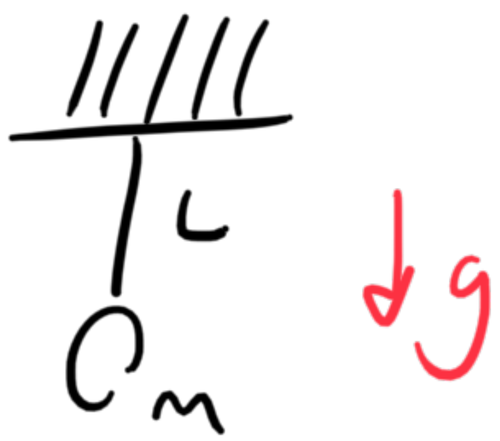
2018B

#4

Momentum: not conserved because of external force

energy: not conserved because of sound, heat, etc.

Dimensional analysis.



What's the period?

A. $\sqrt{g/L}$

B. $\sqrt{L/g}$

C. $\sqrt{mg/L}$

D. $\sqrt{L/mg}$

E. $\sqrt{gL}$

can't have mass $\rightarrow$ $\cancel{C}$, $\cancel{D}$

arg of $\sqrt{\ }$ must be s^2.

can't have $\sqrt{gL}$ $\rightarrow$ $\cancel{E}$

$[g] = m/s^2$ that must be in the denominator.

$\Rightarrow$ $\boxed{B}$

#6

$$x \propto F^{1} I^{-1} E^{\gamma} L^{\delta}$$

$$[F] = \boxed{N}$$

$$[I] = m^{4}$$

$$[E] = \boxed{N} m^{-2}$$

$$[L] = m$$

$$x \propto F^{1} I^{-1} E^{-1} L^{\delta}$$

$$[F^{1} I^{-1} E^{-1} L^{\delta}] = N m^{-4} N^{-1} m^{2} L^{\delta} = [x] = m$$

