and net of the momentum in the PC factors is −5 bps.

The momentum strategy based on the three-factor model residuals is less profitable (44 bps, t-value = 3.83) than that based on CAPM residuals, and the strategy based on the five-factor model residuals is less profitable still (37 bps, t-value = 3.39). The profits decrease because the additional factors that we now expunge—size, value, and so forth—contribute meaningfully to momentum profits. These strategies' alphas are also close to zero when we control for factor momentum. For example, the alpha of the five-factor model residual strategy is zero (t-value = −0.03) when we control for the momentum found in the first 10 PC factors.

The estimates in Table VII are consistent with the momentum in individual stock returns rising from factors. When we remove a factor that is very systematic relative to its autocorrelation, the momentum strategy becomes more profitable, whereas when we remove factors that contribute to momentum profits, the strategy becomes increasingly less profitable. The extent to which residuals display momentum net of factor momentum depends on the properties of the factors that remain outside the factor model used to estimate the residuals. The insignificant alphas in Table VII suggest that whatever these additional factors beyond the five-factor model are, they are still largely the same as those found within the first 10 PC factors.[24]

In Section III of the Internet Appendix, we return to Section III.A's linear-weight decomposition and reach the same conclusion about the tension between factor momentum, residual momentum, and omitted-factor momentum. Under the CAPM, the decomposition attributes almost all of the profits to the autocorrelation in firm-specific returns but in a seven-factor model it attributes most of the returns to the autocorrelations in factor returns.

An additional observation about the nature of residual momentum strategies is in order. These strategies can appear profitable not only because there is momentum in firm-specific returns, but also because these strategies also implicitly bet against betas. To see why, note that a firm's estimated residual return is its return minus the product of the estimated factor loadings and factor returns. Firms with high residual returns have either high returns or low estimated betas while those with low residuals have either low returns or high estimated betas. Thus, residual momentum strategy is also, in part, long low-beta stocks and short high-beta stocks. We show in Section VI of the Internet Appendix that residual momentum strategies do indeed make significant bets against betas. Their five-factor model alphas exceed their average returns because of these bets, and, by controlling for betting-against-beta factors, factor momentum strategies span the three residual momentum strategies.

IV. Momentum vis-à-vis Other Factors

A. Unconditional and Conditional Correlations with the Momentum Factor

The puzzling feature of individual stock momentum is its low correlations with other factors. Over the July 1963 through December 2019 period, the

[24] The absence of residual momentum is also consistent with the predictions of KNS. Only those sentiment-driven demand components that align with covariances distort prices; there are no firm-specific distortions and, by extension, no firm-specific momentum.

adjusted R^2 from regressing UMD on the Fama-French five-factor model is just 9%. These estimates might imply that factors *unrelated* to the market, size, value, profitability, and investment factor must explain the remaining 91% of the variation. Alternatively, these estimates might suggest that momentum is a distinct risk factor.

However, the *unconditional* correlations between UMD and the other factors significantly understate their associations. Consider, for example, the size factor. If size has performed well, UMD will, by construction, be long small-cap stocks and short large-cap stocks. Because both UMD and SMB are now long small-cap stocks and short large-cap stocks, we expect them to correlate positively the next month. If, in contrast, size has performed poorly, UMD will be short small-cap stocks and long large-cap stocks. We would therefore expect UMD and SMB to correlate negatively. The same mechanism should hold for all factors: if a factor has performed well, UMD will be long that factor, and UMD and the factor will positively correlate, but if the factor has performed poorly, UMD will be short that factor and the correlation will be negative.

In Table VIII, we report factors' correlations with UMD. In particular, we report three correlations: the unconditional correlation, the correlation conditional on the factor's return over the prior year being positive, and the correlation conditional on this return being negative. The unconditional correlations between UMD and the factors are low; 11 of the 20 correlations with the individual factors are positive, and the correlation between UMD and the portfolio of all 20 factors is 0.04. The correlations conditional on past returns, however, are remarkably different. Except for the short-term reversals factor, all factors correlate more with UMD when their past returns are positive.[25] For 17 of these 19 factors, the difference is statistically significant at the 5% level. The first row assigns all factors to two groups based on their past returns. The basket of factors with positive past returns has a correlation of 0.45 with UMD; the basket of factors with negative returns has a correlation of −0.51. Table A.III shows that both the long and short legs contribute to this pattern: the short leg drives the positive correlation following a year of positive returns and the long leg drives the negative correlation following a year of negative returns.

