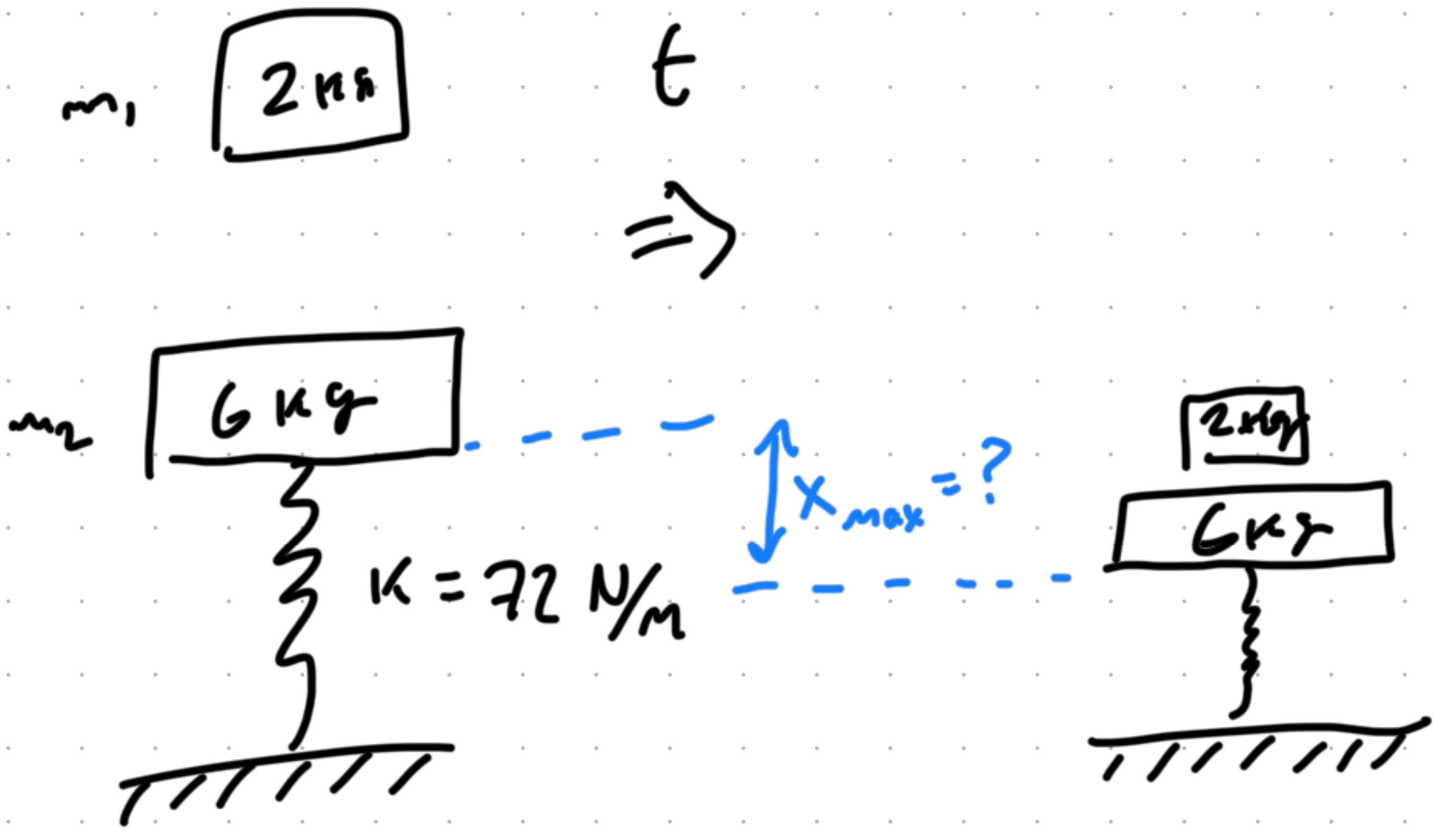
2015 #23

$$\Delta E = \frac{1}{2}k(h+x)^2 - \frac{1}{2}kh^2 = \frac{1}{2}kx^2 + \underline{khx} \; !$$



