Error propagation

$x \pm \Delta x$

$y \pm \Delta y$

Additive

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Multiplicative

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$

Power

$$\Delta(x^a) = |a| x^{a-1} \Delta x$$

Multiple trials & weighting

Xavier : $x \pm \Delta x$

Yvonne : $y \pm \Delta y$

wish to weight Yvonne's twice as much.

$$\Delta(x, 2y) = \frac{1}{\sqrt{3}} \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta y)^2}$$

$$= \frac{1}{\sqrt{3}} \sqrt{(\Delta x)^2 + 2(\Delta y)^2}$$

Don't write Δx^2 !

use $(\Delta x)^2$ or $\Delta(x^2)$.

$$\Delta\left(\frac{x}{y}\right) = ?$$

$$\Delta(y^{-1}) = |-1|\, y^{-2} \Delta y = \frac{\Delta y}{y^2}$$

$$\Delta(x y^{-1}) = \sqrt{(x\Delta(y^{-1}))^2 + (y^{-1}\Delta x)^2}$$

$$= \sqrt{\left(\frac{x\Delta y}{y^2}\right)^2 + \left(\frac{\Delta x}{y}\right)^2}$$

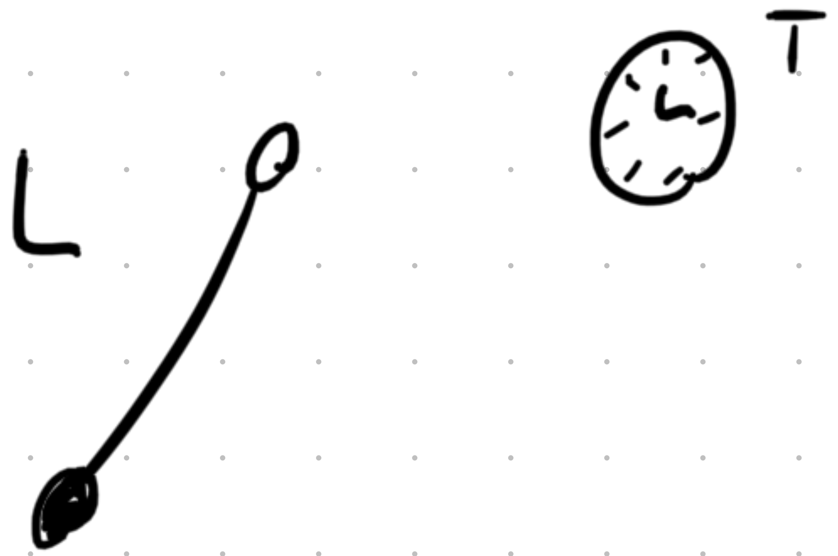
Relative uncertainties

$x \pm \Delta x \qquad t \pm \Delta t$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

multiplied or divided uncertainties.

Ex. measuring g with a pendulum.



$\downarrow g$?

$L \pm \Delta L$ $\quad T \pm \Delta T$

$g \pm \Delta g$?

$$g = 4\pi^2 L/T^2$$

$$\Delta g = 4\pi^2 \Delta(LT^{-2})$$

$$\Delta(T^{-2}) = |-2|\, T^{-3} \Delta T$$

$$= 2T^{-3}\Delta T$$

$$\Delta(LT^{-2}) = \sqrt{[L\Delta(T^{-2})]^2 + [(T^{-2})\Delta L]^2}$$

$$= \sqrt{[2LT^{-3}\Delta T]^2 + (T^{-2}\Delta L)^2}$$

$$\Delta g = 4\pi^2\sqrt{[2LT^{-3}\Delta T]^2 + (T^{-2}\Delta L)^2}$$

2018 B #19

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} = \sqrt{\left(\frac{2}{75}\right)^2 + \left(\frac{0.10}{2.15}\right)^2}$$

