2019 B #3

$U_1/U_2 = ?$

$F_1 = F_2 = F$

$k_1 x_1 = k_2 x_2$

$U_1 = \frac{1}{2} k_1 x_1^2 = \frac{1}{2}(k_1 x_1)^2 \frac{1}{k_1}$

$U_2 = \frac{1}{2} k_2 x_2^2 = \frac{1}{2}(k_2 x_2)^2 \frac{1}{k_2}$

$U \propto \frac{1}{k}$

$\frac{U_1}{U_2} = \frac{k_2}{k_1}$

C

2019B #6

Gauss' Law for gravity: $\oiint g \cdot dA = 4\pi G M_{enc}$

$$2\pi r l g = 4\pi G M_{enc}$$

$M_{enc} = \lambda l$ λ: mass per length

$$g = \frac{2GM_{enc}}{rl} = \frac{2G\lambda}{r}$$

$$g = \omega^2 r$$

$$\Rightarrow \lambda = \frac{\omega^2}{2G} r^2 = \frac{\omega^2}{2\pi G} A$$

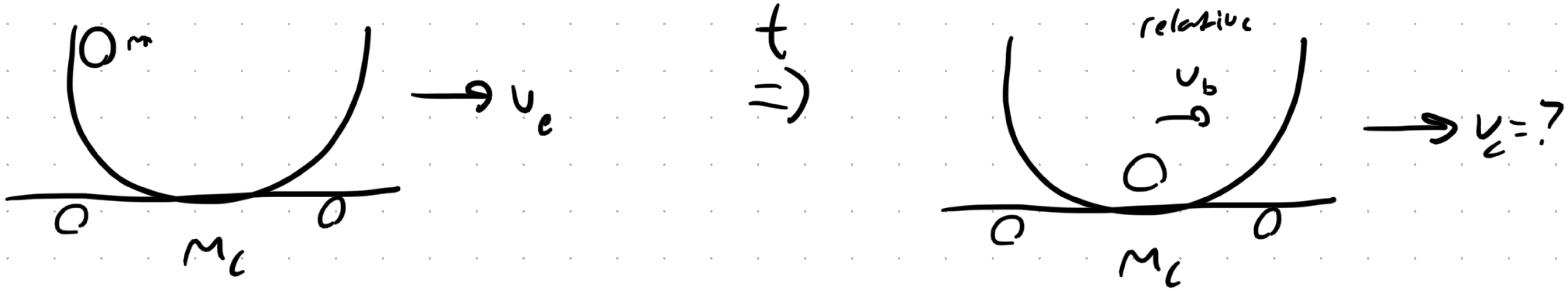
$$A = \pi r^2$$

$$\lambda \propto A$$

$\Rightarrow$ density of any stick of the cylinder is same as all other stick

$\Rightarrow$ C

2019B #10



there are no external forces except gravity ... aka no external horizontal forces!

like a totally inelastic collision in reverse!

$$(M_c + m) v_0 = (M_c + m) v_c + m v_b$$

$$v_c = v_0 - \frac{m}{M_c + m} v_b$$

D

2019 B #13

sparse information...

dimensional analysis!