initial compression $F_s = m_2 g = kh$

$$h = \frac{m_2 g}{k} = 0.83 \text{ m}$$

impact speed $mgy = \frac{mv^2}{2}$ $\quad v = \sqrt{2gy}$

$$= 10 \text{ m/s}$$

post-impact speed $v_f = \frac{m_1 v}{m_1+m_2} = 2.5 \text{ m/s}$

$KE_1 = \frac{1}{2}(m_1+m_2)v_f^2$

$U_{s_1} = \frac{1}{2}kh^2$

$U_{g1} = 0$

$KE_2 = 0$

$U_{s_2} = \frac{1}{2}k(h+x_{max})^2$

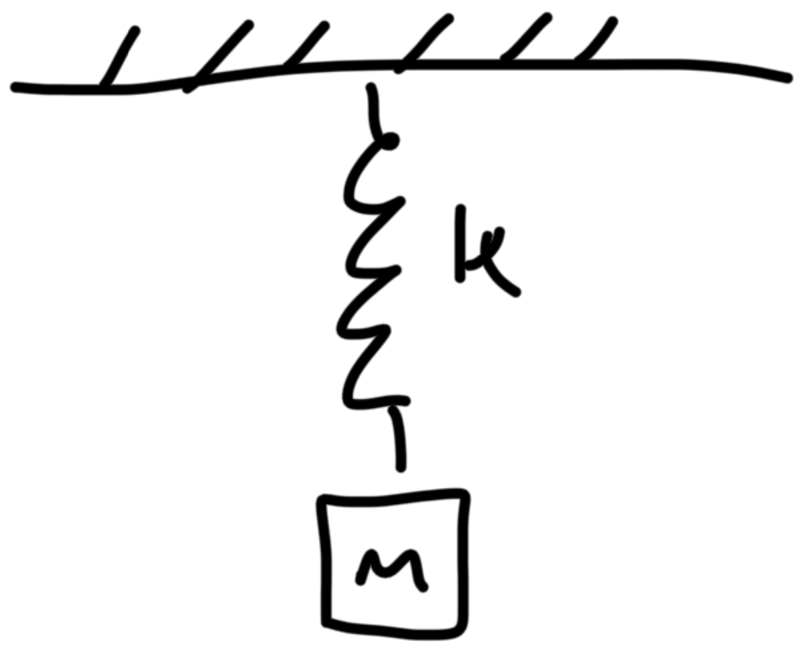
$U_{g_2} = -(m_1+m_2)g x_{max}$

$$(m_1+m_2)v_f^2 + \cancel{kh^2} = k(h+x_{max})^2 - 2(m_1+m_2)g x_{max}$$

$$= \cancel{kh^2} + 2khx_{max} + kx_{max}^2 - 2(m_1+m_2)g x_{max}$$

$$0 = kx_{max}^2 + 2\left[kh - (m_1+m_2)g\right]x_{max} - (m_1+m_2)v_f^2$$

Sol. $x_{max} = 1.16$ m $\quad$ B

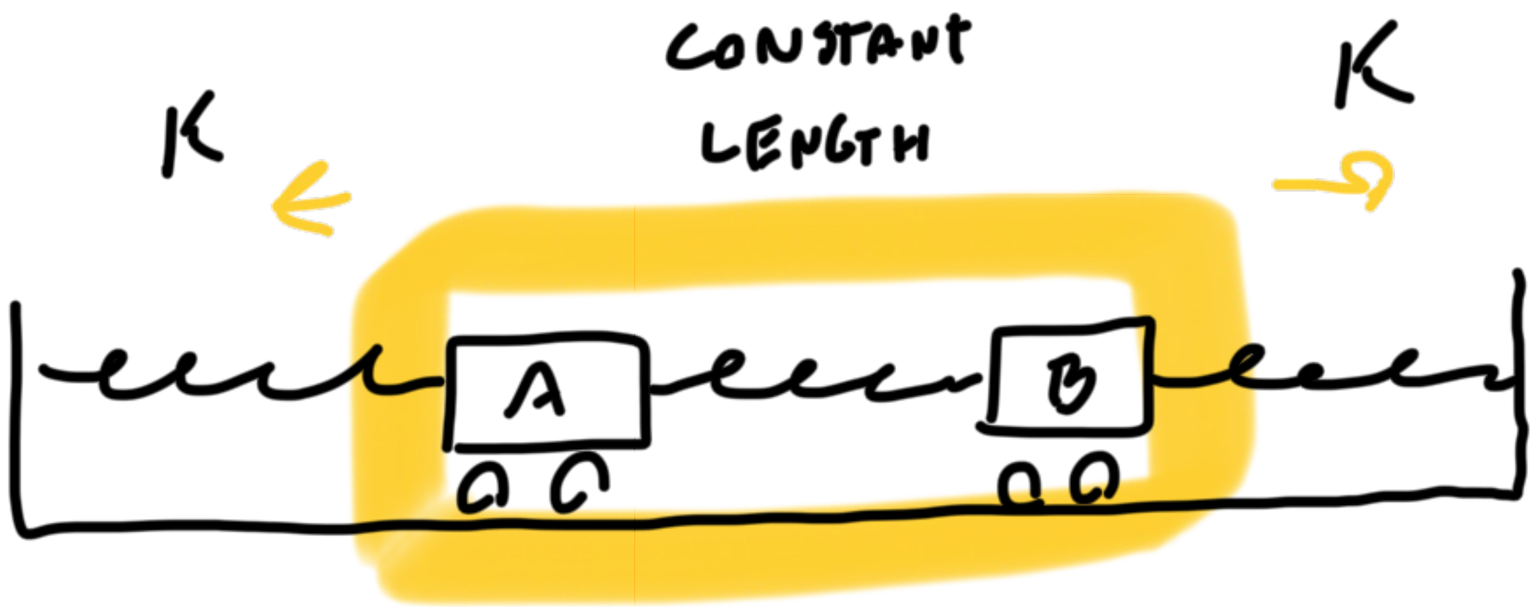


Same ω

different equilibrium lengths

2015 #25

1.