$$= 0.054$$

$$\Delta v = v \cdot 0.054 = 19\ m/s$$ E

$$\Delta v = \Delta\left(\frac{x}{t}\right)$$

2018B #25

$R \pm \Delta R = 1 \pm 0.1$ cm

$L \pm \Delta L = 1.00 \pm 0.01$ m

$\frac{\Delta R}{R} = 10\%$

$\frac{\Delta L}{L} = 1\%$

(1) $\sqrt{(1\%)^2 + (1\%)^2} \simeq 1.4\%$

(2) $\sqrt{(10\%)^2 + (0.1\%)^2} \simeq 10\%$

(3) $\sqrt{(10\%)^2 + (1\%)^2} / \sqrt{10} \simeq 3.3\%$

D

Rotations

(topic 6)

rotations	relations	translations
$\vec{\tau}$	$\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = r_\perp F = r F_\perp$	$\vec{F}$
$\vec{\alpha}$	$\vec{\alpha} = \frac{\vec{a}}{r}$ $\vec{a} = r\vec{\alpha}$	$\vec{a}$
$\vec{\omega}$	$\vec{\omega} = \frac{\vec{v}}{r}$ $\vec{v} = r\vec{\omega}$	$\vec{v}$
$\vec{\theta}$	$\vec{\theta} = \frac{\vec{x}}{r}$ $\vec{x} = r\vec{\theta}$	$\vec{x}$
I	$I \propto mr^2$	m
K	$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$	K