$$= \cancel{N^{(1-1)}} m^{(-4+2+\delta)} = m^{(1)}$$

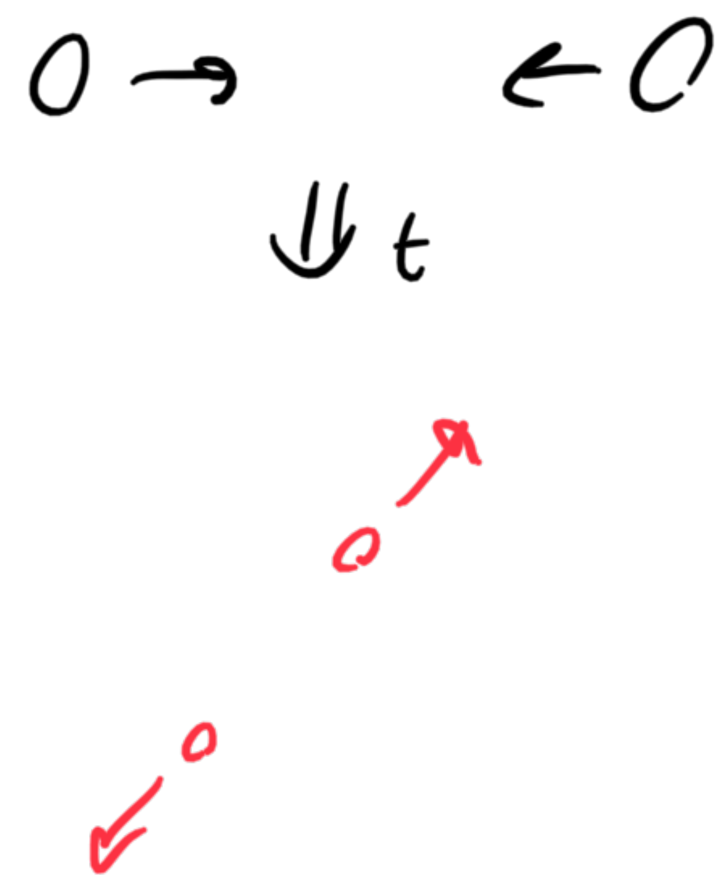
$$m^{(\delta-2)} = m^{(1)}$$

$$\delta - 2 = 1 \Rightarrow \delta = 3 \;\longrightarrow\; x \propto L^{3}$$

#9

$$\vec{P}_i = m_1 v_1 + m_2 v_2 = 0 = \vec{P}_f$$

$$\Rightarrow \quad m_1 \vec{v}_1' + m_2 \vec{v}_2' = 0$$



$$\vec{P}_i = \vec{P}_f$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2$$

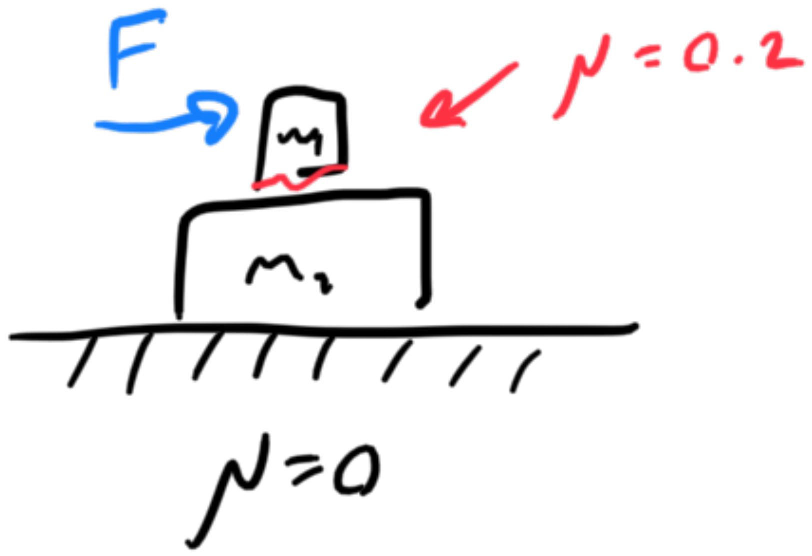
$$\vec{P}_f = 3\,\text{kg}\,(2v)\,\hat{v}_1 + 2\,\text{kg}\,(3v)\,\hat{v}_2$$