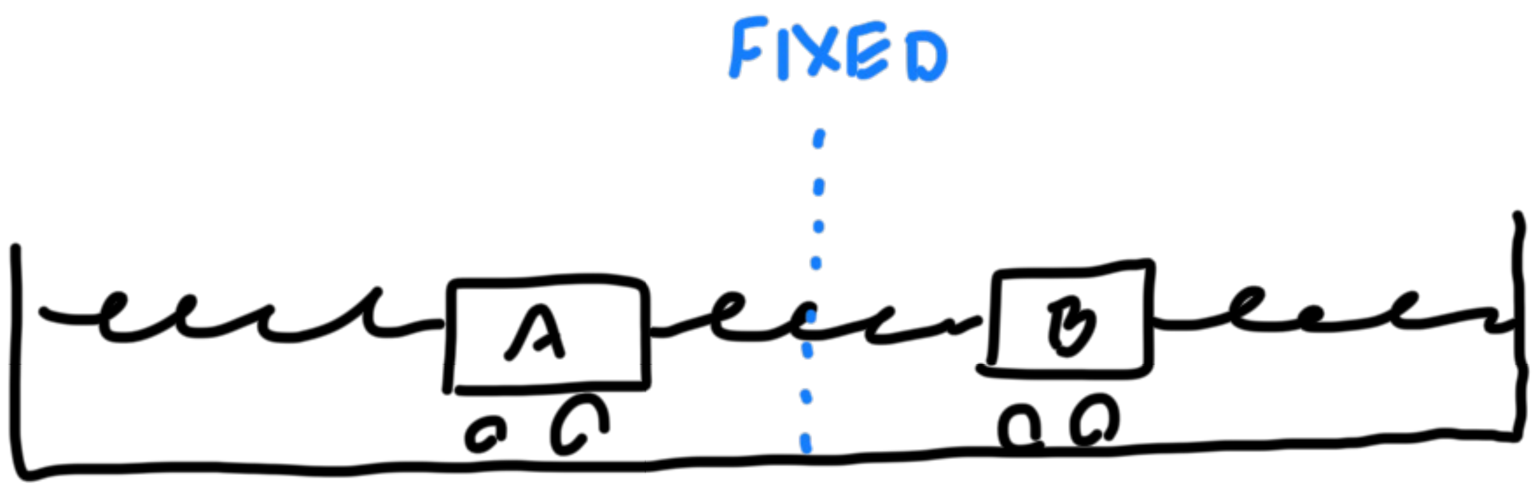


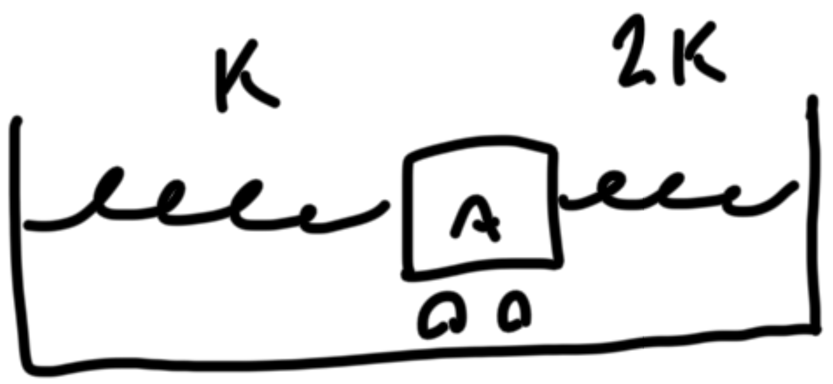
$$\omega_1 = \sqrt{\frac{2k}{2m}} = \sqrt{\frac{k}{m}}$$

