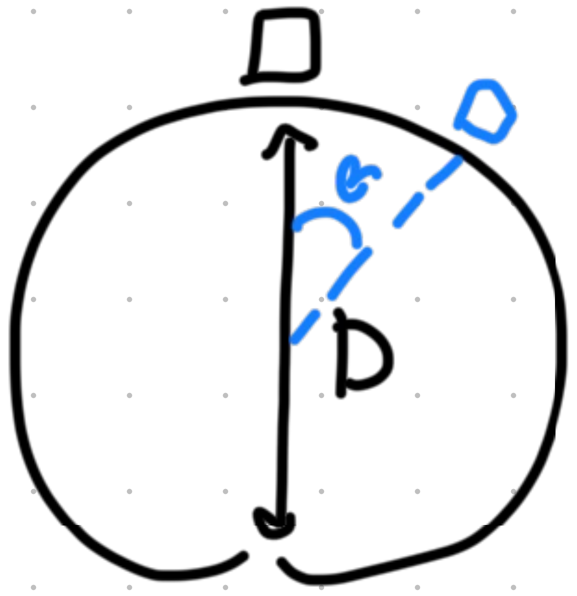
2016 # 9



V = ? when the bead leaves the sphere?

① $mg\cos\theta \geq \frac{mv^2}{R}$ contact force

② $mgh = \frac{mv^2}{2}$ conservation of energy

③ $h = R(1-\cos\theta)$ geometry

①: $\cos\theta = \frac{v^2}{gR}$

③ $h = R\left(1-\frac{v^2}{gR}\right) = R - \frac{v^2}{g}$

② $\cancel{m}g\left(R-\frac{v^2}{g}\right) = \frac{\cancel{m}v^2}{2}$

$gR - v^2 = \frac{v^2}{2}$

$gR = \frac{3v^2}{2}$

$v = \sqrt{\frac{2gR}{3}} = \sqrt{\frac{gD}{3}}$ E

SIDE NOTE ①

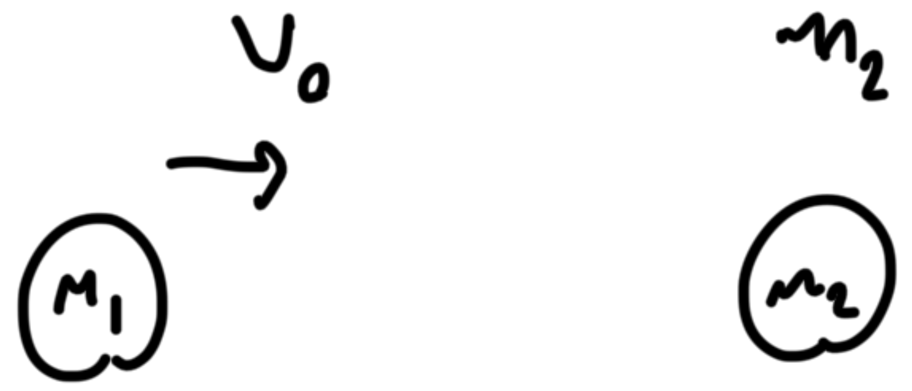
$\cos\theta = \frac{\left(\frac{gD}{3}\right)}{gR}$