Because the unconditional correlations between momentum and the other factors are close to zero, most factor models, such as the five-factor model, explain none of momentum profits. However, this result does not imply that momentum is "unrelated" to the other factors. Table VIII shows that the unconditional correlations are close to zero because these correlations are significantly time-varying. In fact, momentum appears to relate to *all* factors—unconditional correlations close to zero are due to momentum switching between being long and short other factors. This argument of time-varying loadings also suggests a solution to the puzzle that Cochrane (2011, p. 1075) poses when discussing a behavioral explanation for momentum:

[25] The short-term reversals factor has almost 100% turnover per month (Novy-Marx and Velikov (2016)). Any association between past factor returns and current holdings therefore breaks down.

Table VIII

Unconditional and Conditional Correlations with the Momentum Factor

This table reports correlations between UMD and factor returns: ρ is UMD's unconditional correlation with the factor, ρ^+ is the correlation conditional on the factor's return over the prior year being positive, and ρ^- is the correlation conditional on the prior-year return being negative. The first row takes the average of all 20 factors or averages of factors with positive or negative returns over the prior year. The z-value in the last column is from a test that the conditional correlations are equal. This test uses Fisher (1915) z-transformation, $1\Big/\sqrt{\frac{1}{N^+-3}+\frac{1}{N^--3}}(\tanh^{-1}(\hat{\rho}^+)-\tanh^{-1}(\hat{\rho}^-))\sim N(0,1)$, where $\tanh^{-1}(x)=\frac{1}{2}\frac{\ln(1+x)}{\ln(1-x)}$ and where N^+ and N^- are the number of observations used to estimate ρ^+ and ρ^-, respectively.

	Unconditional Correlation	Conditional Correlations		H_0: $\hat{\rho}^+=\hat{\rho}^-$
Factor	$\hat{\rho}$	$\hat{\rho}^+$	$\hat{\rho}^-$	z-Value
Pooled	0.04	0.45	−0.51	18.37
U.S. Factors				
Size	−0.04	0.16	−0.39	7.20
Value	−0.20	0.17	−0.58	10.45
Profitability	0.11	0.46	−0.41	11.22
Investment	−0.03	0.19	−0.37	7.13
Accruals	0.13	0.30	−0.15	5.46
Betting against beta	0.18	0.41	−0.22	6.70
Cash flow to price	−0.13	0.23	−0.59	11.38
Earnings to price	−0.17	0.20	−0.61	11.50
Liquidity	−0.03	0.03	−0.14	2.15
Long-term reversals	−0.09	0.10	−0.43	7.02
Net share issues	0.11	0.36	−0.42	10.44
Quality minus junk	0.28	0.46	−0.41	11.00
Residual variance	0.21	0.67	−0.56	18.44
Short-term reversals	−0.30	−0.39	−0.19	−2.28
Global Factors				
Size	0.07	0.09	0.05	0.35
Value	−0.16	0.15	−0.48	5.81
Profitability	0.27	0.33	−0.02	2.60
Investment	0.06	0.40	−0.43	7.99
Betting against beta	0.22	0.24	0.15	0.73
Quality minus junk	0.42	0.48	−0.17	4.87

> "For example, 'extrapolation' generates the slight autocorrelation in returns that lies behind momentum. But why should all the momentum stocks then rise and fall together the next month, just as if they are exposed to a pervasive, systematic risk?"

Momentum stocks do indeed comove because of pervasive, systematic risks. Winners, for example, are stocks that positively load on factors that have performed well and negatively on those that have done poorly.[26]

[26] The five-factor model and the 9% adjusted R^2 that it gives to UMD illustrates this issue. Suppose that instead of regressing UMD on the five factors, we split each factor into two parts:

B. Momentum in Momentum-Neutral Factors

Does momentum reside in factors or individual stocks? Because factors are portfolios of stocks, factor returns ultimately arise from individual stock returns. The argument that factor momentum drives individual stock momentum is that the individual stocks that make up the factor are inconsequential—momentum is about the factor loadings (or characteristics) associated with the factors, not about specific companies.

