Rotation.

1. Rotation is analagous to linear motion
2. Need to treat simultaneously.

$$\left.\begin{array}{l} x \to \theta \\ v \to \omega \\ a \to \alpha \end{array}\right\} \ 1/r$$

$F \to N$: $N = r \times F$

$m \to I$: $I = mr^2$

$K \to K$: $K = \frac{1}{2} I \omega^2$

$\frac{1}{2}(m\cancel{r^2}) \cdot \left(\frac{v}{\cancel{r}}\right)^2$

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

Ex.

m, R

$\overset{t}{\Rightarrow}$

$v = ?$

$I_{sph} = \frac{2}{5} mR^2.$

$= \frac{2}{3} mR^2 \; ?$

No energy loss.

$E_0 = E_f$

$\omega = v/R$

$E_0 = \frac{1}{2}\left(\frac{2}{5} mR^2\right)\omega_0^2$

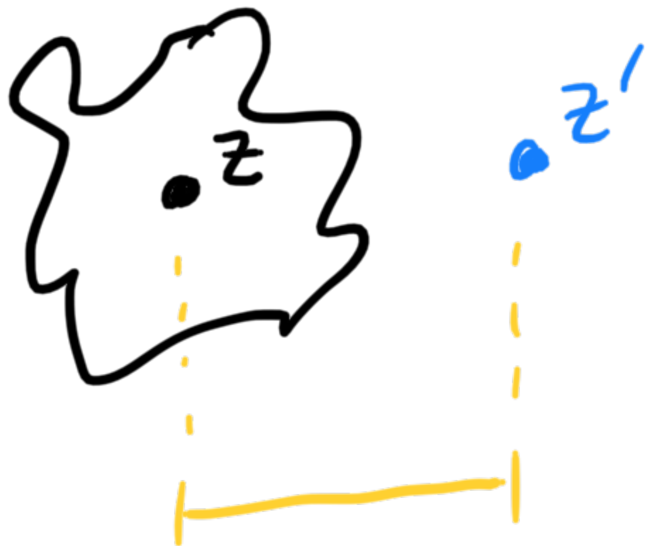
$\parallel$

$E_f = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{5} mR^2\right)\omega^2$

$= \frac{1}{2} m \left(v^2 + \frac{2}{5} v^2\right) = \frac{7}{10} m v^2$

$\frac{1}{5} \cancel{m} R^2 \omega_0^2 = \frac{7}{10} \cancel{m} v^2 \Rightarrow v = \sqrt{2/7}\, R\omega_0$

Parallel axis thm.



$$I_{z'} = I_z + m d^2$$

$$I_{r.c.} = \frac{1}{12} m L^2 = I_z$$

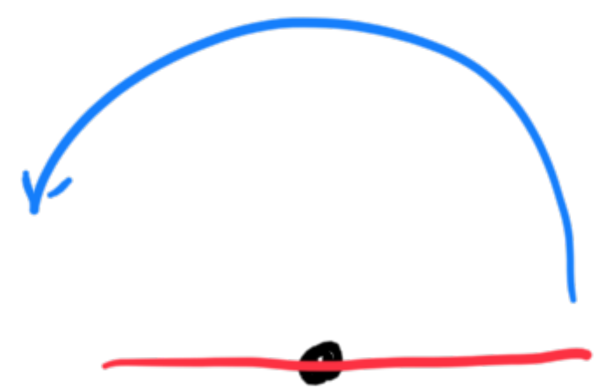
$$I_{r.e.} = \frac{1}{3} m L^2 = I_{z'}$$

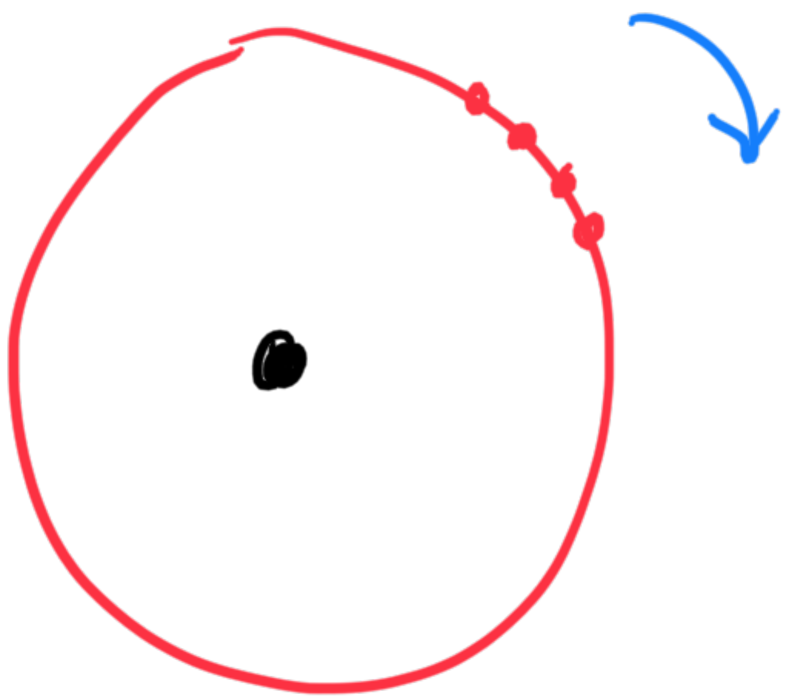
$$I_z + m\left(\frac{L}{2}\right)^2 = \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2$$

