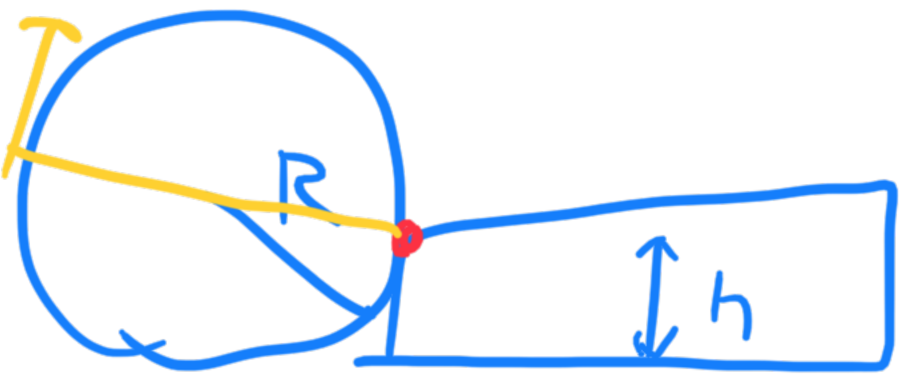
2019 A #5



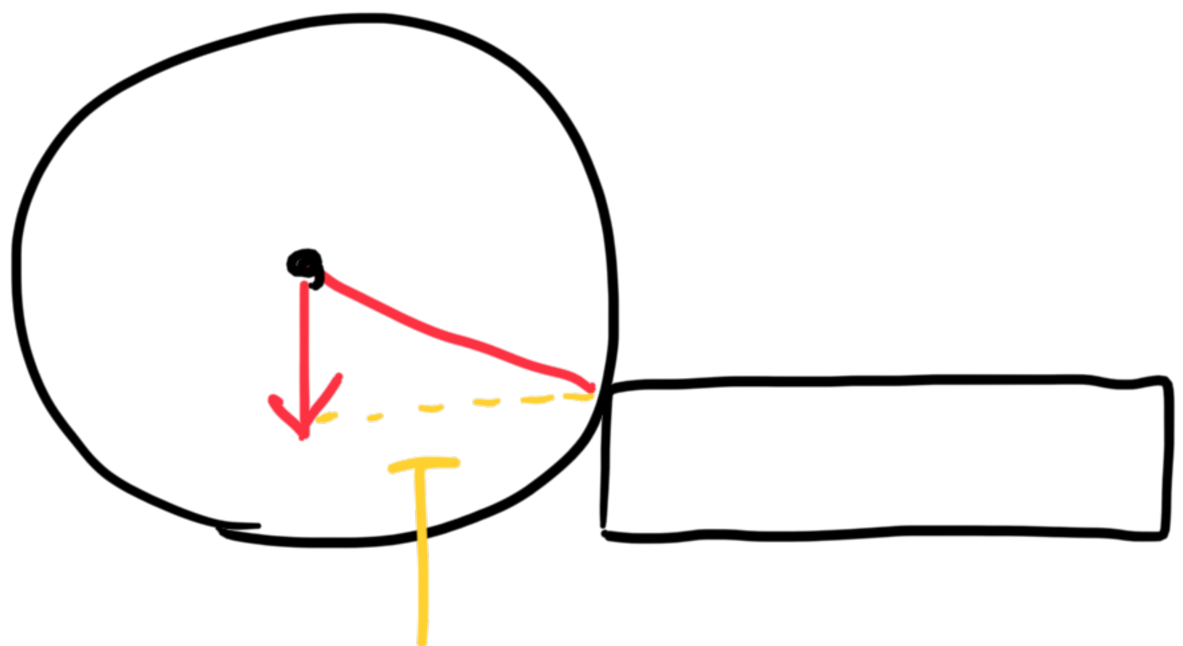
G

min $F = ?$

$$\tau_g = G\boxed{\sqrt{R^2-(R-h)^2}} = G\sqrt{2Rh-h^2}$$