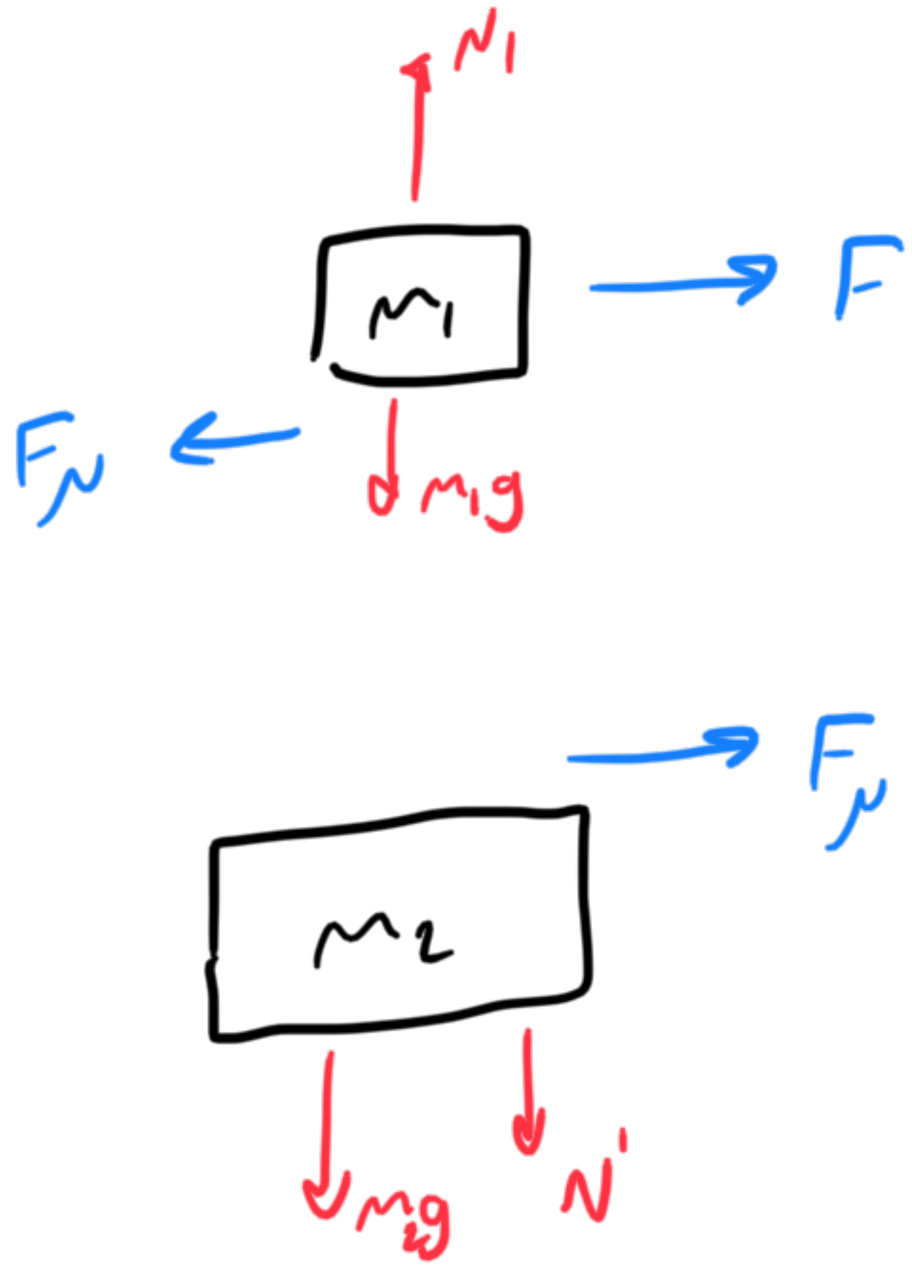
$$\Downarrow$$

$$V = 5$$

So $|v_1'| = 10\ \text{m/s}$ $\quad |v_2'| = 15\ \text{m/s}$

#13





$$F_\mu \le \mu N$$

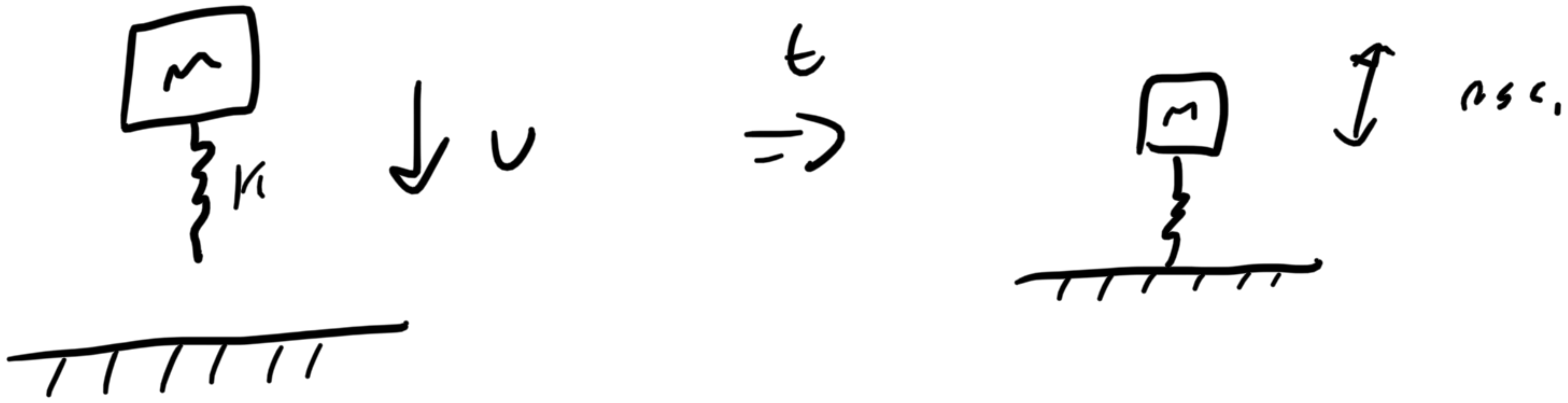
$$F_{\mu_{max}} = \mu N$$

$$F_\mu = m_1 g \mu = 4\,N$$

$$a_2 = \frac{F_\mu}{m_2} = 4\,m/s^2$$

$$\boxed{a_2 = a_1} = \frac{\Sigma F_1}{m_1} = \frac{F - F_\mu}{m_1} = \frac{F - 4N}{2\,kg} = 4\,m/s^2 \Rightarrow F = 12N$$

#17



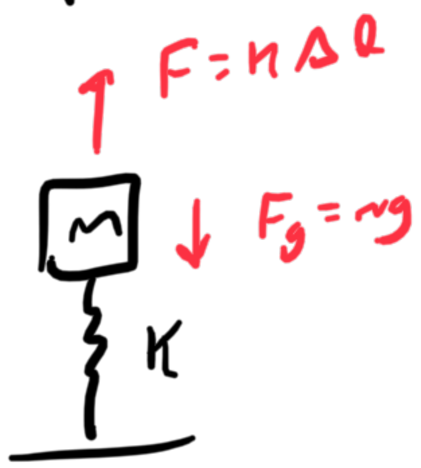
$$KE = \frac{1}{2} m v^2$$

$$PE = \frac{1}{2} k (mg/k)^2$$