$= \frac{2}{3} \Rightarrow \theta = \cos^{-1}\frac{2}{3}$

$\boxed{\theta = 48.2^\circ}$

UNIVERSAL RESULT

2016 #13



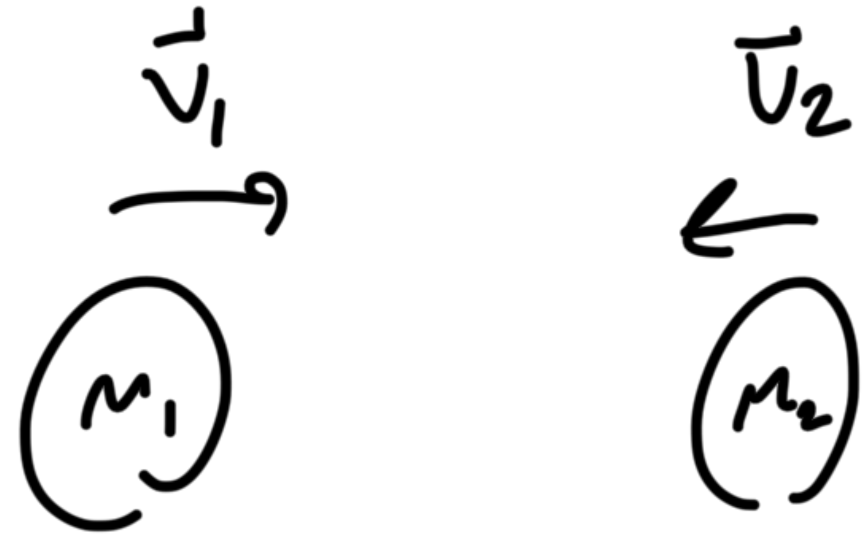
$m_2 = \alpha m_1 < m_1$

elasticity unknown

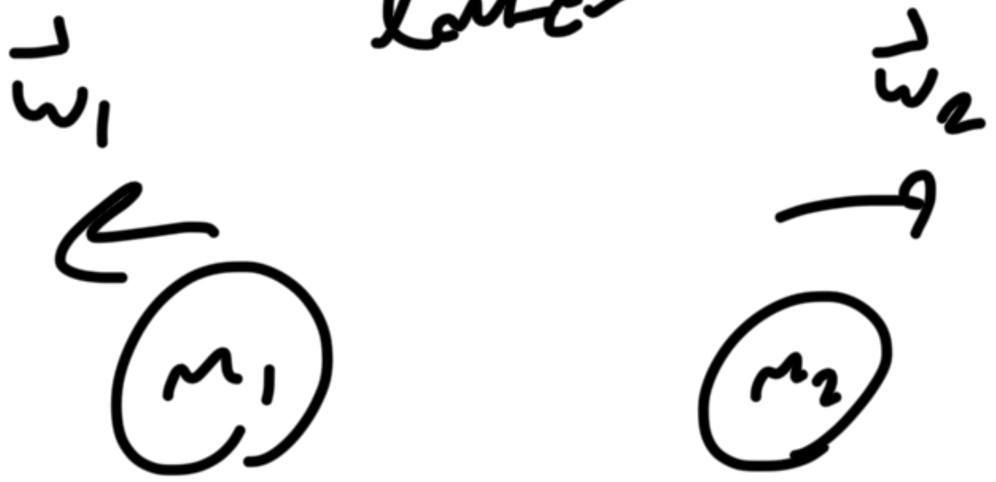
v_1 v_2

range of allowed $r_2 = v_2/v_0$?

general case:



later



elastic:

$$m_1 v_1 + m_2 v_2 = m_1 w_1 + m_2 w_2$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 w_1^2 + m_2 w_2^2$$

$$\Rightarrow \boxed{v_1 + w_1 = v_2 + w_2}$$ USEFUL!

$$\boxed{w_1 = \frac{v_1(m_1 - m_2) + 2m_2 v_2}{m_1 + m_2}} \qquad \boxed{w_2 = \frac{v_2(m_2 - m_1) + 2m_1 v_1}{m_1 + m_2}}$$

completely inelastic:

$$m_1 v_1 + m_2 v_2 = m_1 w_1 + m_2 w_2$$

$$w_1 = w_2 = w$$

$$\boxed{w = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = v_{com}}$$

KNOW THESE!

returning to original question...

