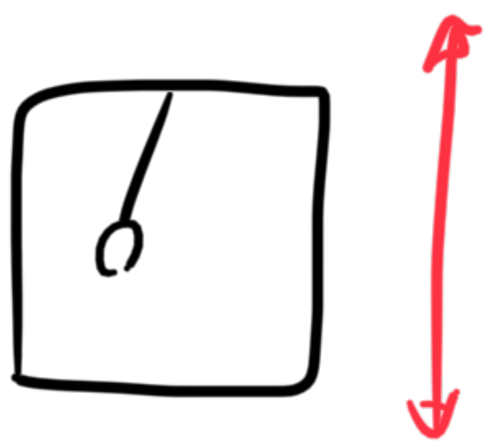
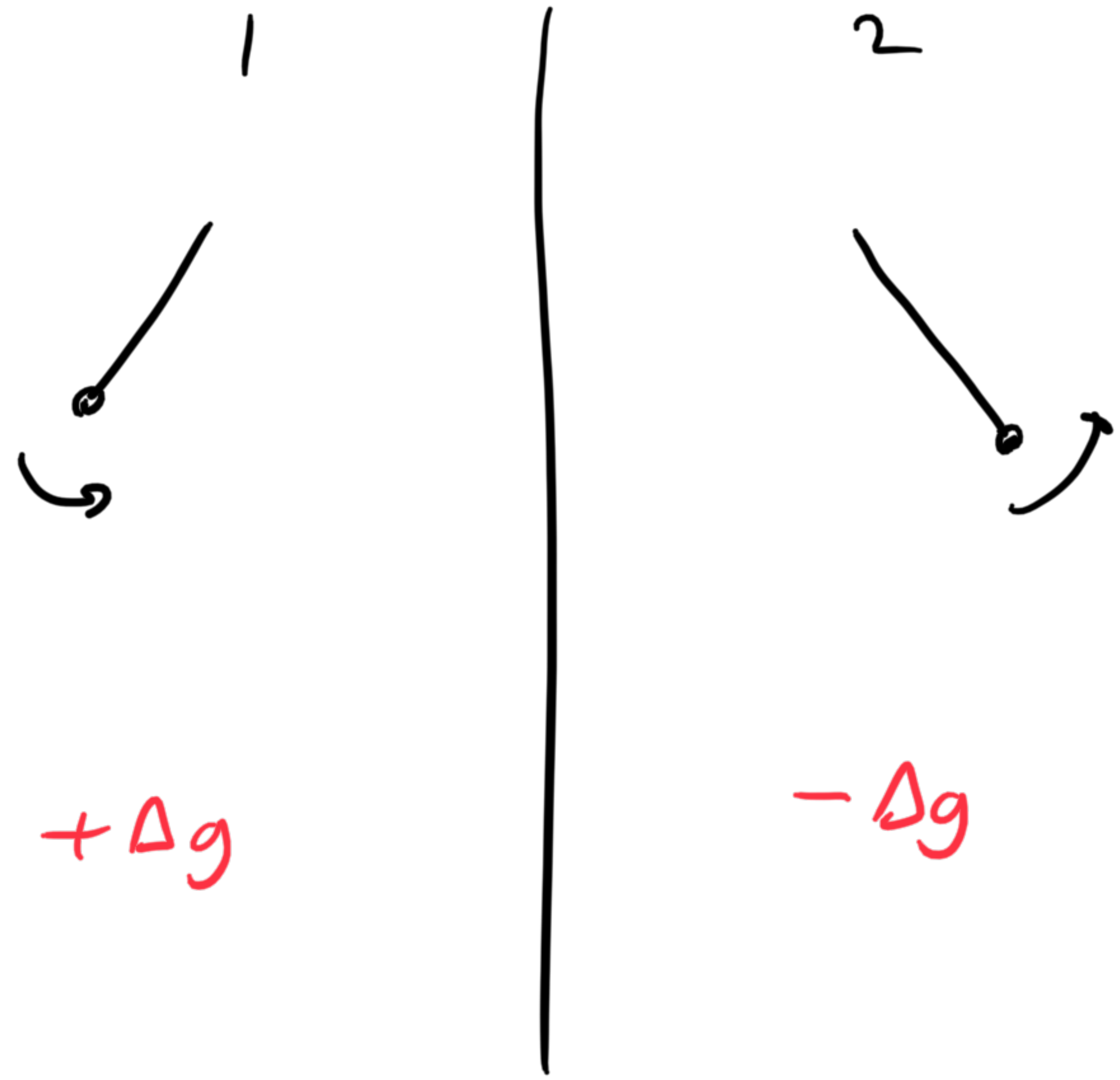
2018B

#7

$\Omega_p = \sqrt{g/L}$

$\Omega_p' \approx \sqrt{g/L}$





$g \to g(t)$

Δg changes sign twice per pendulum swing.