A source of some ambiguity in showing causality is that individual stock momentum may induce incidental momentum in factor returns. If the size factor, for example, has performed well over the prior year, then the stocks in this factor's long leg have, by definition, higher past returns than those in its short leg. The existence of individual stock momentum alone would then predict that the size factor should continue to perform well. This incidental momentum effect could give rise to what looks like factor momentum even if momentum does not reside in factors. In this section, we quantify the extent to which this mechanism contributes to factor momentum profits. Whereas the tests in Section III capture whether individual stock momentum survives when we control for factor momentum—it does not—we now approach this question about the connection between individual stock and factor momentum from the opposite direction: how much of factor momentum survives when we control for individual stock momentum?

We examine the origins of momentum by measuring factor momentum in *momentum-neutral* factors. A factor has incidental momentum if $\sum_{i=1}^{N} w_{i,t} r_{i,t-12,t-2} \neq 0$, that is, if the factor's past return, computed using its *current* weights $w_{i,t}$, is nonzero. An investor who invests in such a factor may indirectly benefit from the momentum in stock returns. We construct momentum-neutral factors by taking the Kozak, Nagel, and Santosh (2020) factors and twisting the factor weights as little as possible to render them orthogonal with respect to past returns. The objective is to find new weights x_i such that

$$\min_{x_i} \sum_i (w_i - x_i)^2 \quad \text{s.t.} \quad \sum_{i=1}^{N} x_i = 0 \;\; \text{and} \;\; \sum_{i=1}^{N} x_i r_{i,t-12,t-2} = 0. \tag{16}$$

In Section VII of the Internet Appendix, we show that the weights x_i are equivalent to the residuals from a cross-sectional regression of the original factor weights on past returns:

$$w_{i,t} = a + b\, r_{i,t-12,t-2} + x_{i,t}. \tag{17}$$

HML_t^{up}, $\text{HML}_t^{\text{down}}$, SMB_t^{up}, $\text{SMB}_t^{\text{down}}$, and so forth, where

$$\text{HML}_t^{\text{up}} = \begin{cases} \text{HML}_t & \text{if HML's prior-year return is positive,} \\ 0 & \text{otherwise,} \end{cases}$$

and similarly for the other factors. This *conditional* five-factor model explains 49% of the variation in UMD's returns.

We call the factors with weights $x_{i,t}$ *momentum-neutral* factors. The idea of momentum-neutral factors applies to all factors. An investor investing in value may notice, for example, that the returns on the stocks in the long and short legs over the prior year differ. To avoid an incidental bet on stock-level momentum, the investor could alter the weights to render the value and growth portfolios perfectly indistinguishable from each other based on past returns: the value portfolio that the investor buys has performed exactly as well (or poorly) as the growth portfolio he sells. Momentum-neutrality means that at any point in time, $\sum_{i=1}^{N} x_i r_{i,t-12,t-2} = 0$. This condition does *not* mean that the factor's past return is zero: a factor's return is based on time t weights and future returns while momentum neutrality is based on time t weights and *past* returns.

The weights of the standard and momentum-neutral factors are close to each other. Consider, for example, the value factor, which invests in stocks based on their book-to-market ratios. Most of the cross-sectional return variation in this factor's weights are unrelated to past returns. The average and median R^2s from the regression in equation (17) are 2.9% and 1.5%. These numbers imply that the original factor weights and the momentum-neutral weights are very close: the average correlation between them is $\sqrt{1-0.029} = 0.99$. The average R^2s of the 47 factors range from 0.4% (growth in long-term net operating assets) to 10.2% (Asness and Frazzini (2013) monthly version of value); the average across all factors is 2.4%.

Table A.II shows the annualized CAPM alphas for momentum-neutral versions of the 47 factors. Momentum-neutral factors typically earn similar premiums as the original factors but with lower volatility, and thus they often earn higher information ratios. Consider, for example, the 37 original factors whose premiums are statistically significant at the 5% level. Momentum-neutral versions of 30 of these factors earn higher information ratios than the original factors.

Table IX shows that the factor momentum strategy that trades momentum-neutral factors is *more profitable* than the one that trades the original factors. The strategy that trades the 10 high-eigenvalue PCs extracted from the standard factors earns a five-factor alpha that is significant with a t-value of 6.51; this estimate corresponds to an annualized information ratio of 0.96. This t-value increases to 7.53 when we construct the strategy using momentum-neutral PC factors; this estimate corresponds to an information ratio of 1.10. Controlling for momentum-neutral factors, the strategy that trades the PCs based on the original factors has an alpha of 3 bps (t-value = 1.45). The strategy that trades momentum-neutral factors, however, remains significant with a t-value of 3.91 when we reverse this regression.