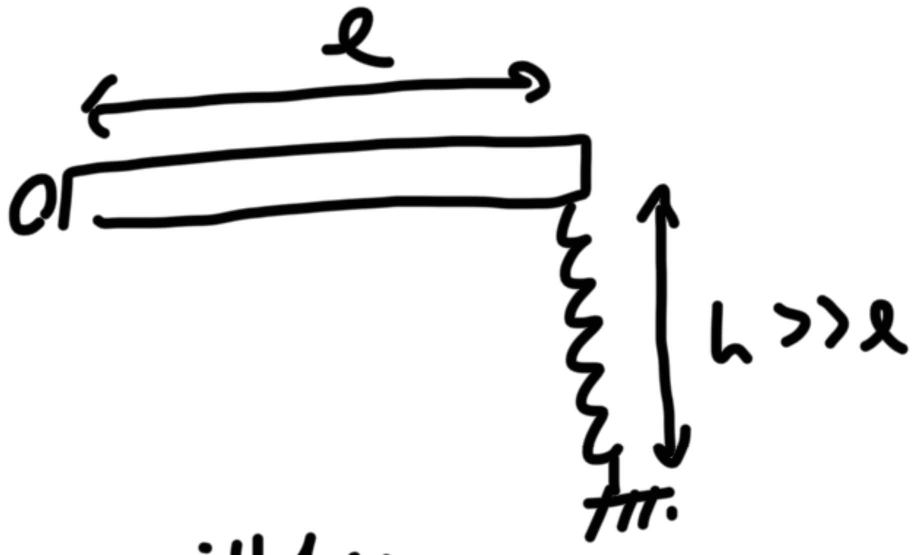
$v_2 = 0$

elastic: $r_2 = \left|\dfrac{w_2}{v_1}\right| = \dfrac{2m_1}{m_1+m_2} = \dfrac{2}{1+\alpha}$

c.i.: $r_2 = \left|\dfrac{w_2}{v_1}\right| = \dfrac{m_1}{m_1+m_2} = \dfrac{1}{1+\alpha}$

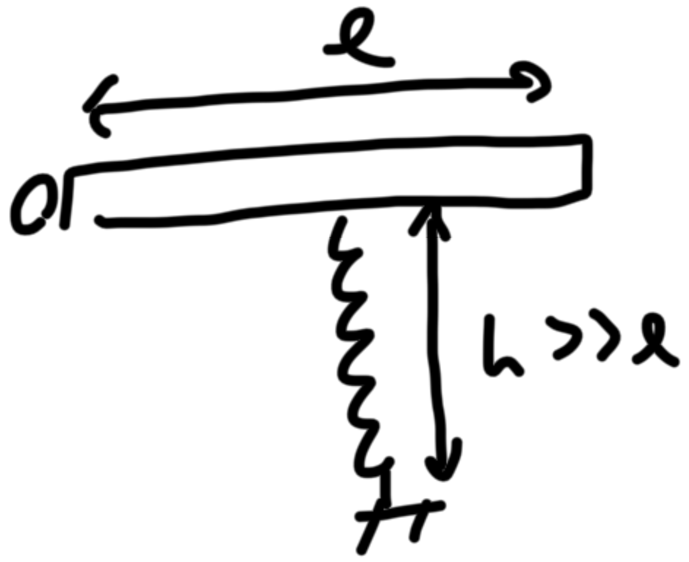
$\Rightarrow \quad \dfrac{1}{1+\alpha} \le r_2 \le \dfrac{2}{1+\alpha} \quad$ E

2016 #14



oscillates

w/ freq f



new frequency?