$$\frac{\omega_2}{\omega_1} = \sqrt{3}$$

A

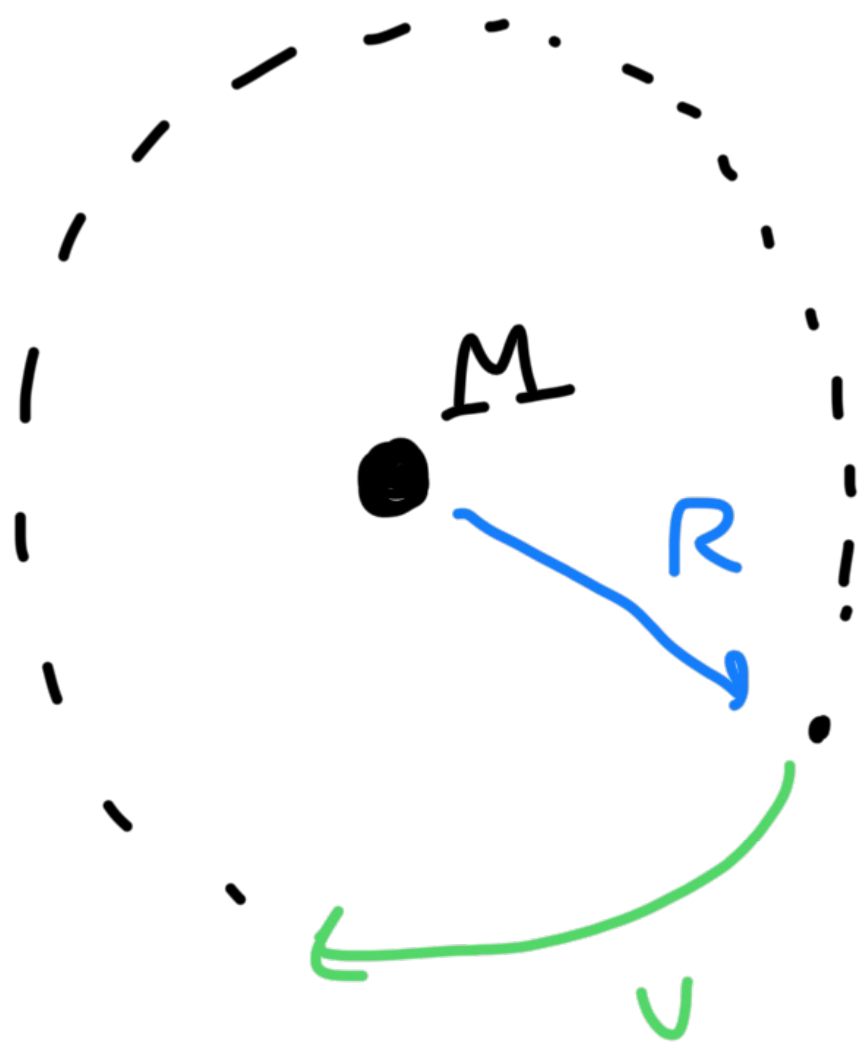
2.





$$\omega_2 = \sqrt{\frac{3k}{m}}$$

Orbits



$$F_{cent} = \frac{mv^2}{R} = \frac{GMm}{R} = F_{grav}$$

given R, $v = \sqrt{\frac{GM}{R}}$

$$T = 2\pi \frac{R}{v} = 2\pi \frac{R}{\sqrt{GM/R}} = 2\pi \sqrt{\frac{R^3}{GM}}$$

Non-gravitational orbits

$F \propto R^{-2}$

let $F = qR^{-2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{q}{mR}}$

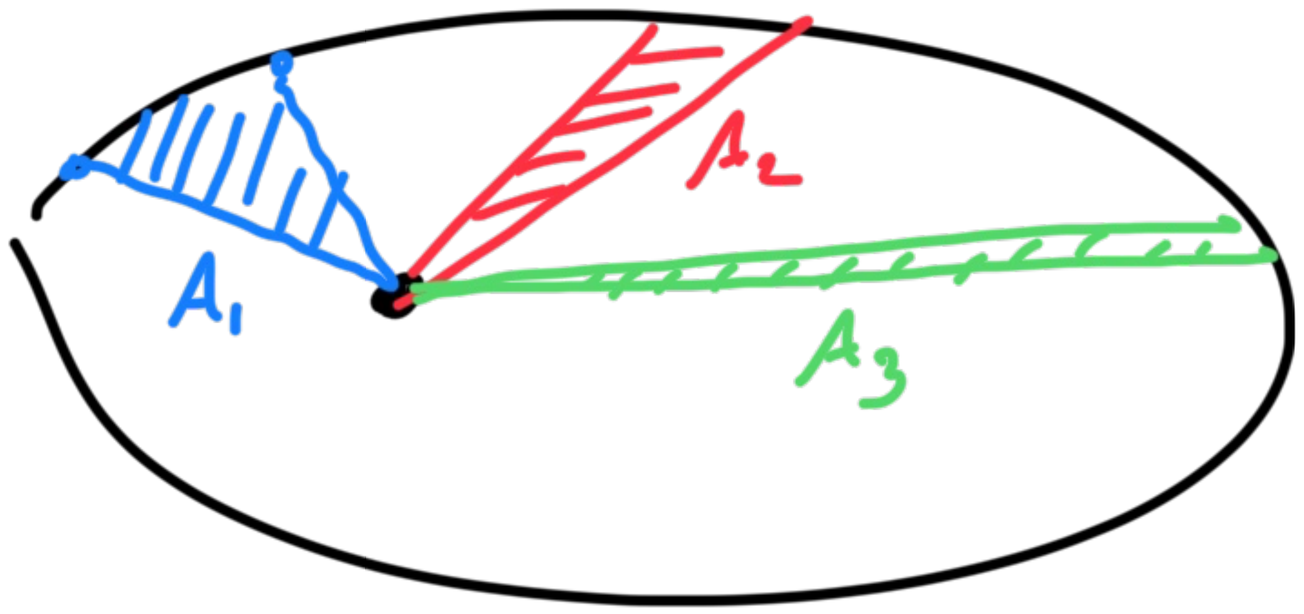
$q = GMm \Rightarrow v = \sqrt{\frac{GM}{R}}$

Kepler's Laws

1. all bound orbits are elliptical with the orbited body at a focus.