The finding that the factor momentum in the momentum-neutral factors subsumes that in the original factors rejects the possibility that factor momentum is merely incidental momentum. In fact, the results indicate that incidental momentum explains *none* of the factor momentum profits.

Table IX

Factor Momentum in Momentum-Neutral Factors

This table reports estimates from time-series regressions in which the dependent variable is the return on a factor momentum strategy. We construct time-series factor momentum strategies from the 47 factors listed in Table A.II, using either the original or momentum-neutral versions of these factors. Momentum-neutral factors adjust factor weights so that they are orthogonal to individual stock returns from month $t-12$ to $t-2$. We extract the PC factors from either the original or momentum-neutral factors and trade momentum in the first 10 high-eigenvalue PC factors. The independent variables are the five factors of the Fama-French model and the other factor momentum strategy. The sample begins in July 1973 and ends in December 2019.

	Dependent Variable			
	Momentum in Original Factors		Momentum in Momentum-Neutral Factors	
Independent Variable	(1)	(2)	(3)	(4)
Alpha	0.18	0.03	0.15	0.06
	(6.51)	(1.45)	(7.53)	(3.91)
Momentum in original factors				0.52
				(24.83)
Momentum in momentum-neutral factors		1.01		
		(24.83)		
FF5 factors	Y	Y	Y	Y
N	558	558	558	558
R^2	2.4%	53.9%	5.9%	55.5%

V. Conclusion

Positive autocorrelation is a pervasive feature of factor returns. Factors with positive returns over the prior year earn significant premiums; those with negative returns earn premiums that are indistinguishable from zero. Factor momentum is a strategy that bets on these autocorrelations in factor returns.

Factor momentum transmits into the cross section of stock returns through variation in stocks' factor loadings. Consistent with this mechanism, we show that factor momentum explains the "standard" momentum of Jegadeesh and Titman (1993), industry-adjusted momentum, industry momentum, intermediate momentum, Sharpe momentum, and three versions of residual momentum. By contrast, these other momentum factors do not explain factor momentum. An empirical model that controls for momentum found in high-eigenvalue PC factors describes the data well. Our results imply that momentum is not a distinct factor; rather, it is the sum of the autocorrelations found in the other factors. An investor trading momentum bears systematic risk because all winners and all losers have similar factor exposures. We are left with the impression that momentum is unrelated to the other factors only because these loadings change over time.

Factor momentum may stem from mispricing. We show that KNS model with sentiment investors produces factor momentum when sentiment is sufficiently persistent. This model predicts that, in this case, momentum should concentrate in factors that explain more of the cross section of stock returns. The data support this prediction: factor momentum, particularly during the second half of the sample, concentrates in high-eigenvalue factors. Some of our results, however, appear to challenge the interpretation that all momentum is high-eigenvalue momentum. In the early part of the sample, for example, momentum does not concentrate only in the high-eigenvalue factors in which we would expect to find it—it spreads out to some lower-eigenvalue factors. Although this pattern is consistent with the entry of arbitrageurs, we have not devised a test for testing whether this mechanism drives our results.

We leave two questions for future research. First, although factor momentum is consistent with KNS model of sentiment investors, this consistency does not imply that factor momentum must stem from mispricing. KNS's point, after all, is that the extent to which covariances align with premiums provides no clues as to whether factor premiums compensate for risk or reflect mispricing. The finding that momentum resides in the high-eigenvalue factors is a more general implication of the absence of near-arbitrage opportunities. If we were to write down a rational model with time-varying risk premiums, would such a model provide additional—and distinguishing—predictions about factor momentum? Second, although we find no residual momentum net of factor momentum, this result does not conclusively prove that firm-specific returns are serially uncorrelated. Because of complications stemming from omitted factors and estimated betas, we cannot settle this issue if we do not observe the true asset pricing model. To pass the final judgment on the divide between factor and firm-specific momentum, one would need to devise a method for extracting—or find a natural experiment for identifying—firm-specific returns.