$$KE_{max} = \frac{1}{2} m \underbrace{\left(v^2 + mg^2/k \right)}_{= v_{max}^2} = \frac{1}{2} m v_{max}^2$$

$$\boxed{v_{max} = \sqrt{v^2 + mg^2/k}}$$

equil:



$$mg = k \Delta l$$

$$\Delta l = \frac{mg}{k}$$

$$l_{equil} = l - \frac{mg}{k}$$

Springs in Series & Parallel

See 2020 Hw 1 #11 about cutting springs

$k' = ?$

$\Delta x = ? = {}^{2F}/_{k}$

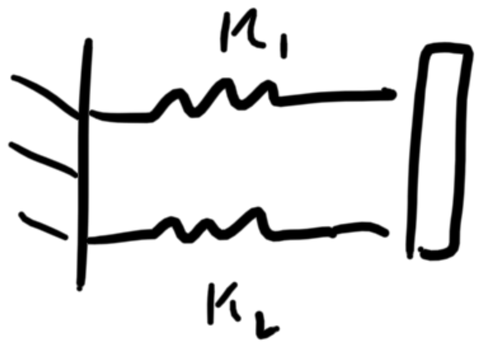
$F = k' \Delta x$

$\Delta x_1 = {}^{F}/_{k}$

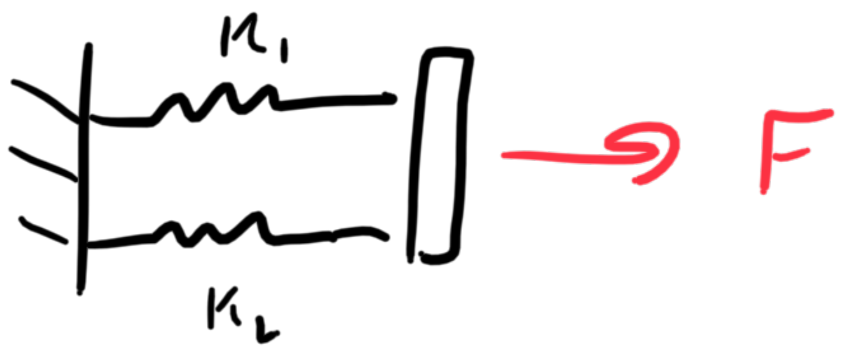
$\Delta x = {}^{F}/_{k'} = {}^{2F}/_{k}$