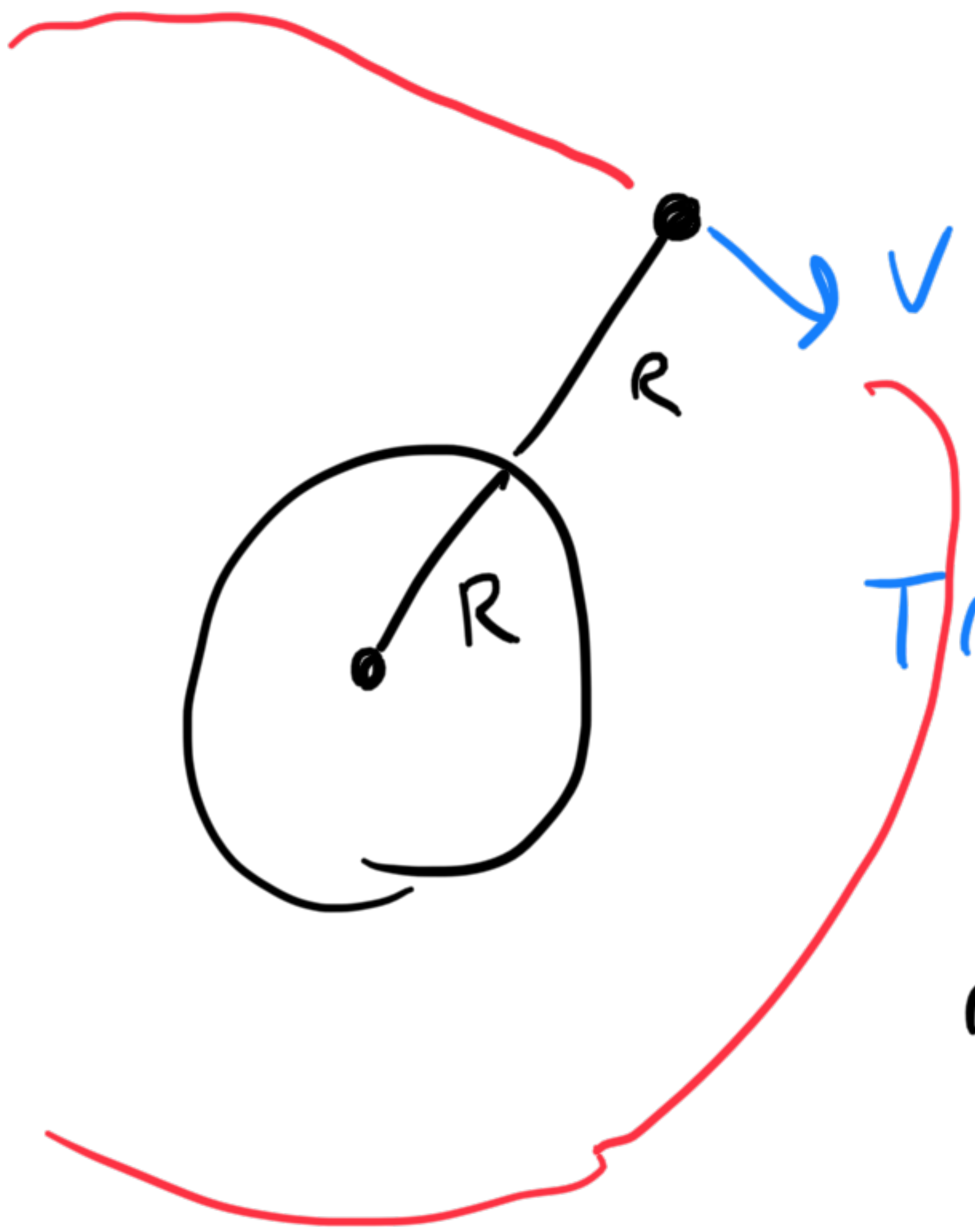
$$F = \frac{\tau_g}{2R} = \frac{G\sqrt{2Rh-h^2}}{2R}$$

C



$$\sqrt{R^2-(R-h)^2}$$

#8



$V = \sqrt{\frac{2GM}{3R}}$

Trajectory?