2. equal areas in equal times

$$T^2 \cdot \frac{r^2}{2} \cdot \frac{d\theta}{dt} = \pi ab$$



$A_1 = A_2 = A_3$

$t_1 = t_2 = t_3$

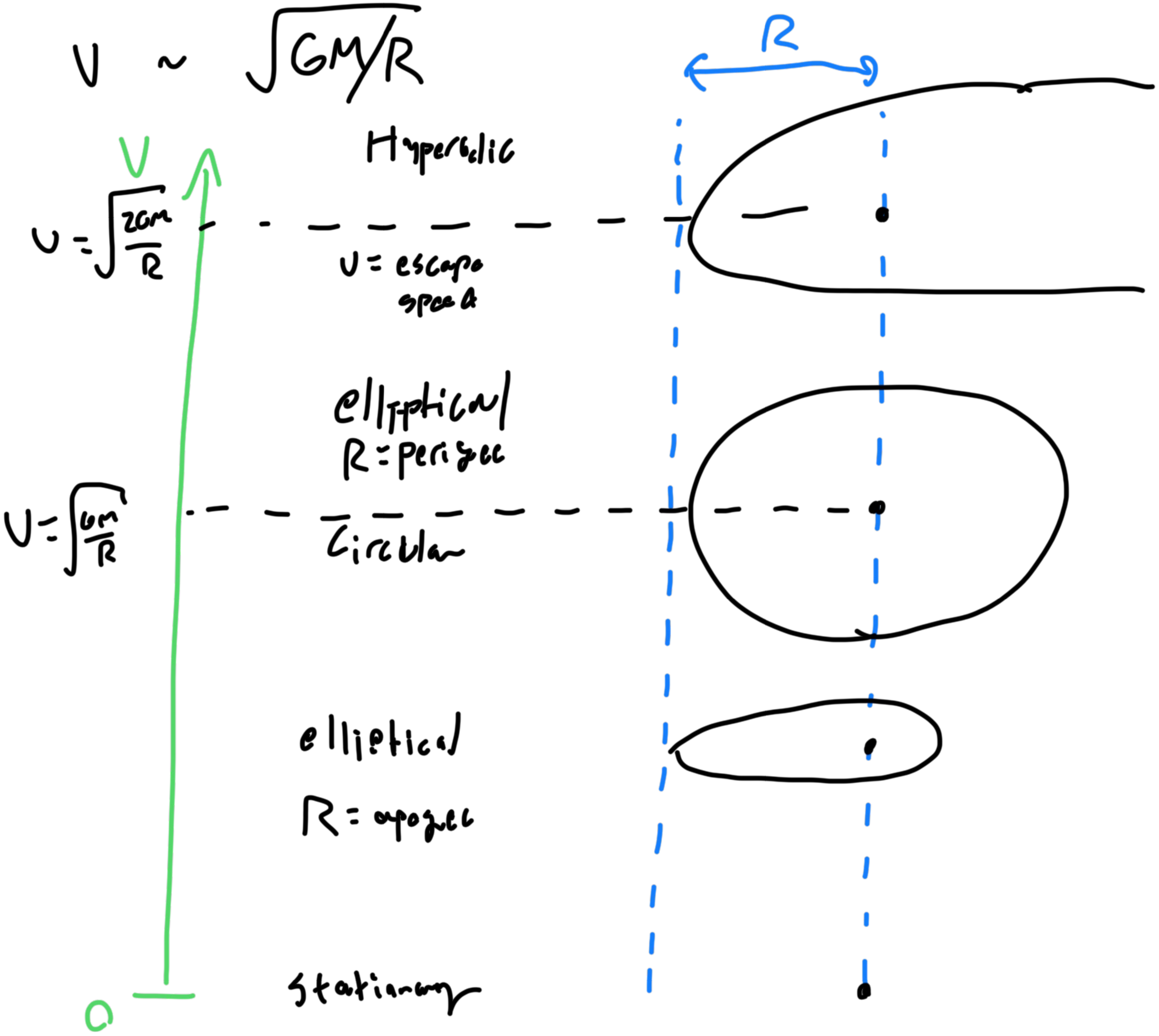
$\dot{\theta}_1 < \dot{\theta}_2 < \dot{\theta}_3$

