2019 B #3

$\rightarrow F = F_1 = F_2$

$$\frac{U_1}{U_2} = ?$$

$$x \propto k^{-1}$$

$$F_1 = F_2$$

$$k_1 x_1 = k_2 x_2$$

$$U = \frac{1}{2} k x^2$$

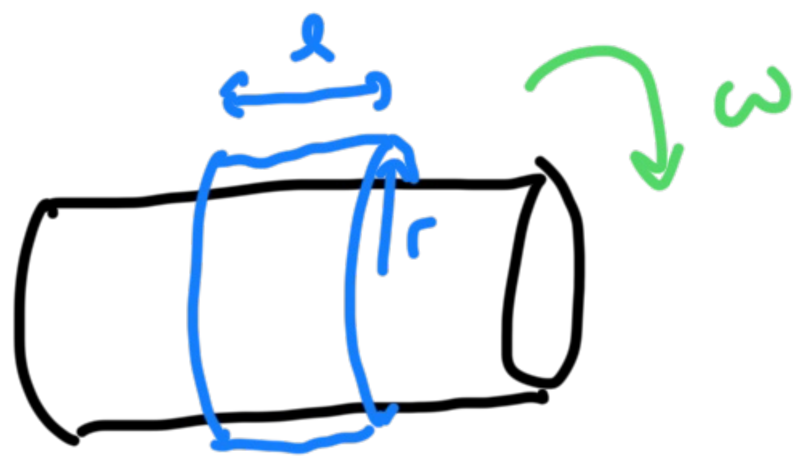
$$= \frac{1}{2k} (kx)^2$$

$$U \propto k^{-1}$$

$$\frac{U_1}{U_2} = \frac{k_2}{k_1}$$

C

2019 B #6



$\rho(r) = ?$

$\oiint g \cdot dA = 4\pi G M_{enc}$

$2\pi r \ell g = 4\pi G M_{enc}$

gravitational accel. at r

$\lambda = \frac{\text{mass}}{\text{unit length}}$

$M_{enc} = \lambda \ell$

$g = \frac{2GM_{enc}}{r\ell} = \frac{2G\lambda}{r} \propto r$

$g = \omega^2 r$

$\omega^2 r = \frac{2G\lambda}{r}$

$\lambda = \frac{\omega^2}{2G} r^2 = \frac{\omega^2}{2\pi G} A$

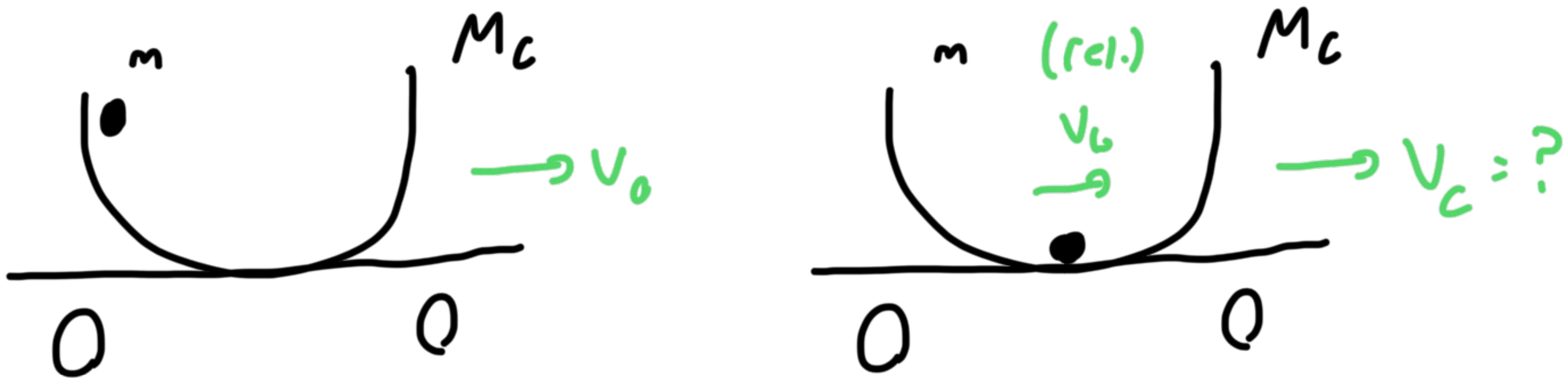
$A = \pi r^2$

$\lambda \propto A$

independent of r!

C

2019 B #10



Inelastic collision in reverse!

No external horizontal forces!

$$P_0 = (M_c + m) v_0 = P_f = (M_c + m) v_c + m v_b$$

$$v_c = v_0 - \frac{m}{M_c + m} v_b$$

D

2019 B #13

$v \to 2v$

how much of the wall gets wet?

Kinematics:

1. going straight up

$$H = \frac{1}{2}vt = \frac{v^2}{2g} \propto v^2$$

2. going to side

$$D = v\sin\theta \cdot \frac{2v\cos\theta}{g} = \frac{v^2\sin 2\theta}{g} \propto v^2$$

both scale as v^2!

sparse information...