$v = [L]/[T]$

$g = [L]/[T]^2$

$[L] = \frac{v^2}{g} \propto v^2$

aka all lengths $\propto v^2$

D

$KE \propto v^2$

$GPE \propto h$

$KE \rightarrow GPE: \quad h \propto v^2$

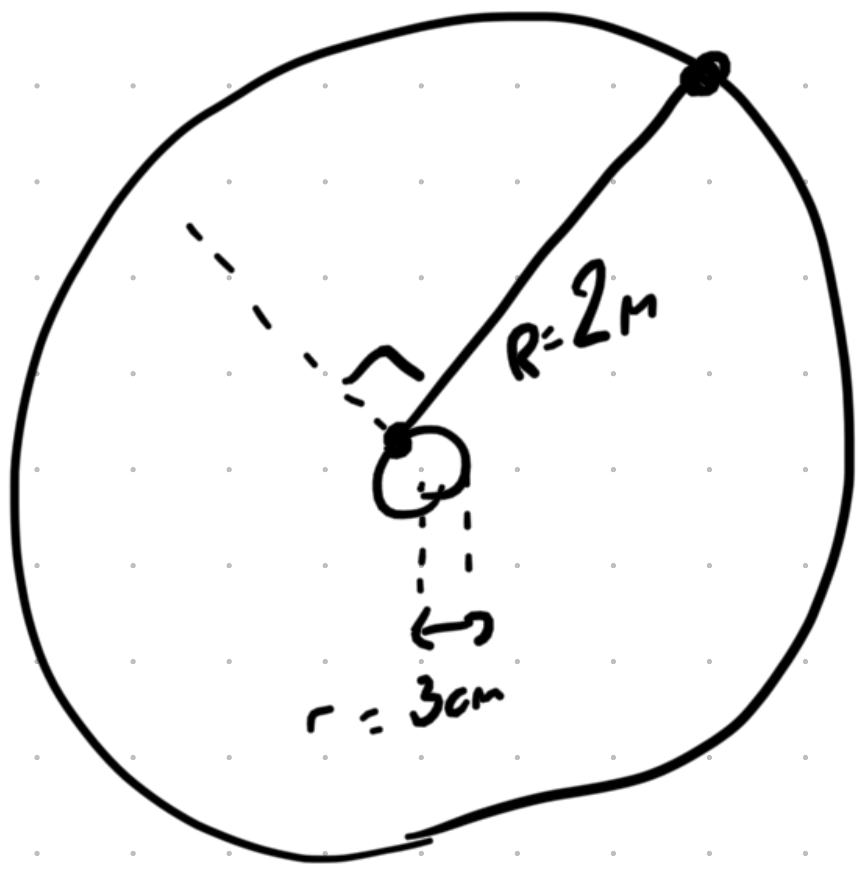
OR

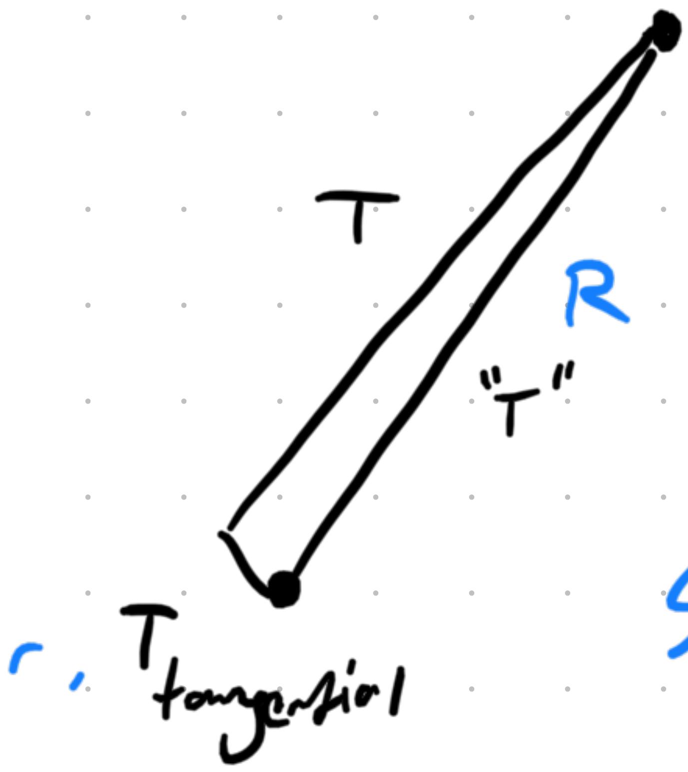
$H = \frac{1}{2} v t = \frac{v^2}{2g} : \quad h \propto v^2$

⋮

etc.