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Appendix

Table A.I
Autocorrelations in Factor Returns

Table II in the main text reports estimates from regressions in which the dependent variable is a factor's return in month t and the explanatory variable is an indicator variable that takes the value of one if the factor's return over the prior year is positive and zero otherwise. This table reports estimates from regressions in which the dependent variable is a factor's monthly return and the independent variable is the factor's average return over the prior year. We estimate these regressions using pooled data (first row) and separately for each anomaly (remaining rows). We cluster standard errors by month in the pooled regression.

	Intercept		Slope	
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
Pooled	0.27	5.27	0.25	2.59
U.S. Factors				
Accruals	0.22	2.82	−0.03	−0.23
Betting against beta	0.41	2.22	0.50	3.38
Cash flow to price	0.23	2.14	0.15	0.92
Investment	0.21	2.59	0.24	1.43
Earnings to price	0.24	2.12	0.18	1.12
Book-to-market	0.21	1.75	0.26	1.60
Liquidity	0.35	2.44	0.09	0.54
Long-term reversals	0.12	1.18	0.41	3.26
Net share issues	0.15	1.64	0.34	1.92
Quality minus junk	0.28	3.27	0.27	1.76
Profitability	0.20	1.71	0.26	1.08
Residual variance	0.10	0.51	0.20	1.10
Market value of equity	0.17	1.45	0.29	1.80
Short-term reversals	0.50	2.76	−0.01	−0.04
Momentum	0.64	3.91	0.00	−0.02
Global Factors				
Betting against beta	0.59	3.05	0.31	1.82
Investment	0.11	1.00	0.31	1.26
Book-to-market	0.19	1.34	0.44	2.26
Quality minus junk	0.44	3.82	0.12	0.55
Profitability	0.29	3.46	0.16	0.82
Market value of equity	0.09	0.81	0.20	1.02
Momentum	0.75	3.86	−0.09	−0.45

Table A.II

Standard and Momentum-Neutral Factors Based on the Kozak, Nagel, and Santosh (2020) Characteristics

This table reports annualized CAPM alphas and t-values associated with these alphas for 47 factors. These factors are based on the Kozak, Nagel, and Santosh (2020) characteristics except for the seven characteristics related to momentum. Each characteristic is converted into a centered cross-sectional rank normalized by the average absolute deviation from the mean. A factor's return in month t is the product of stock returns in month t and these characteristics ("weights") in month $t-1$. The factors are not re-signed; the size factor, for example, is long large stocks and short small stocks. Each month, we exclude stocks with market values less than 0.01% of the total market value of all common stocks traded on NYSE, Amex, and Nasdaq. Original factors use raw firm characteristics. Momentum-neutral factors use weights orthogonal to stocks' returns over the prior year (see Section VII in the Internet Appendix). Freq. is the frequency at which the characteristics are recomputed: A = annual, Q = quarterly, and M = monthly.

			Original		Momentum-Neutral	
#	Factor	Freq.	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\alpha}$	$t(\hat{\alpha})$
1	Size	A	−0.58%	−0.99	−0.13%	−0.28
2	Value	A	2.45%	3.48	2.44%	3.95
3	Gross profitability	A	0.77%	1.39	0.62%	1.20
4	Cash flow duration	M	−2.55%	−3.65	−2.58%	−4.11
5	Value-Profitability	M	3.11%	5.84	2.79%	5.90
6	Piotroski's F-score	A	1.62%	3.98	1.34%	4.97
7	Debt issuance	A	0.13%	0.28	0.18%	0.56
8	Share repurchases	A	1.59%	3.47	1.45%	4.80
9	Share issuance, annual	A	−2.57%	−5.52	−2.47%	−6.30
10	Accruals	A	−0.98%	−3.13	−0.80%	−2.68
11	Asset growth	A	−2.29%	−5.03	−2.33%	−5.69
12	Asset turnover	A	0.97%	1.90	0.75%	1.55
13	Gross margins	A	−0.21%	−0.50	−0.12%	−0.32
14	Dividend yield	A	1.75%	3.28	1.87%	3.78
15	Earnings/Price	A	3.19%	4.51	2.97%	5.09
16	Cash flow/Market value of equity	A	2.77%	4.09	2.62%	4.40
17	Net operating assets	A	−2.02%	−5.15	−1.81%	−5.03
18	Investment	A	−2.10%	−5.31	−2.01%	−5.57
19	Investment-to-capital	A	−2.28%	−3.26	−2.32%	−3.99
20	Investment growth	A	−1.76%	−5.25	−1.63%	−5.39
21	Sales growth	A	−1.66%	−3.30	−1.84%	−4.19
22	Leverage	A	1.84%	2.34	1.77%	2.62
23	Return on assets	A	0.73%	1.64	0.65%	1.57
24	Return on equity	A	0.82%	1.79	0.68%	1.70
25	Sales-to-Price	A	2.73%	3.72	2.46%	3.97
26	Growth in LTNOA	A	−0.26%	−0.86	−0.20%	−0.72
27	Dividend growth	A	−0.70%	−2.31	−0.76%	−2.62
28	Abnormal investment	A	−0.19%	−0.32	−0.40%	−0.68
29	Short interest	M	−0.95%	−2.05	−0.96%	−2.25
30	Long-term reversals	M	−1.73%	−3.02	−1.76%	−3.34
31	Value	M	1.94%	2.23	2.80%	4.91
32	Share issuance	M	−2.62%	−5.01	−2.76%	−6.70