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

2018 B #5

$$\omega_f^2 - \omega_i^2 = 2\alpha\theta$$

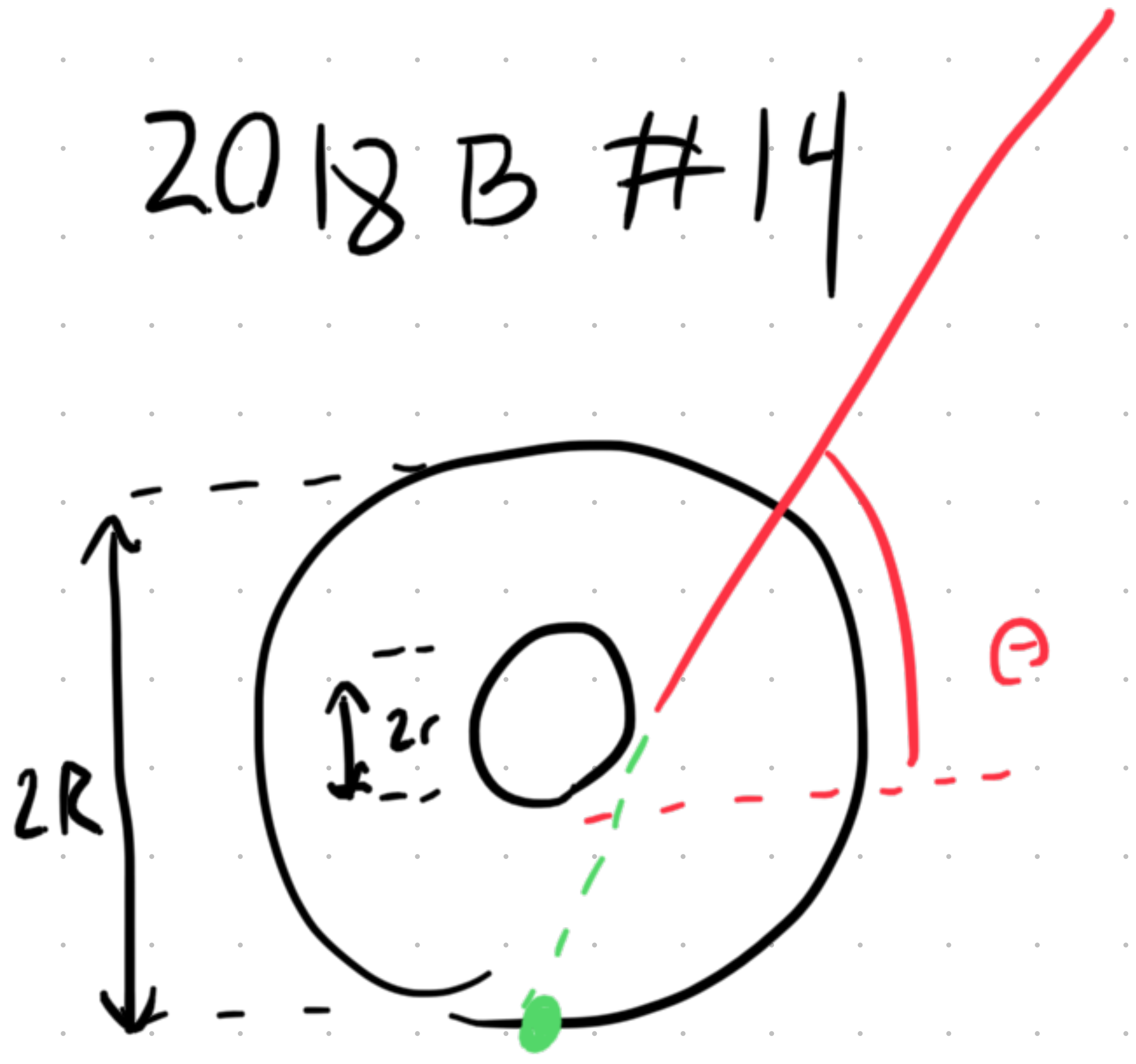
$$v_f^2 - v_i^2 = 2ax$$

$$2\alpha_+\theta_+ = 2\alpha_-\theta_-$$

$$\frac{\theta_+}{\theta_-} = \frac{1}{5} \qquad \frac{\alpha_+}{\alpha_-} = \frac{5}{1} = 5$$

D

2018 B #14



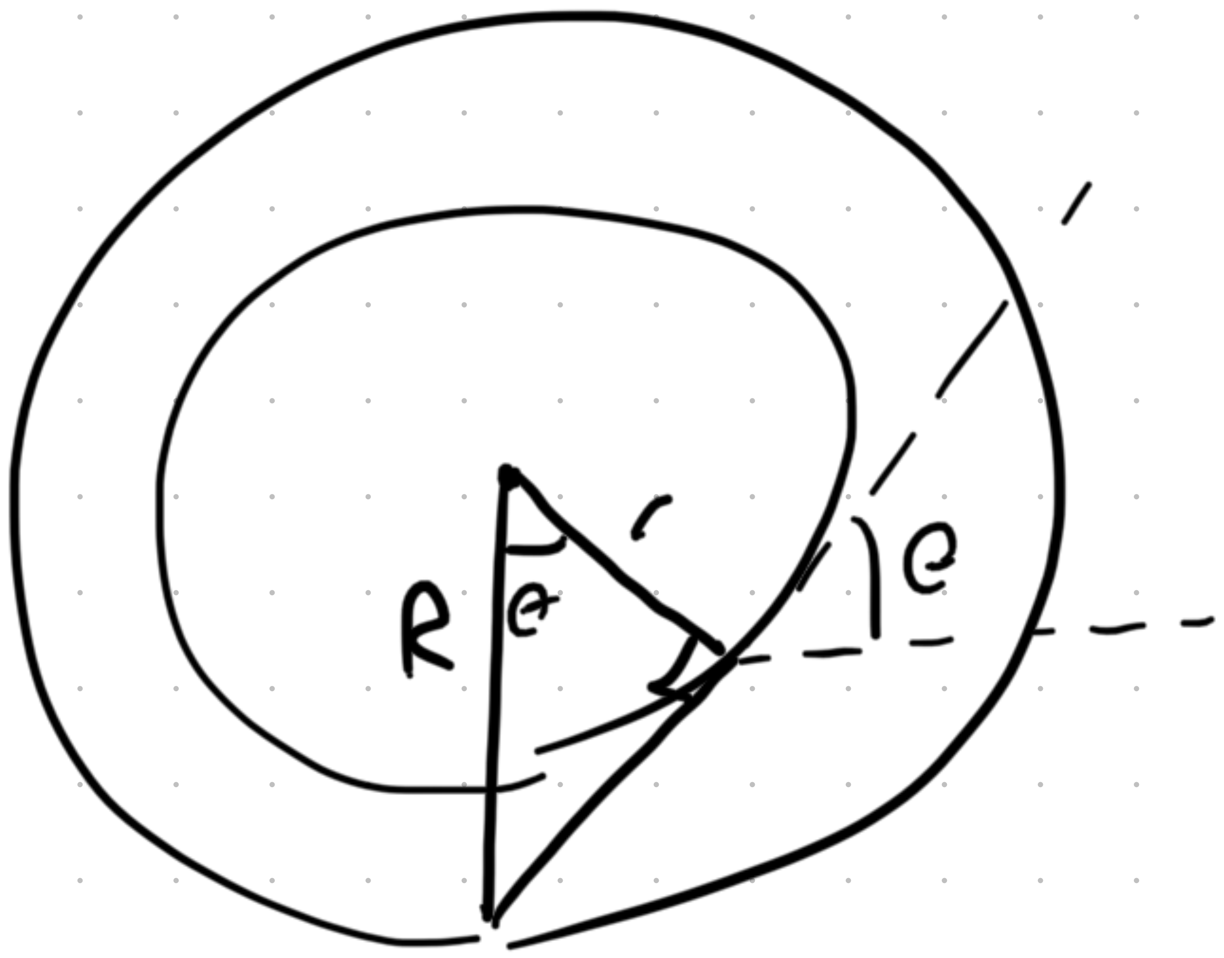
$\cos\theta = \frac{r}{R} \Rightarrow \theta = 41.4°$