$w = 2\Omega_p = \sqrt{4g/L}$

A

#10

$F_b = \rho V g$

$B_0 = \rho V_1 g$ $\quad (h)$

$B' = \rho V_2 g$ $\quad (2h)$

$B' = \rho \frac{V_1}{2} g$

$= B_0/2$

$P \propto h$

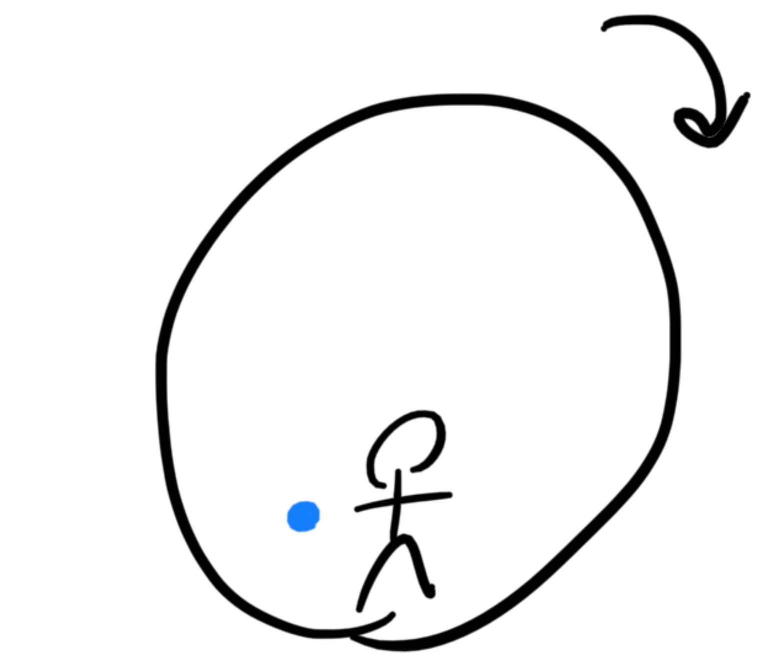
$P_2 = 2P_1$

$PV = \underbrace{nRT}_{\text{cst.}}$

$P_1 V_1 = P_2 V_2$

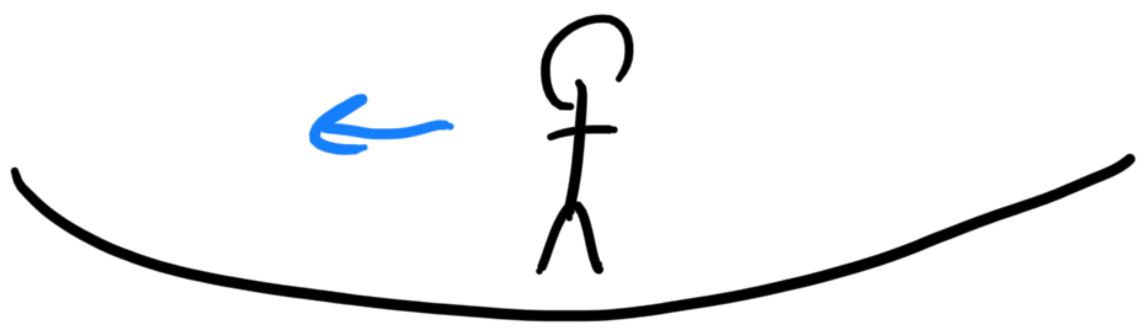
$V_2 = \frac{P_1}{P_1} V_1 = \frac{V_1}{2}$