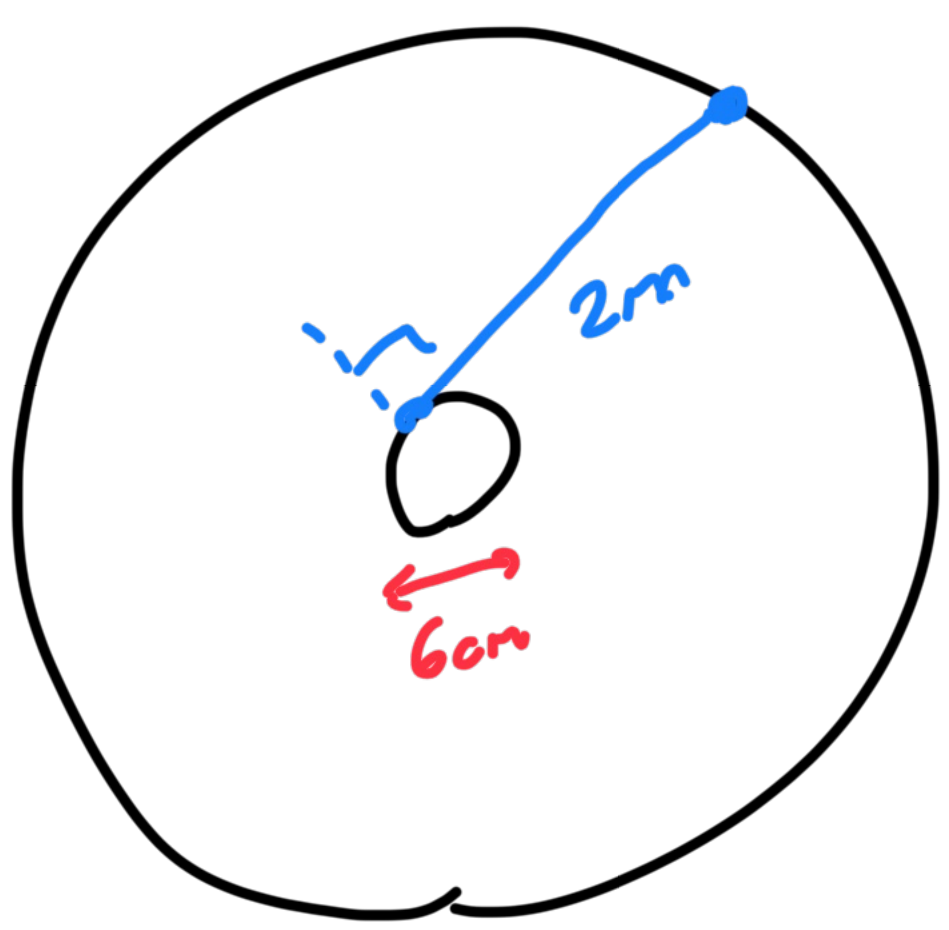
Dimensional Analysis!

$[v] = \frac{m}{s}$ $[g] = \frac{m}{s^2}$

$$\text{length} = \frac{[v]^2}{[g]} \propto v^2$$

D

2019 B #15



$F_{air} = ?$

ΣF = purely radial due to U.C.M.