(*Continued*)

Table A.II—*Continued*

#	Factor	Freq.	Original $\hat{\alpha}$	Original $t(\hat{\alpha})$	Momentum-Neutral $\hat{\alpha}$	Momentum-Neutral $t(\hat{\alpha})$
33	PEAD (SUE)	M	2.62%	4.42	1.57%	3.61
34	Return on book equity	M	2.43%	4.08	1.53%	3.45
35	Return on market equity	M	4.58%	5.30	4.03%	6.75
36	Return on assets	Q	2.21%	3.48	1.29%	2.50
37	Short-term reversals	M	−2.19%	−3.09	−2.42%	−4.53
38	Idiosyncratic volatility	M	−3.40%	−3.75	−3.27%	−5.16
39	Beta arbitrage	M	−2.92%	−3.73	−3.08%	−4.74
40	Seasonality	M	1.74%	3.90	1.49%	3.72
41	Industry relative reversals	M	−3.81%	−7.79	−3.85%	−9.75
42	Industry relative reversals (low vol)	M	−5.00%	−15.18	−5.04%	−16.45
43	Composite issuance	M	−2.73%	−6.45	−2.70%	−7.30
44	Price	M	0.55%	0.75	−0.51%	−1.13
45	Firm age	M	1.69%	2.46	1.79%	4.06
46	Share volume	M	−2.29%	−2.78	−2.66%	−4.35
47	Initial public offering	M	−6.87%	−2.99	−5.59%	−4.92

Table A.III

Conditional Covariances with the Momentum Factor: Decomposition

This table reports covariances between UMD and factor returns. It is similar to Table VIII except that (i) we report covariances instead of correlations and (ii) we decompose covariances into two components: the covariance between UMD and each factor's long leg (L) and that between UMD and the *negative* of each factor's short leg ($-S$). These covariances add up to the total covariance between UMD and the factor. We compute the covariance between the month t returns of UMD and the factors, conditioning on the sign of the factor's average return from month $t-12$ to $t-1$. Data are for the 12 U.S. factors with portfolio-level data. The data begin in July 1963 and end in December 2019.

	Conditional on a Year of Positive Factor Returns Cov. between UMD and:			Conditional on a Year of Negative Factor Returns Cov. between UMD and:		
Factor	L	$-S$	$L-S$	L	$-S$	$L-S$
Pooled	−3.51	6.16	2.64	−4.28	−1.18	−5.46
Size	−5.54	6.94	1.39	1.36	−6.43	−5.08
Value	−3.62	5.42	1.80	−6.37	−2.29	−8.67
Profitability	−3.91	7.50	3.59	−0.64	−3.90	−4.54
Investment	−3.80	5.31	1.51	−2.39	−0.78	−3.17
Accruals	−0.49	2.69	2.20	−5.34	4.00	−1.34
Cash flow to price	−2.98	5.21	2.23	−5.32	−1.93	−7.25
Earnings to price	−3.76	5.68	1.91	−4.70	−3.20	−7.90
Long-term reversals	−4.81	5.94	1.14	−4.81	0.96	−3.84
Net share issues	−4.71	8.97	4.26	−0.09	−3.04	−3.12
Quality minus junk	7.31	−4.75	2.56	−11.39	6.40	−4.99
Residual variance	−5.14	21.22	16.05	2.07	−11.74	−9.67
Short-term reversals	−5.12	1.47	−3.64	−10.33	5.56	−4.77

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.