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{a}{x}}$$

original: x causes $a = \alpha l$

new: x causes $a = \alpha l/2$

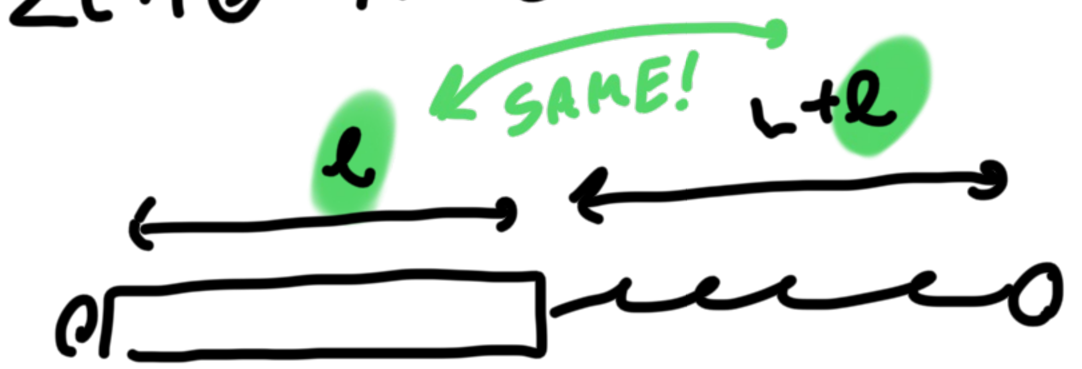
$$I\alpha = \tau = kxr$$

$$\alpha = kxr^2/I$$

$$\omega = \sqrt{\frac{kr^2}{I}}$$ scales as r

$$\omega' = \frac{\omega}{2} \Rightarrow f' = \frac{f}{2}$$ A

2016 #15



$$\omega = \sqrt{\frac{a}{x}}$$

$$x = \ell \sin\theta \approx \ell\theta \iff a = \alpha\ell$$

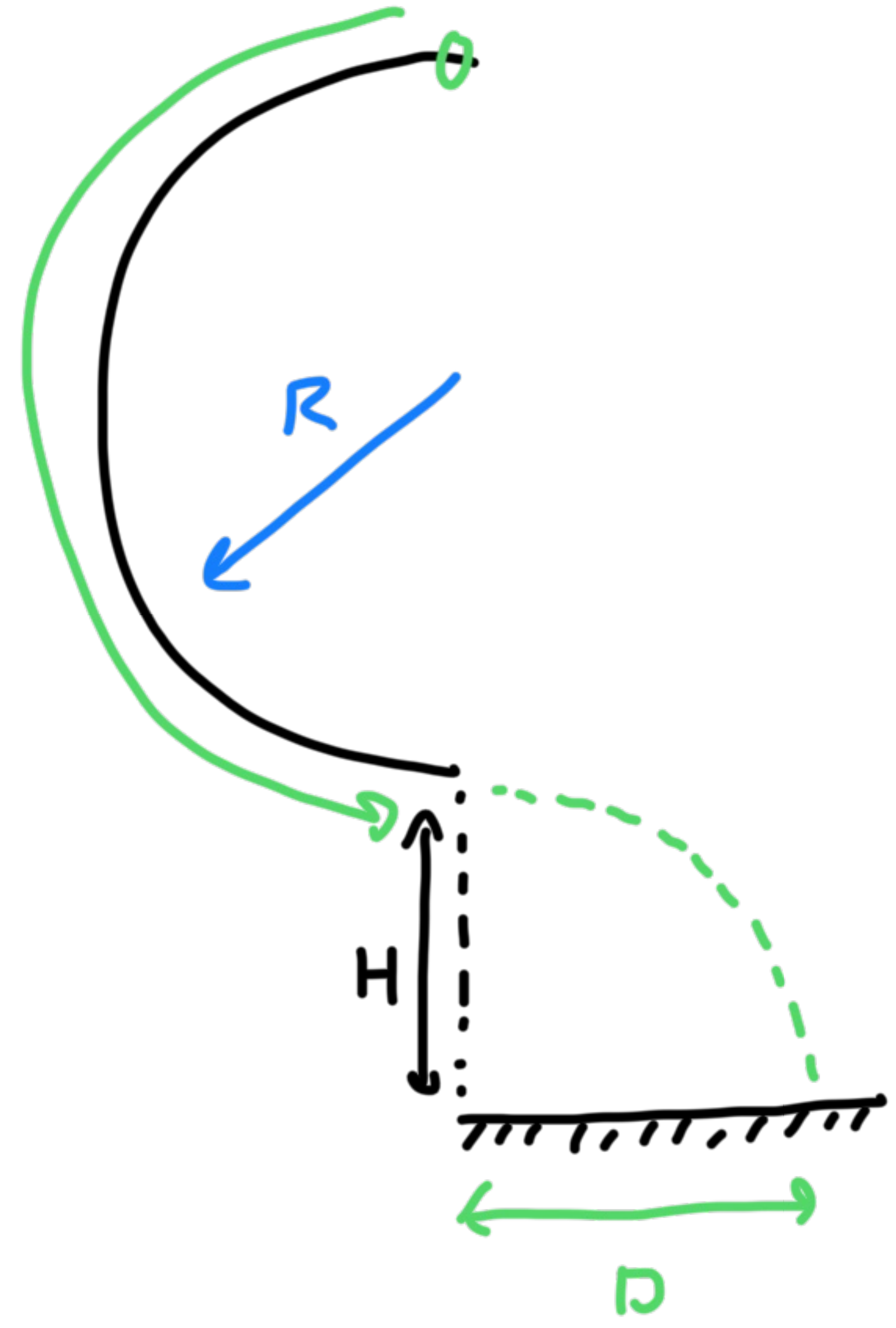
$$I\alpha = \tau = F\sin\theta\,\ell \approx k\ell^2\theta = kx\ell$$