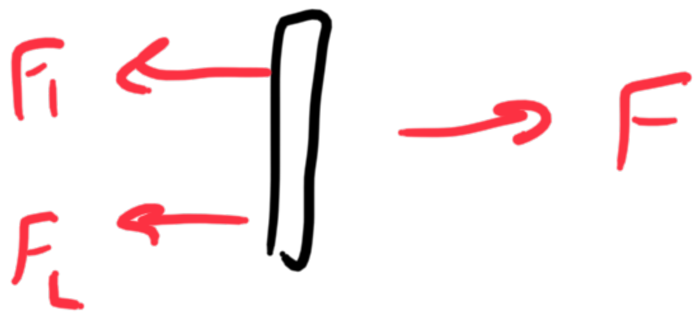
$\Delta x_{outer} = {}^{F}/_{k}$

$$\boxed{k' = \frac{k}{2}}$$



$k' = ?$





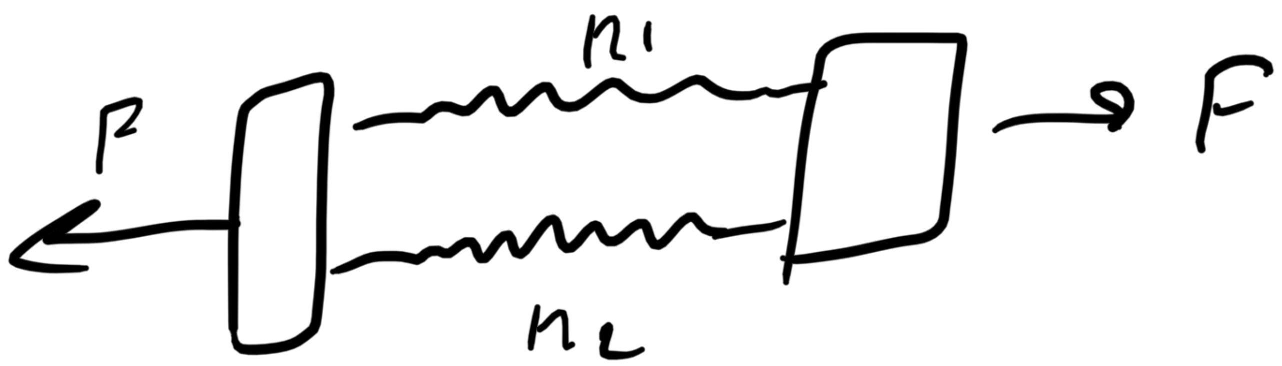
$F_1 + F_2 = F$

$\Delta x_1 = \Delta x_2$

$F = k_1 \Delta x_1 + k_2 \Delta x_2 = (k_1 + k_2)\Delta x$ $\quad F = k' \Delta x,$ $\quad k' = k_1 + k_2$

#18

$k = k_1 + k_2$ (from before)