$\dot{\theta} = \frac{d\theta}{dt} \neq \omega$

3. $T^2 \propto R^3$

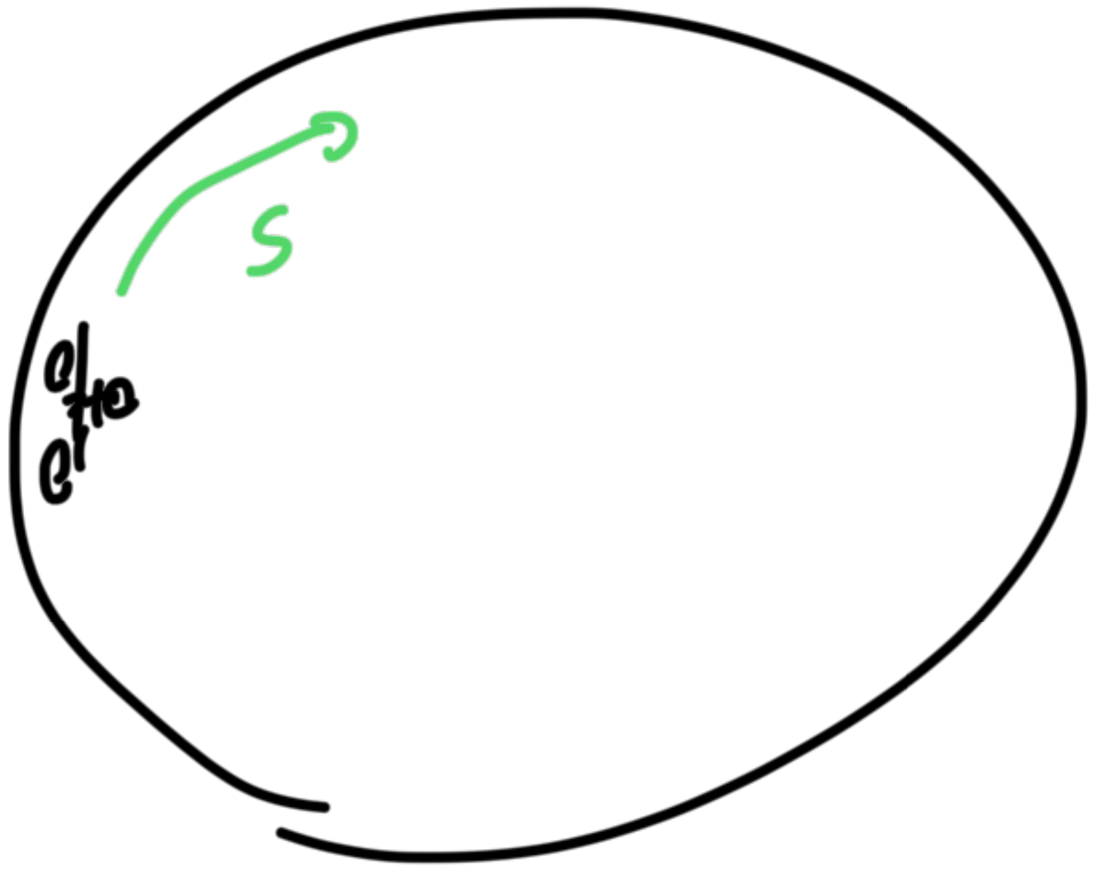
Non-circular orbits

$v \sim \sqrt{GM/R}$



Topic 7.4

2017 #1



$\otimes g$

into page

$\mu \propto s^k \quad k = ?$

to prevent sliding down wall

Friction is up, to counter gravity.