B

$\Sigma \tau = 0, \quad \alpha = 0$

no rotation.

$\vec{\tau} = \vec{r} \times \vec{F}$

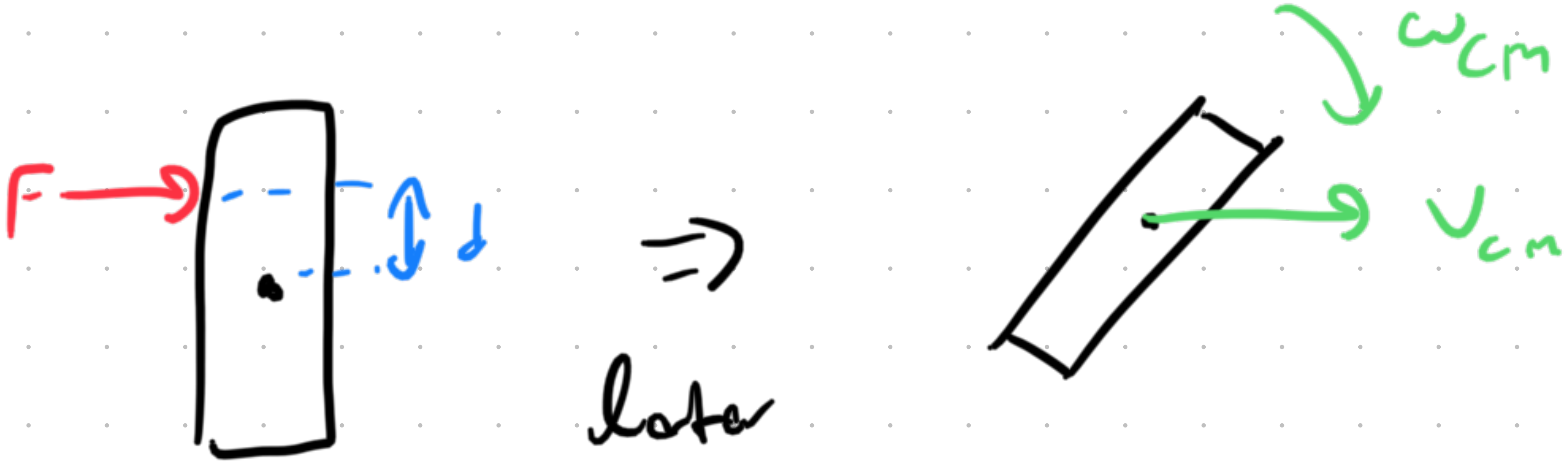


$r = R\cos\theta$

$\cos\theta = \frac{r}{R}$

go read topic 6.2 !