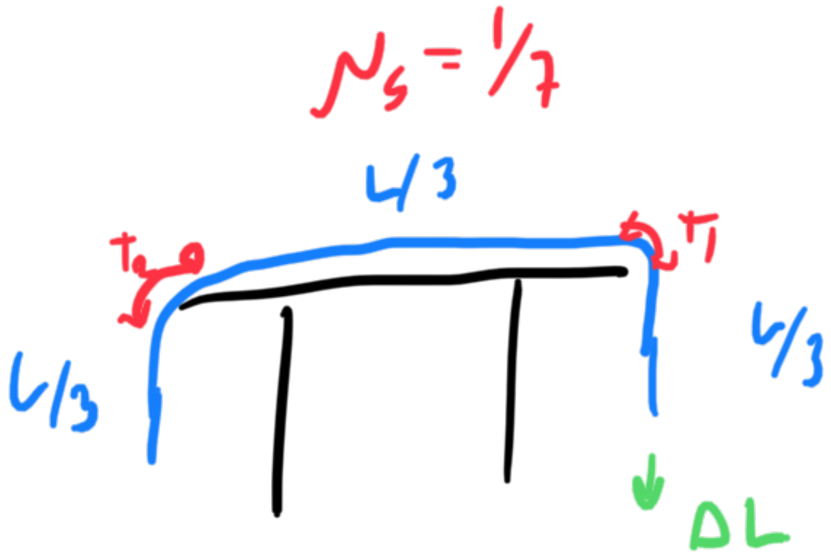


$\Sigma F = 0,$ $\quad 0 = k_1(l - l_1) + k_2(l - l_2)$

$$\Rightarrow k_1 l_1 + k_2 l_2 = (k_1 + k_2) l$$

$$l = \frac{k_1 l_1 + k_2 l_2}{k_1 + k_2}$$

#20



$$N_T = mg/3$$

$$T_1 = \left(\frac{1}{3} + \frac{\Delta L}{L}\right) mg$$

$$T_2 = \left(\frac{1}{3} - \frac{\Delta L}{L}\right) mg$$

$$\Sigma F = |T_1 - T_2| - F_N$$

$$= \frac{2\Delta L}{L} mg - F_N$$

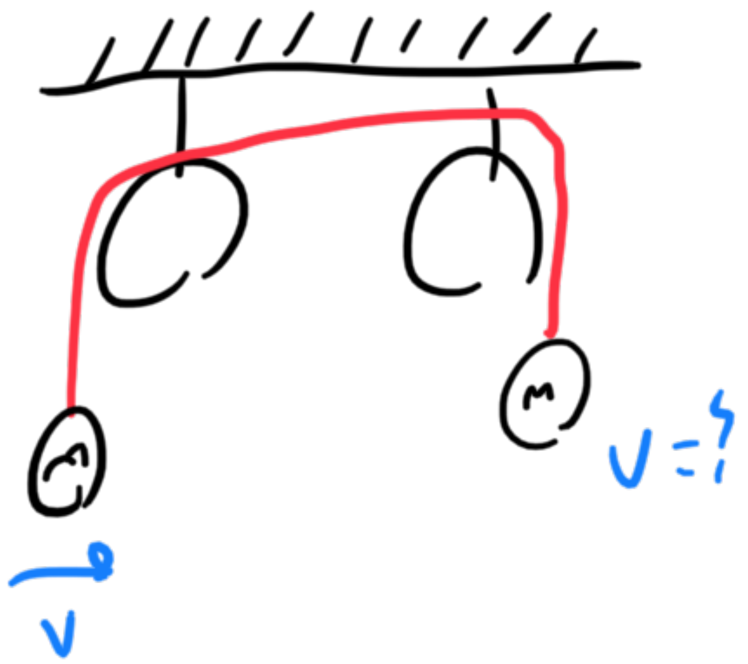
$$\frac{2\Delta L}{L} mg = F_N \leq \mu mg/3$$

max to not slip:

$$\frac{2\Delta L}{L} \cancel{mg} = \mu \cancel{mg}/3$$

$$\frac{2\Delta L}{L} = \frac{\mu}{3} \Rightarrow \Delta L = \frac{\mu L}{6} = L/42$$

#22



vertical only!
not at rest.

$T_L = ?$

assume $\langle T_{L_y} \rangle = mg$ (average)

then $\langle T_{L_x} \rangle \neq 0$ so that's a contradiction

with. $\langle T_L \rangle = mg$.

← contradiction!

$\langle T_L \rangle > mg$

mass on right ascends