$$\alpha = kx\ell/I \implies a = kx\ell^2/I$$

Same as original!

C

2016 #19



RH vs D^2 ?

energy conservation:

$$\frac{1}{2}mv^2 = 2mgR$$

(bottom of wire)

$$v^2 = 4gR$$

how long is the bead in the air?

$$H = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2H/g}$$

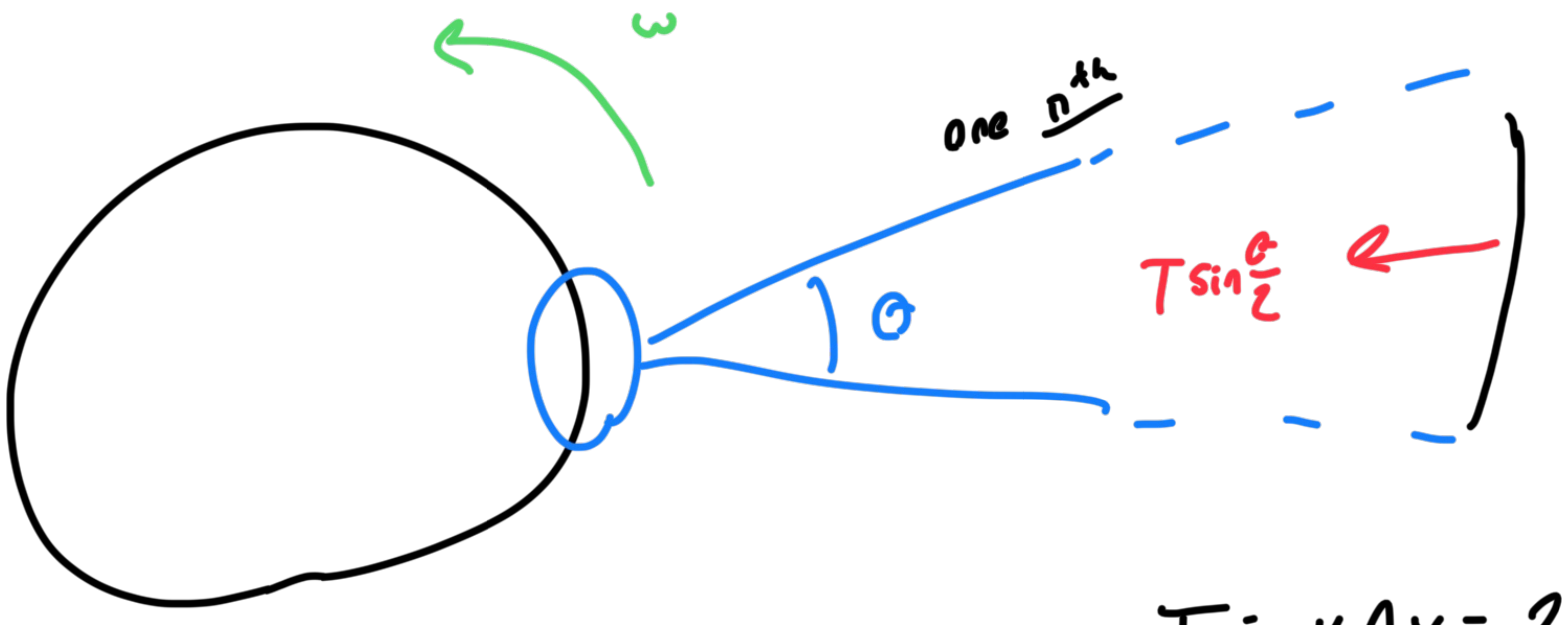
$$D = vt \Rightarrow D^2 = v^2t^2 = 4gR\,\frac{2H}{g} = 8RH$$

$$RH = \frac{1}{8}D^2.$$

RH is linear in D^2 with zero y-intercept.