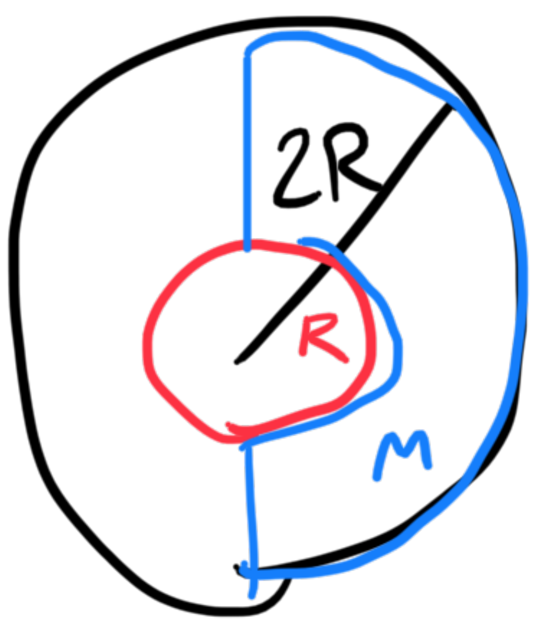
$r = 2R$

$V = \sqrt{\frac{4GM}{3r}}$

V:

$0 \to \sqrt{GM/R}$	elliptical R = apogee	
$\sqrt{GM/R}$	circular	
$\sqrt{GM/R} \to \sqrt{2GM/R}$	elliptical R = perigree	
$\sqrt{2GM/R}$	parabolic	
$> \sqrt{2GM/R}$	hyperbolic	

#10



m

$$M = \frac{1}{2}\left(m - \frac{m}{4}\right)$$

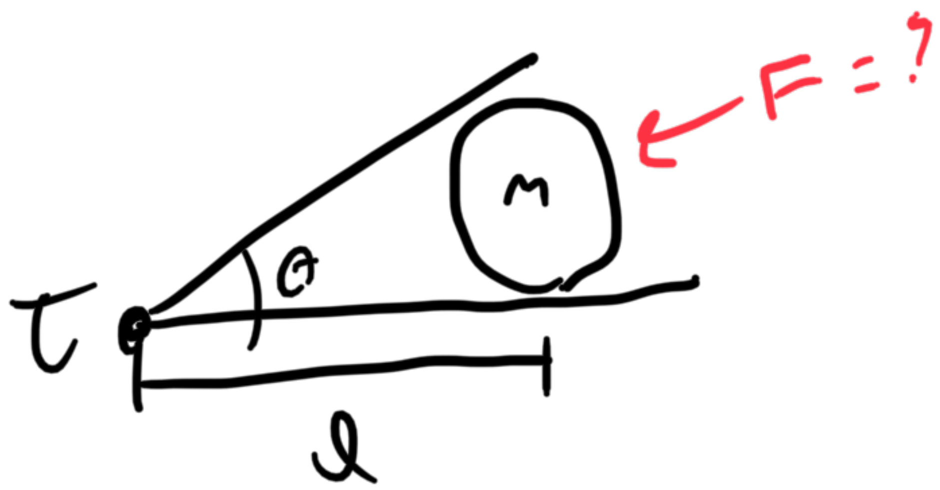
$$\Rightarrow \quad m = \frac{8}{3} M$$

(c) $I_d = \frac{1}{2} m r^2$

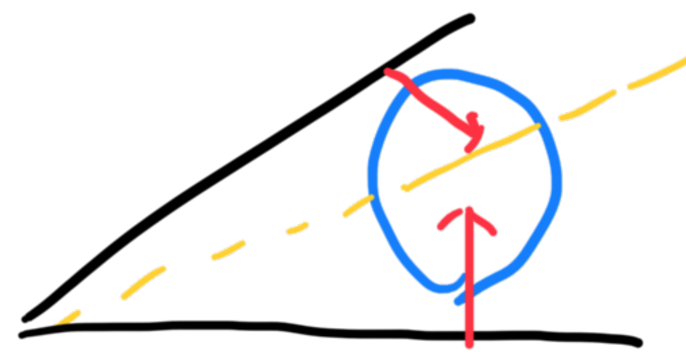
(a) $I_a = \frac{1}{2} m (2R)^2 - \frac{1}{2} \frac{m}{4} R^2 = \frac{15}{8} m R^2 = 5 M R^2$

