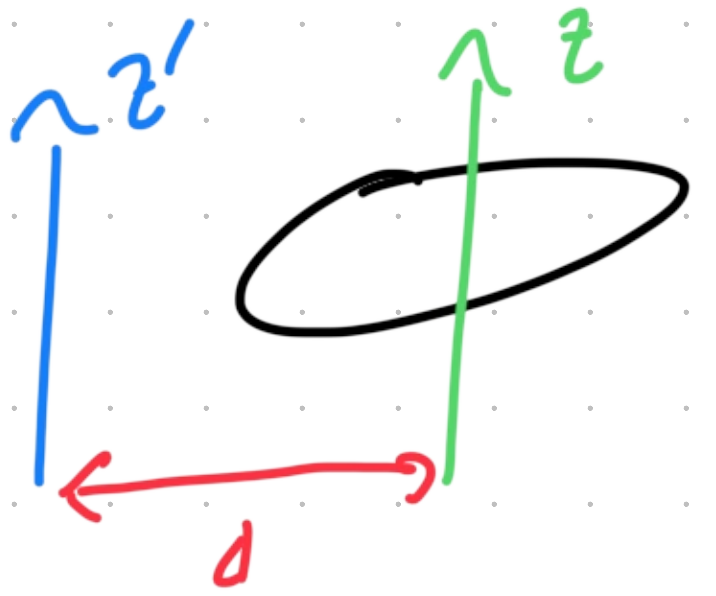
2020 B #13

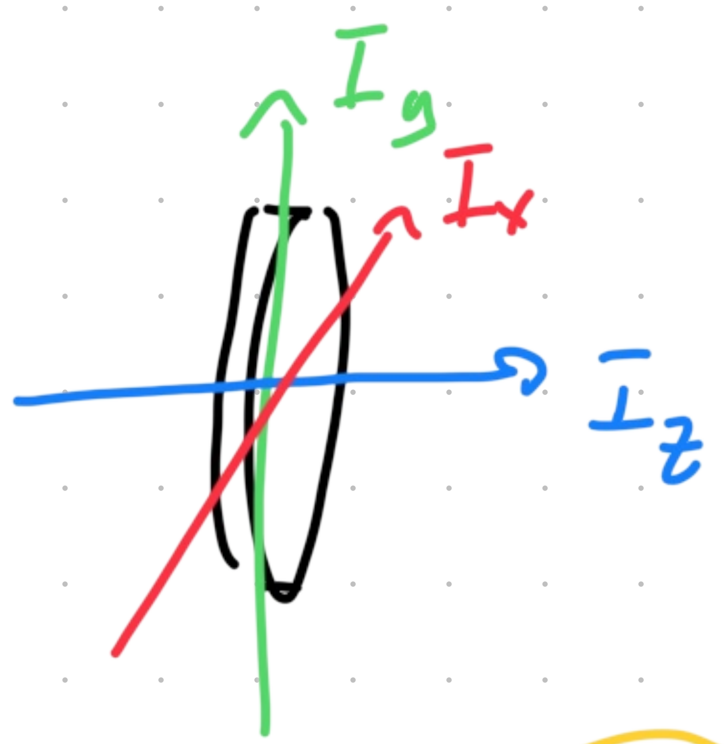
Topic 5.2

Parallel Axis Theorem



$$I_{z'} = I_z + md^2$$

Perpendicular Axis Theorem (for planar objects)



$$I_z = I_x + I_y$$

derivation

$$I_x = \int x^2\,dm \qquad I_y = \int y^2\,dm$$

$$I_z = \int z^2\,dm \qquad z^2 = x^2 + y^2$$

$$= \int (x^2+y^2)\,dm = I_x + I_y$$

$$I_\perp = I_x + I_y \qquad I_x = I_y$$

$$I_x = I_\perp/2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right) = \frac{1}{4}mr^2$$

$$I = I_\perp + mr^2 = \frac{5}{4}mr^2$$

← key to solving

$\Rightarrow$ now use Topic 5.3

$$\omega = \sqrt{\frac{mgr}{I}} = \sqrt{\frac{mgr}{\frac{5}{4}mr^2}} = \sqrt{\frac{4g}{5r}} \qquad T = \frac{2\pi}{\omega} = \pi\sqrt{5r/g}$$

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2020B #19

$I_0 \to I' > I_0$

$\Sigma F = \Sigma \tau = 0$

So, L is conserved!

$L = I_0 \omega_0 = I' \omega'$

$K_0 = \frac{1}{2} I_0 \omega_0^2$

$K' = \frac{1}{2} I' (\omega')^2$

$I' \equiv c I_0 \qquad c > 1$ (constant)

$\omega' = \omega_0 / c < \omega_0$

$K' = \frac{1}{2} c I_0 \left(\frac{\omega_0}{c}\right)^2$

$= K_0 / c < K$

So, $\omega' < \omega_0$

$K' < K_0$

A

2020 B #22

both slipping

$F_1 = -\mu m g$

$F_2 = +\frac{2}{3}\mu m g$

$a_1 = -\mu g$

$a_2 = +\frac{1}{3}\mu g$

$a_{rel} = \frac{4}{3}\mu g$

$v_{1_0} = p/m$

$t_{slip} = v_{1_0}/a_{rel}$

$v_{avg} = v_{1_0}/2 = p/2m$

$\Delta x = v_{avg}\, t_{slip}$

$= \frac{v_{1_0}}{2} t_{slip} = \frac{v_{1_0}^2}{2a_{rel}}$

$= \frac{p^2}{m^2} \cdot \frac{1}{2} \cdot \frac{3}{4\mu g}$

$= 3p^2/8m^2\mu g = \frac{3}{8}\, p^2/m^2\mu g$ A