D

2016 #23



$R' = ?$

$$T = k\Delta x = 2\pi k(R'-R)$$

$$2T\sin\frac{\theta}{2} = m\omega^2 R'$$

$$\simeq T\theta \simeq m\omega^2 R'$$

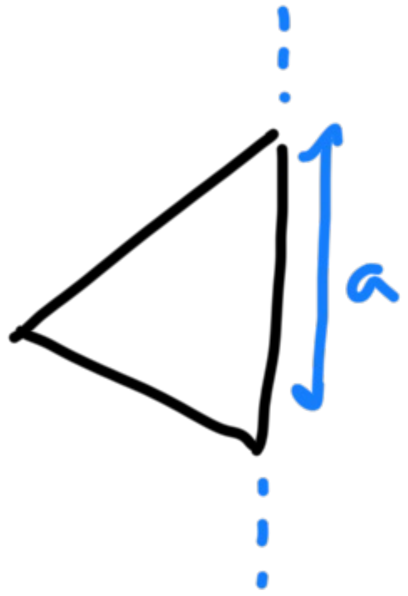
$$2\pi T \simeq M\omega^2 R'$$

$$4\pi^2 k(R'-R) \simeq M\omega^2 R'$$

$$R' \simeq \frac{4\pi^2 k R}{4\pi^2 k - M\omega^2}$$

D

2016 #24



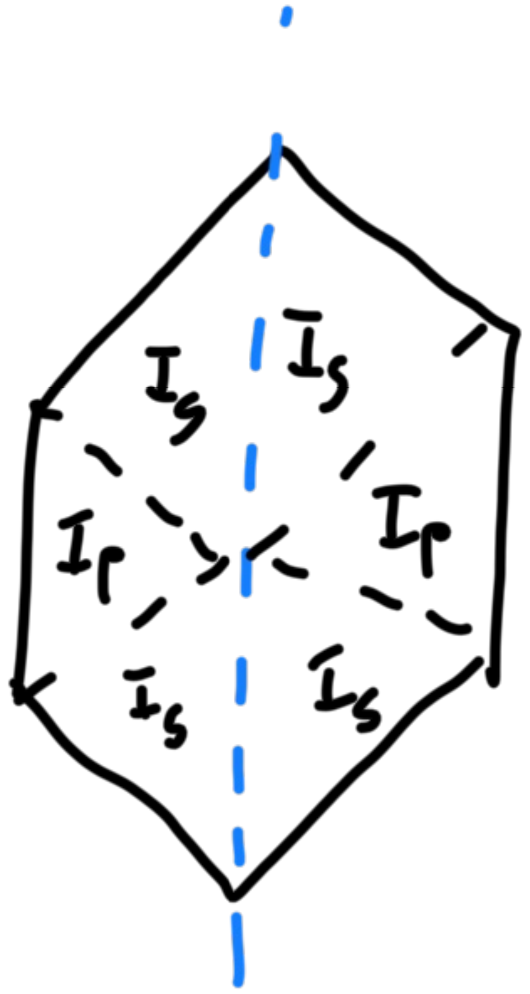
$I_s = \frac{1}{8} m a^2 \Rightarrow$

$$I_c = I_s - m\left(\frac{1}{3}\,\frac{a\sqrt{3}}{2}\right)^2 = \frac{1}{24} m a^2$$

$\Rightarrow$

$$I_p = I_c + m\left(\frac{2}{3}\,\frac{a\sqrt{3}}{2}\right)^2 = \frac{3}{8} m a^2$$

$I_{hexagon} = ?$



each with $m/6$

$$I_{hex} = 4\left[\frac{1}{8}\left(\frac{m}{6}\right)a^2\right] + 2\left[\frac{3}{8}\left(\frac{m}{6}\right)a^2\right]$$

$$= \frac{5}{24} m a^2$$

B