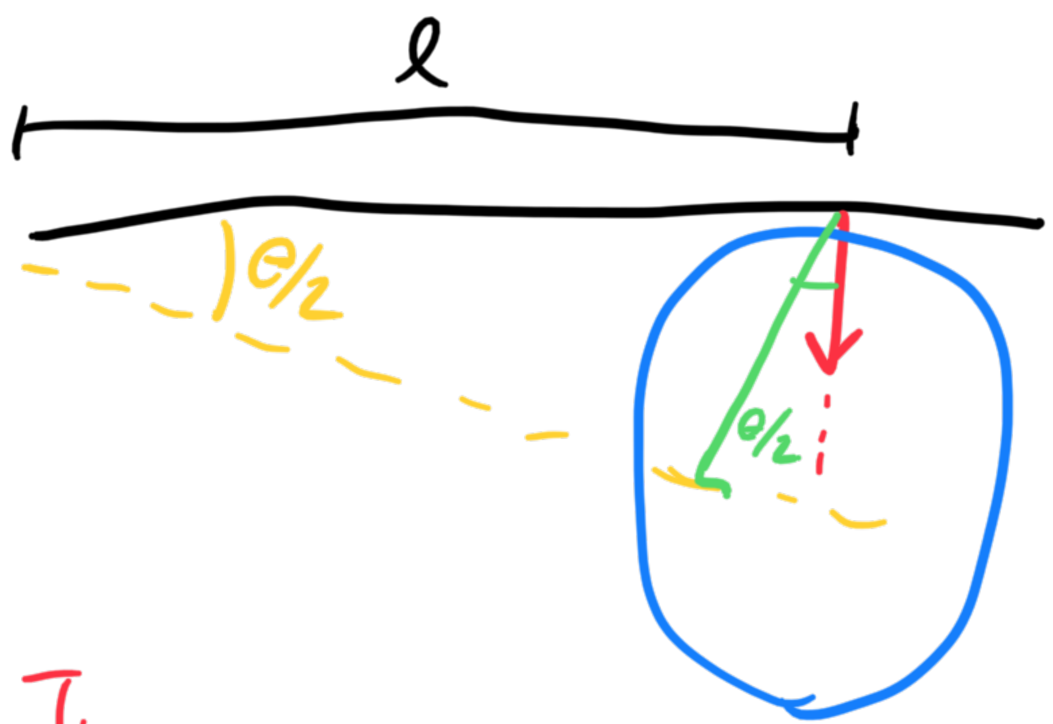
(b) $I_{final} = \frac{5}{2} M R^2$

#17



$F = \tau / R$



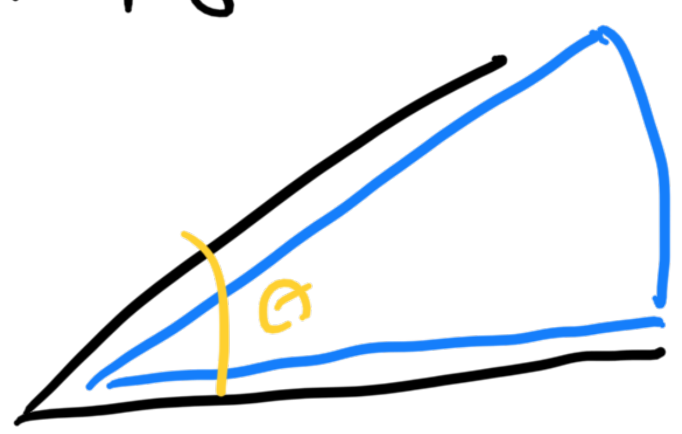


$$F_b = \frac{\tau}{\ell}$$

$$F_{b\text{-out}} = \frac{\tau}{\ell} \sin\left(\frac{\theta}{2}\right)$$

$$F = 2F_{b\text{-out}} = 2\frac{\tau}{\ell} \sin\left(\frac{\theta}{2}\right)$$

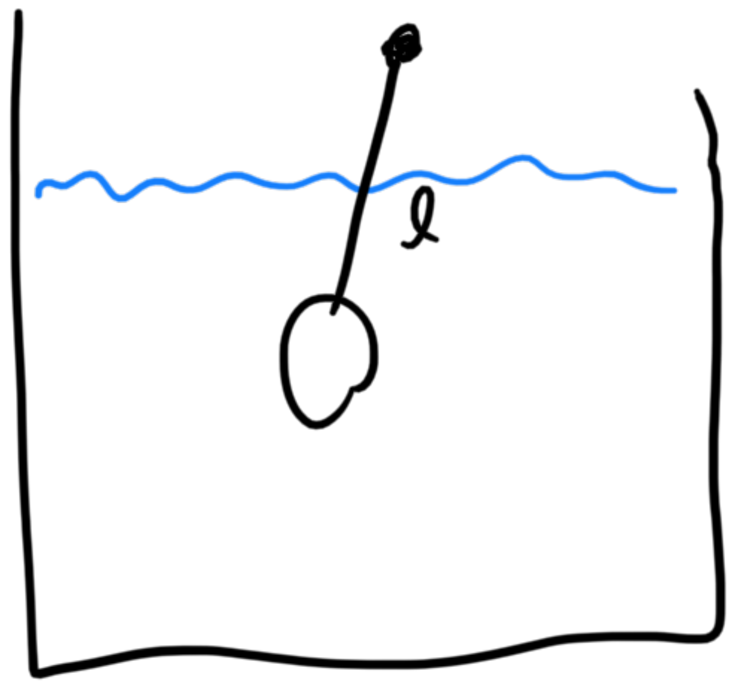
#18