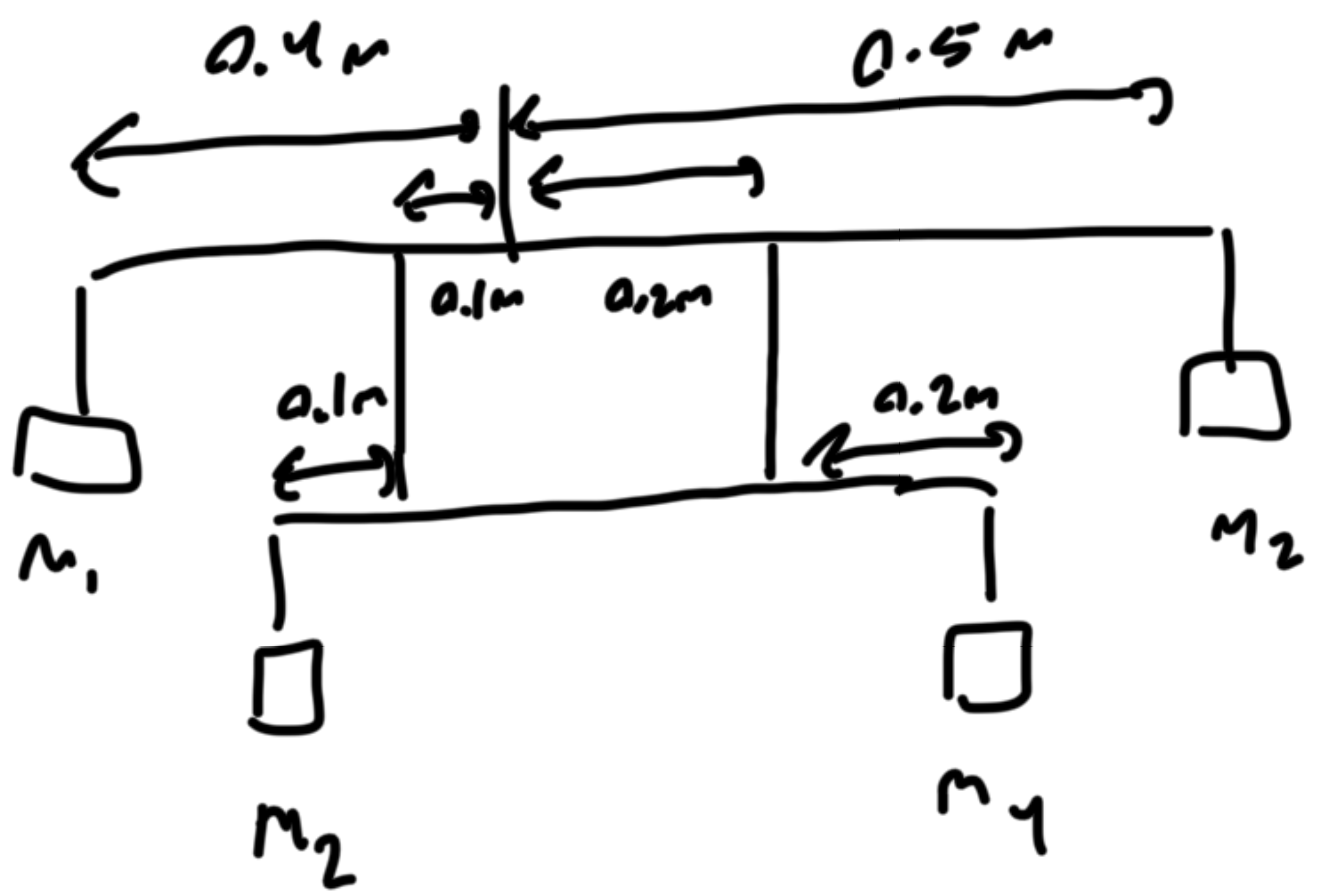
$$F_N = \frac{ms^2}{R}$$

$$F_\mu \le \mu F_N = \mu \frac{ms^2}{R} = mg$$

$$\mu_{min} = \frac{gR}{s^2} \propto s^{-2}$$

D

2017 #6



$M_1 = 400\,g$

$M_2 = 200\,g$

$M_4 = 500\,g$

$M_3 = ?$

CCW positive

$$\sum \tau_{top} = (0.4\,m) M_1 g + (0.1\,m) T_1 - (0.2\,m) T_2 - (0.5\,m) M_2 g = 0$$

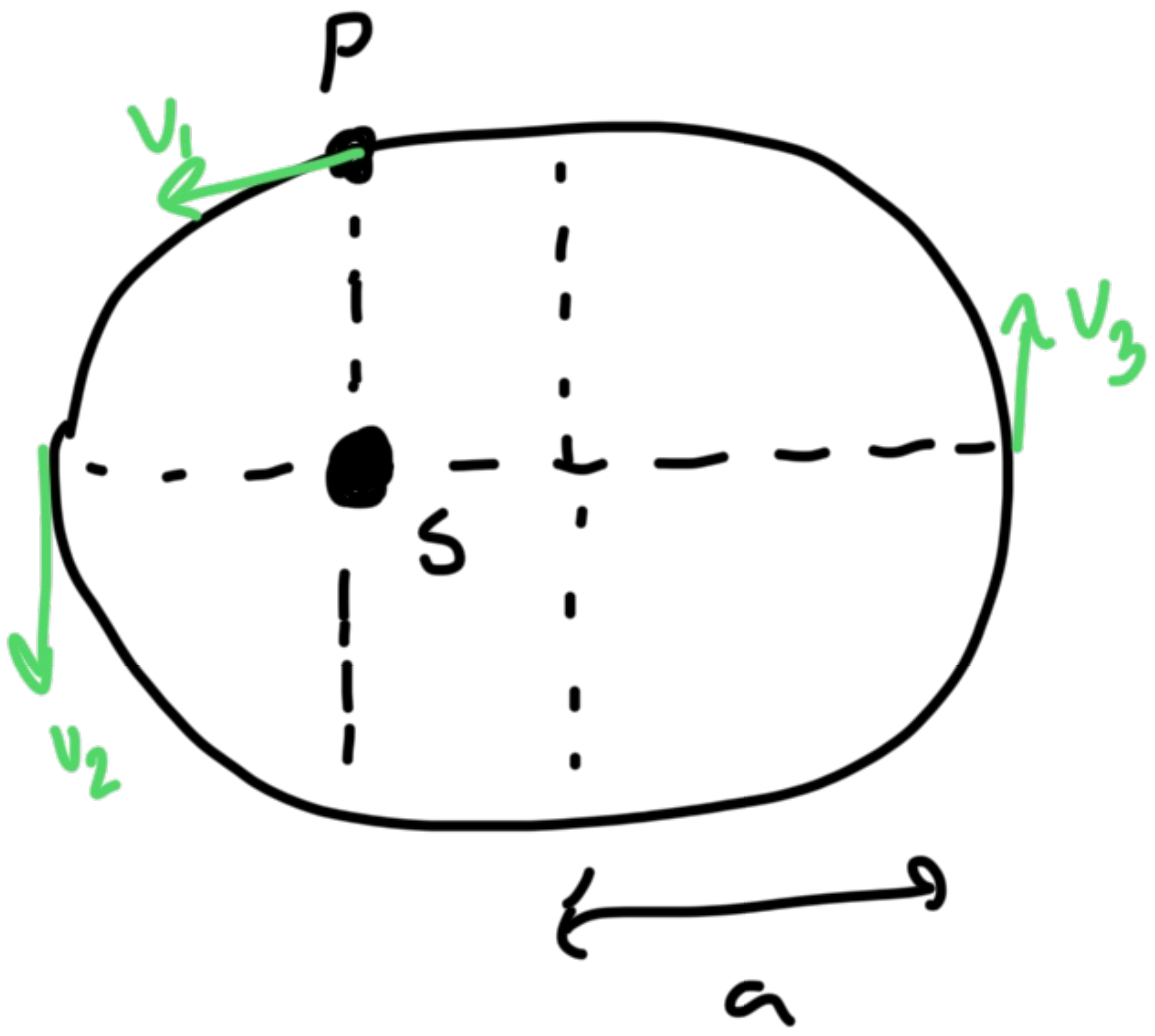
$$\sum \tau_{bot} = (0.2\,m) M_3 g - (0.1\,m) T_1 + (0.2\,m) T_2 - (0.4\,m) M_4 g = 0$$