Moments of Inertia

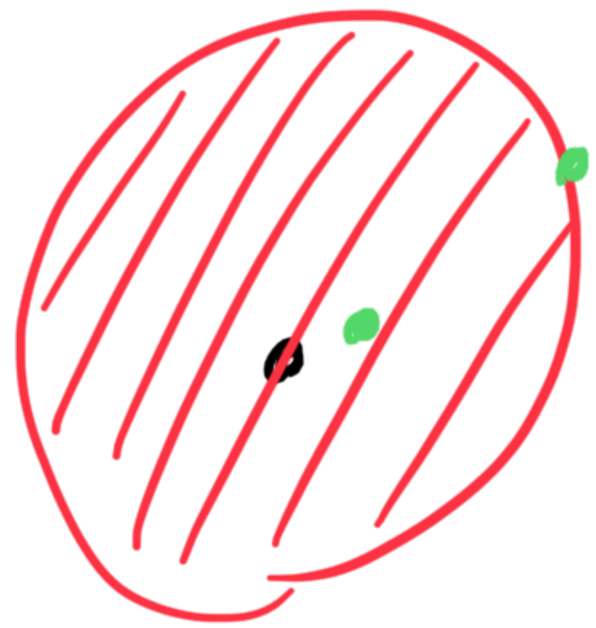
$I = mr^2$

$I = \frac{1}{12} m L^2$



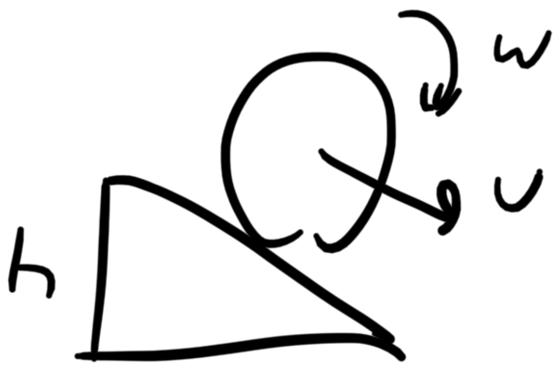


$I = MR^2$



$I = \frac{1}{2} MR^2$

2017 #14



$$\boxed{I = cmR^2}$$

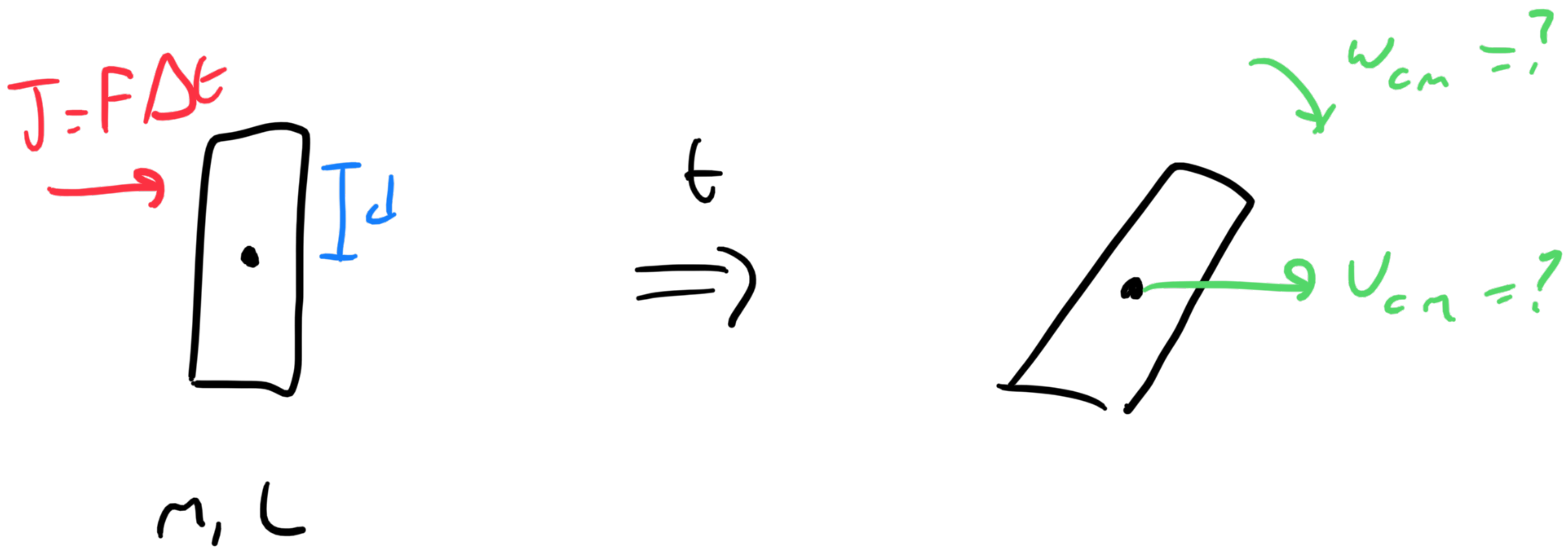
$$\Delta K = mgh = \frac{1}{2}mv^2 + \frac{1}{2}(cmR^2)w^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}cmv^2$$