$$E = Fd = \tau\theta$$

$$\frac{mv^2}{2} = \tau\theta \Rightarrow v = \sqrt{\frac{2\tau\theta}{m}}$$

#19



$l = 5\,m$

$\omega = ?$

$\rho_{rock} = 2\rho_{H_2O}$

$\omega = \sqrt{g/l}$

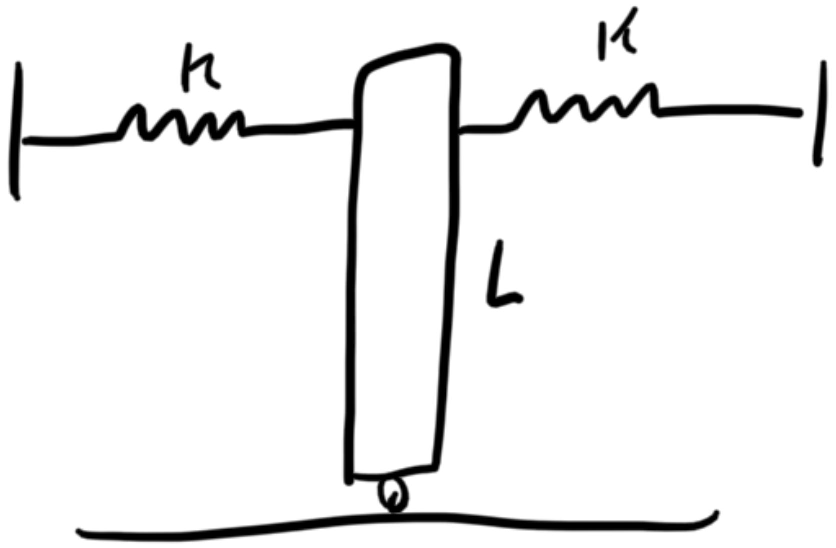
$|F_b| = \left|\frac{mg}{2}\right|$

$g_{eff} = g/2$

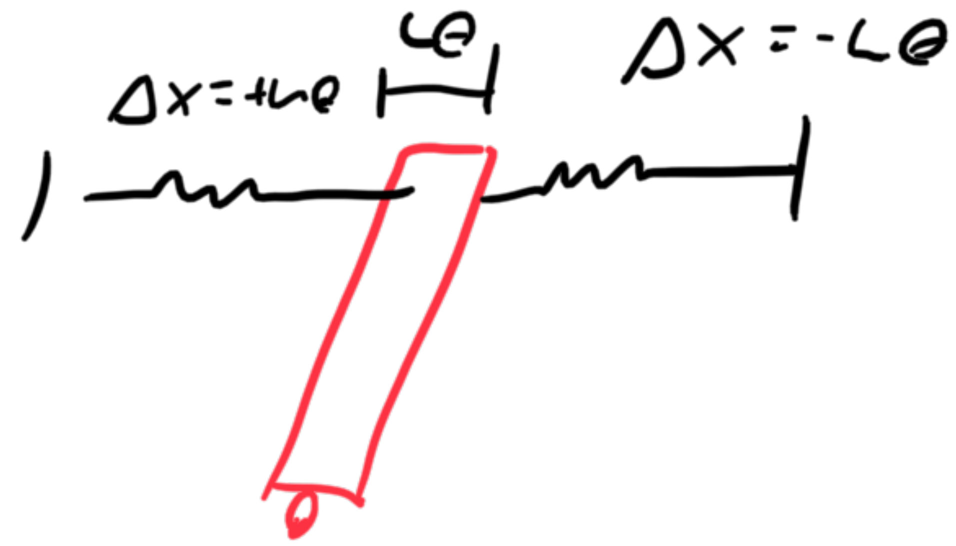
$\omega = \sqrt{g'/l} = \sqrt{g/2l} = 1\,\text{rad/s}$