2019 B #15





Similar triangles!

$F_{drag} = ?$

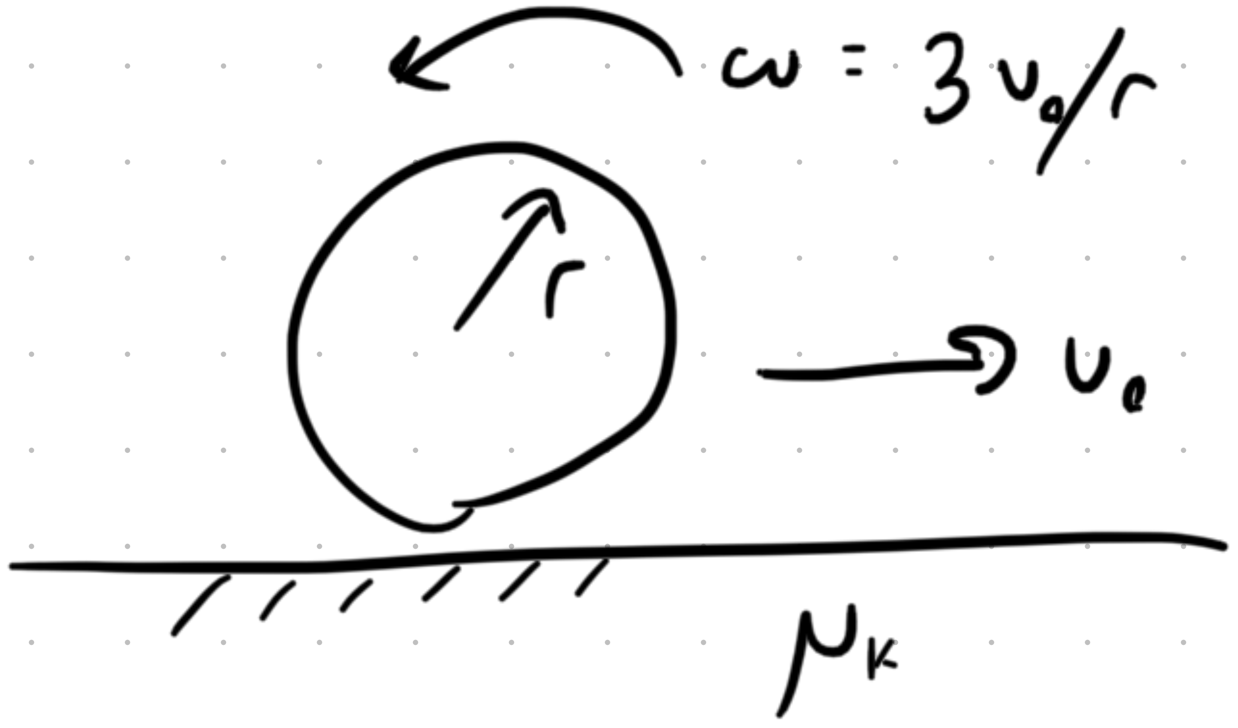
$$T = MR\omega^2 = 20N$$

$$F_g = T_{tangential} = \frac{r}{R}T = 0.3N$$

by similar triangles

D

2019 B #16



how long to return to starting position?

at first, slipping

$F_k \Rightarrow v_c(t)$ and $\omega(t)$

$a = F_k/m \qquad \alpha = \tau/I = -F_k r/I$

$v_c = v_0 + at \qquad \omega = \omega_0 + \alpha t$

$\omega_0 = -3v_0/r \qquad a = -\mu g \qquad \alpha = \mu g/r$

when slipping ends at T

$v_c(T) = r\omega(T) \Rightarrow T = 2v_0/\mu g$

$v_c(T) = v_0 - \mu g T = -v_0$

$\omega(T) = -v_0/r$

since acceleration was constant, that means we're back to our starting point!