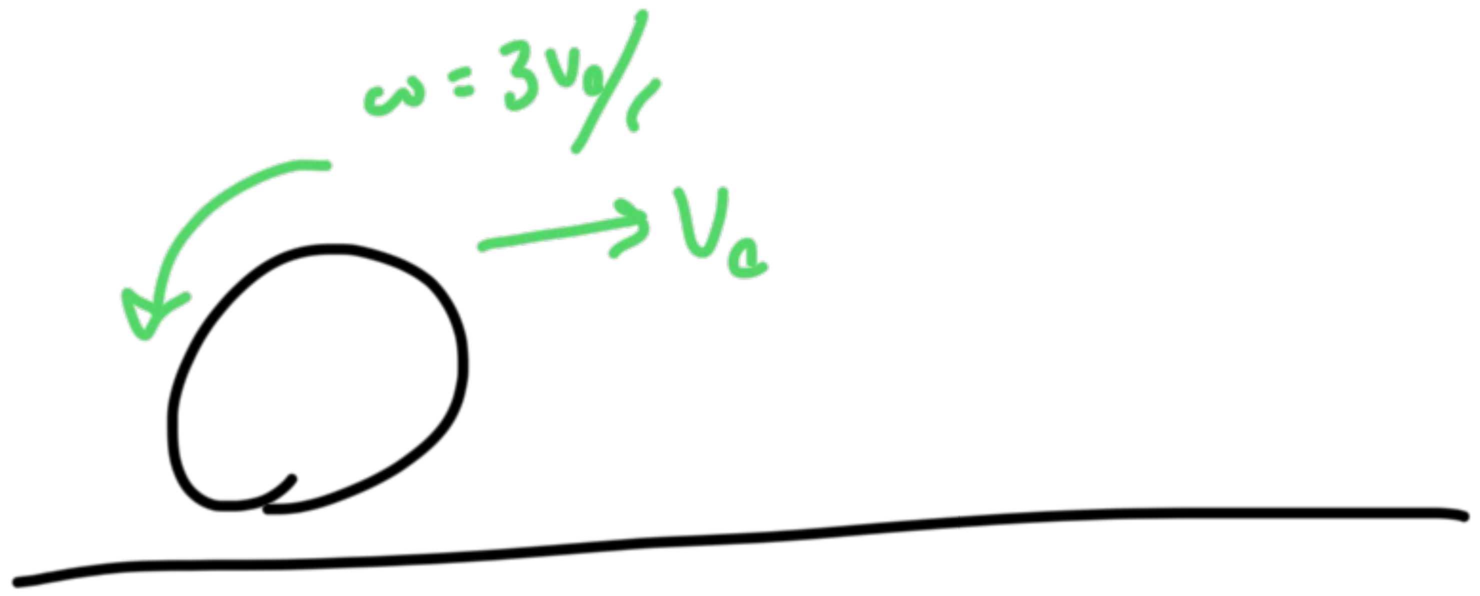
D

$T = MR\omega^2 = 20\,N$

$F_{air} = T_{tangential} = T\sin\theta \simeq T\frac{r}{R} = 0.3\,N$

small angle approx.

2019 B #16



How long for hoop to return to starting position?

① while slipping

$$v_c(t) = v_0 + at$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\omega_0 = -3v_0/r \qquad a = F_k/m = -\mu g$$

$$\alpha = \tau/I = -\mu g/r$$

slipping ends at T

$$v_c(T) = \omega(T) r$$

$$\Rightarrow T = 2v_0/\mu g$$

$$v_c(T) = -v_0 \qquad \omega(T) = -\frac{v_0}{r}$$

returning to negative initial velocity under constant acceleration means 0 displacement!

So,

$\Delta x = 0$ at $t = T$ (B)

$$x(T) = v_0 T + \frac{1}{2} a T^2 = v_0 \frac{2v_0}{\mu g} - \frac{1}{2}\mu g \left(\frac{2v_0}{\mu g}\right)^2 = 0$$

2019 B #17

Hoop → Disk ... what changes?

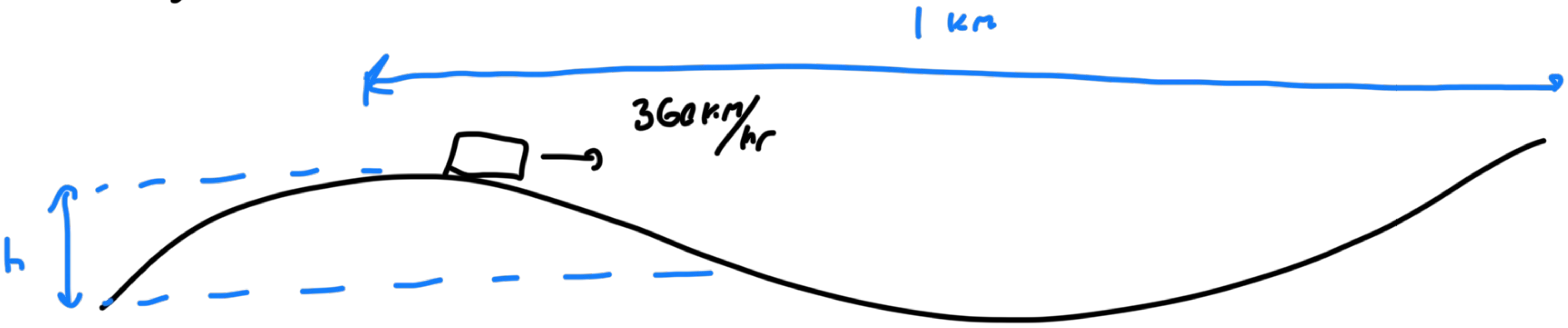
$I \to I/2$

less than T

stops slipping $T' = \frac{4v_0}{3\mu g} \Rightarrow v_c(T) = \frac{-v_0}{3}$ $x(T) = \frac{4v_0^2}{9\mu g}$

$T_{return} = T' + \frac{x(T)}{|v(T)|} = \frac{4v_0}{3\mu g} + \frac{4v_0}{3\mu g} = \frac{8v_0}{3\mu g} \neq T$

2019 B #23



$a_{max} = 0.1\ m/s^2$ $\qquad h_{max} = ?$

SHO!

$A = h$

$T = \frac{d}{v} = \frac{1\ km}{360\ \frac{km}{hr}} = 10\ s$

$T = 2\pi\sqrt{\frac{k}{m}} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$a_{max} = \frac{F}{m} = \frac{kA}{m} = \frac{4\pi^2 h}{T^2}$

$0.1\ m/s^2 \geq 4\pi^2 h / T^2$

$h \leq \frac{T^2}{4\pi^2} a_{max} = \frac{(10\ s)^2}{4\pi}(0.1\ m/s^2) = 0.25\ m$

B