A

#20



m, k, L for stable equilib?



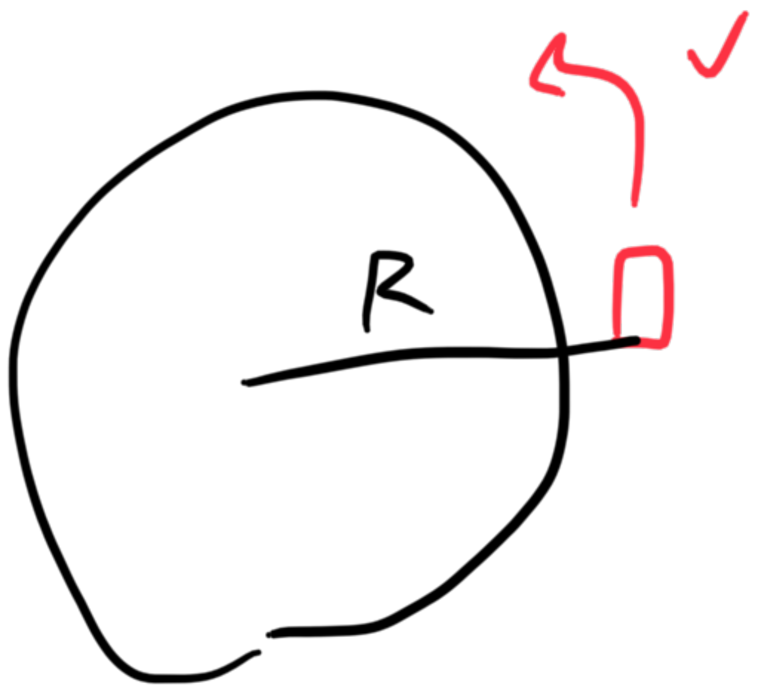
$$\tau_s = 2kL^2\theta = FL = 2kL\theta \cdot L$$

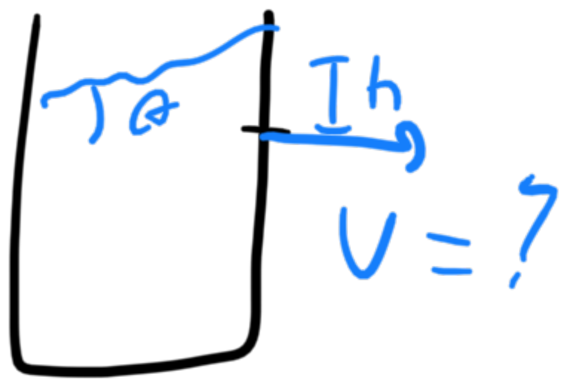
$$\tau_g = mgL\theta/2$$

$$\tau_s > \tau_g$$

$$mg < 4kL$$

#25





$$a_c = \frac{v^2}{r}\hat{i}$$

$$g = -g\hat{j}$$

$$g_{eff} = a_c + g = \frac{v^2}{r}\hat{i} - g\hat{j}$$

$$\theta = \arctan\left(\frac{v^2}{gr}\right)$$

$$h_{eff} = h\cos\theta$$

$$P = \rho g_{eff} h_{eff}$$

$$= \rho g_{eff} h\cos\theta$$

$$= \rho g h$$

Bernoulli's principle $\boxed{\rho g h = \frac{\rho v^2}{2}} \Rightarrow v = \sqrt{2gh}$