$$v = \frac{2\cancel{m}gh}{\cancel{m}(1+c)}$$

smaller c $\rightarrow$ larger v

MIT Ex.



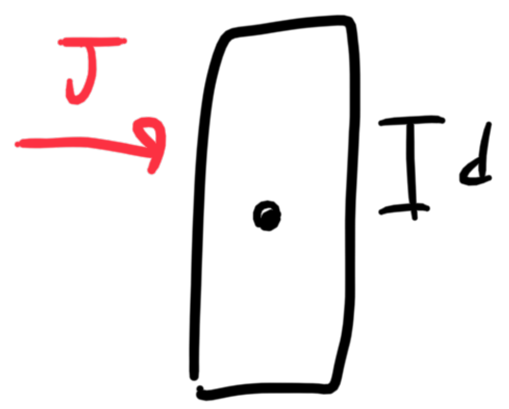
$$J = F\Delta t = Ma\Delta t = M v_{cm}$$

$$\Rightarrow \quad v_{cm} = J/M$$

$\frac{1}{12} mL^2$

$$J_N = F\Delta t\, d = I\alpha\Delta t = I w_{cm}$$

$$w_{cm} = 12\, Jd / mL^2$$



$$E_0 = 0$$

$$E_f = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega_{cm}^2$$

does this depend on d?

$$v_{cm} = J/m$$

$$\omega_{cm} = 12 J d / m L^2$$

2017 #3.

$$X_4 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m \cdot 0 + m \cdot R}{2m} = \frac{R}{2}$$

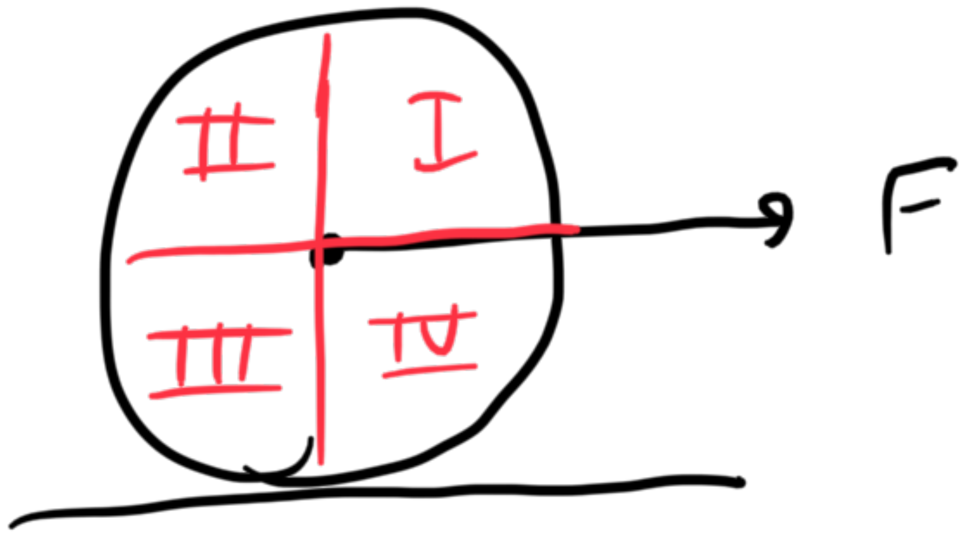
#11, 12

Strike at C.O.M., no rotation.

Mass does not affect rotation if we strike at c.o.m.

#18

any point with no acceleration?



$$\vec{a}_c + \vec{a}_t \approx -\vec{a}_F$$

IV only

$$\| a_c + a_t \| = \sqrt{\|a_c\|^2 + \|a_t\|^2} = r\sqrt{\omega^4 + \alpha^2} = \alpha R ?$$

true for some

$r \in [0, R]$

$$\| a_F \| = \alpha R$$