$X(T) = v_0 T + \frac{1}{2}aT^2 = v_0 \frac{2v_0}{\mu g} - \frac{1}{2}\mu g \left(\frac{2v_0}{\mu g}\right)^2 = 0$

B

2019 B #17

replace hoop with disk, does anything change?

$I' = I/2$

$\alpha' = 2\mu g/r$

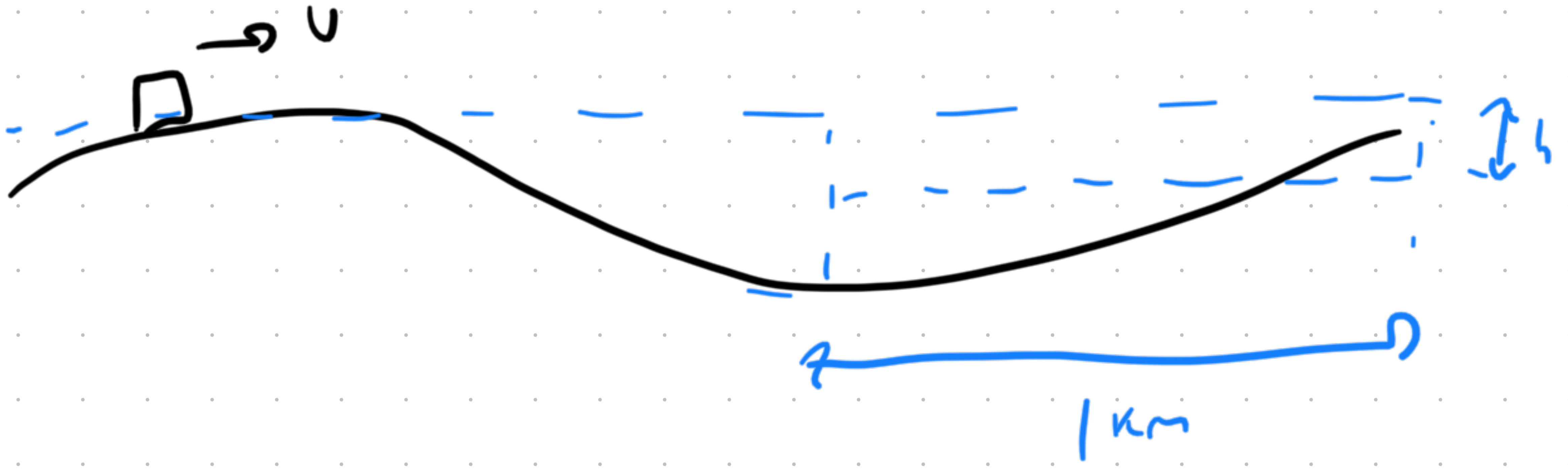
$T' = \frac{4v_0}{3\mu g}$

$v_c(T) = -v_0/3 \quad x(T) = \frac{4v_0^2}{9\mu g}$

$$T_{total} = T' + \frac{x(T)}{|v(T)|} = \frac{4v_0}{3\mu g} + \frac{4v_0}{3\mu g} = \frac{8v_0}{3\mu g} \neq T$$

D

2019B #23



$a_{max} = 0.1\ \text{m/s}^2$ $h_{max} = ?$

Clue: sinusoidal

Model as SHO

$A = h$ $T = \frac{1\text{km}}{360\text{km/h}} = 10\text{s}$

spring analogy:

$T = 2\pi\sqrt{\frac{k}{m}} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$a = \frac{F}{m} = \frac{kA}{m} = \frac{4\pi^2 h}{T^2}$

$a \leq 0.1\text{m/s}^2$

$h \leq \frac{T^2}{4\pi^2} a_{max} = \frac{(10\text{s})^2}{4\pi^2}(0.1\text{m/s}^2) = 0.25\text{m}$

B