Rotational impulse.



Δt

Translation

$F = ma = \frac{\Delta p}{\Delta t}$ so that $F\Delta t = \Delta p = J$

$J = F\Delta t = ma\Delta t = mv_{cm} \Rightarrow v_{cm} = \frac{F\Delta t}{m}$

independent of d!

Rotation

$J_T = F\Delta t\, d = I\alpha\Delta t = I\omega_{cm}$

$= \frac{1}{12}ML^2\omega_{cm}$

$\Rightarrow \omega_{cm} = 12\frac{Jd}{ML^2}.$

does the work done depend on d?

$$\Delta E = F\Delta y = J\frac{\Delta y}{\Delta t}$$

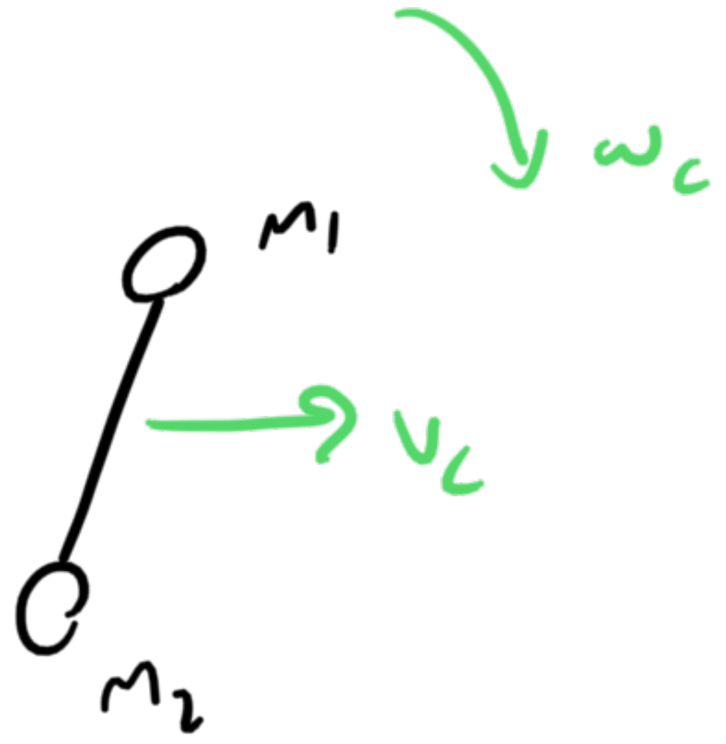
Δt is small, then $\Delta y \simeq d\Delta\theta + \Delta x$

$$\alpha = \frac{Jd}{I\Delta t} \qquad a = \frac{J}{M\Delta t}$$

$$\Delta\theta = \frac{1}{2}\alpha\Delta t^2 = \frac{Jd\Delta t}{2I} \qquad \Delta x = \frac{1}{2}a\Delta t^2 = \frac{J\Delta t}{2M}$$

$$\Delta y = \frac{J\Delta t}{2M}\left(1+\frac{d^2}{L^2}\right) \Rightarrow \Delta E = \frac{J^2}{2M}\left(1+\frac{d^2}{L^2}\right)$$

2018 B #23



$$p = (m_1+m_2)v_c \Rightarrow v_c = \frac{m_1 v}{m_1+m_2}$$

$$r_1 = \frac{m_2 L}{m_1+m_2} \qquad r_2 = \frac{m_1 L}{m_1+m_2}$$

$$v = v_c + r_1 \omega = \frac{m_1 v}{m_1+m_2} + \frac{m_2 L \omega}{m_1+m_2}$$

$$\Rightarrow \omega = v/L$$

$$v_{m_2}(t=0) = 0$$

$$\Rightarrow |r_2 \omega| = |v_c|$$

next time m_2 is at rest is after 1 rotation.

$$T = \frac{2\pi}{\omega} = \frac{2\pi L}{v}$$

A