$M_3 = 700\,g$

$$\sum \tau_{top} + \tau_{bot} = (0.4\,m) M_1 g + (0.2\,m) M_3 g - (0.5\,m) M_2 g - (0.4\,m) M_4 g = 0$$

E

2017 #25



perigee = 0.5 a

$v_2 = ?$

$\overline{SP} = ?$

$$SP + \sqrt{SP^2 + a^2} = 2a$$

$$SP = 3a/4$$

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_3^2 - \frac{GMm}{3a/2}$$

$$\boxed{\frac{m v_2 a}{2} = \frac{3 m v_3 a}{2} \Rightarrow v_3 = \frac{v_2}{3}}$$

can't do K2L w/o v_3!

$$\frac{4}{9} m v_2^2 = \frac{GMm}{a/2} - \frac{GMm}{3a/2} = \frac{4GMm}{3a}$$

$$v_2^2 = \frac{3GM}{a}$$

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_1^2 - \frac{GMm}{3a/4}$$

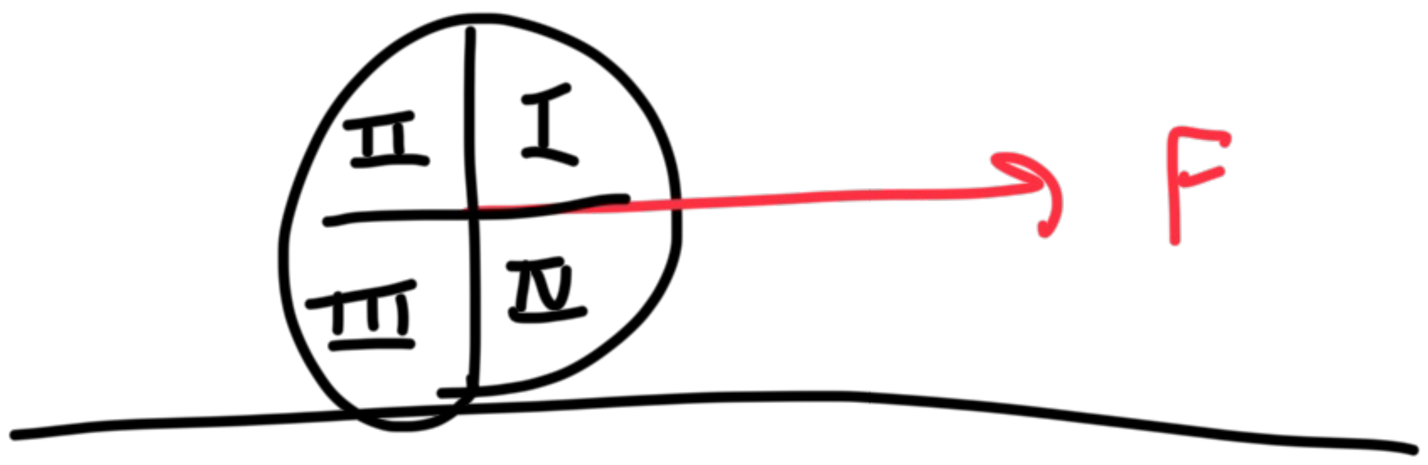
$$v_1^2 = v_2^2 - \frac{4GM}{a} + \frac{8GM}{3a} = GM\left(\frac{3}{a} - \frac{4}{a} + \frac{8}{3a}\right)$$

$$= \frac{5GM}{3a}$$

$$\frac{v_2^2}{v_1^2} = \frac{3GM/a}{\frac{5}{3}GM/a} = \frac{9}{5} \Rightarrow \frac{v_2}{v_1} = \frac{3}{\sqrt{5}}$$

A

2017 #18



Any points with $|a| = 0$?

$a_c = \omega^2 r$ towards center

$a_t = \alpha r$ direction of rotation

a (translational) $\longrightarrow F$

I. $\vec{a}_c + \vec{a}_t$ is downwards ✗

II. $\vec{a}_c + \vec{a}_t$ is rightwards ✗

III. $\vec{a}_c + \vec{a}_t$ is upwards ✗

IV. $\vec{a}_c + \vec{a}_t$ is leftward maybe?

$\|\vec{a}_c + \vec{a}_t\| = r\sqrt{\omega^4 + \alpha^2}$ $\|a\| = \alpha R$

So, yes! D