#12



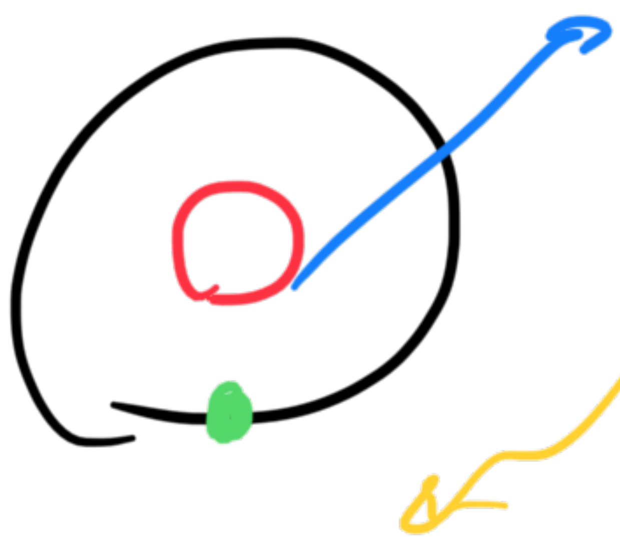
Returns to child ~~A~~ ~~E~~

~~D~~



#14

No rolling



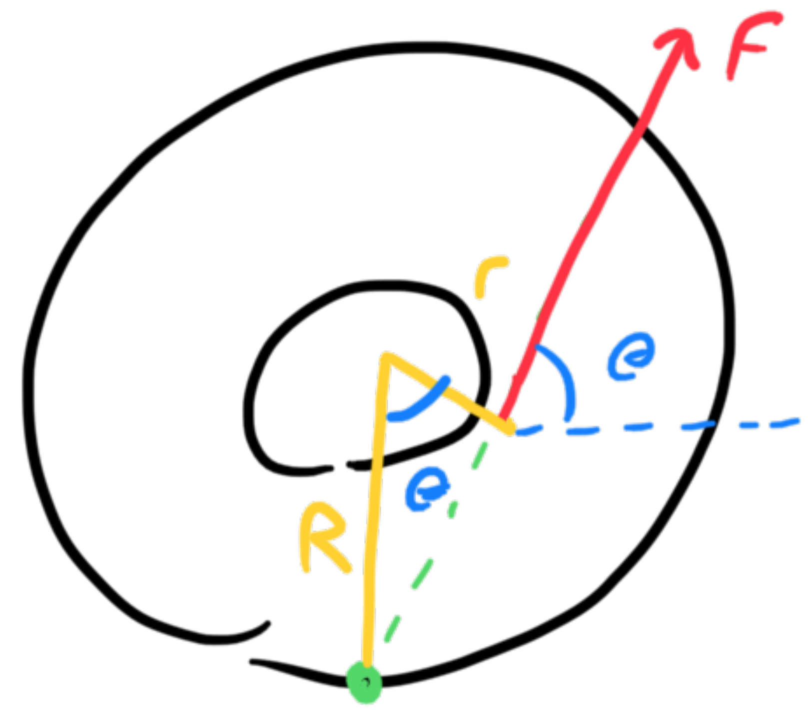
$$\Sigma \tau = 0 = \vec{\tau}_N + \vec{\tau}_g + \vec{\tau}_F + \vec{\tau}_N$$

$\tau_N = 0$ b/c $r = 0$

$\tau_N = 0$ ''

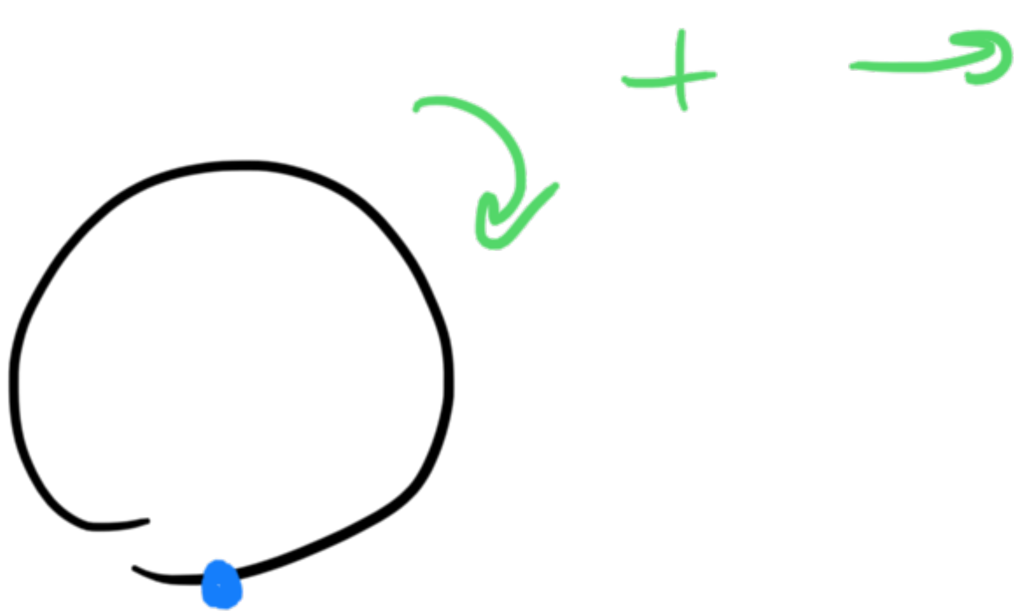
$\tau_g = 0$ b/c $\vec{F}_g \parallel \vec{r}_g$

$\rightarrow \vec{\tau}_F = 0$



$$\theta = \cos^{-1}\left(\frac{r}{R}\right)$$

$$= 41.4^\circ$$



#16

$F_\ell \sim v^2 \theta$ $\downarrow$ $\Rightarrow$ $\theta \uparrow$

$F_d \sim v^2$

$\boxed{P \propto Fv} \propto v^3$ $\downarrow$ $\Rightarrow$ $P \downarrow$

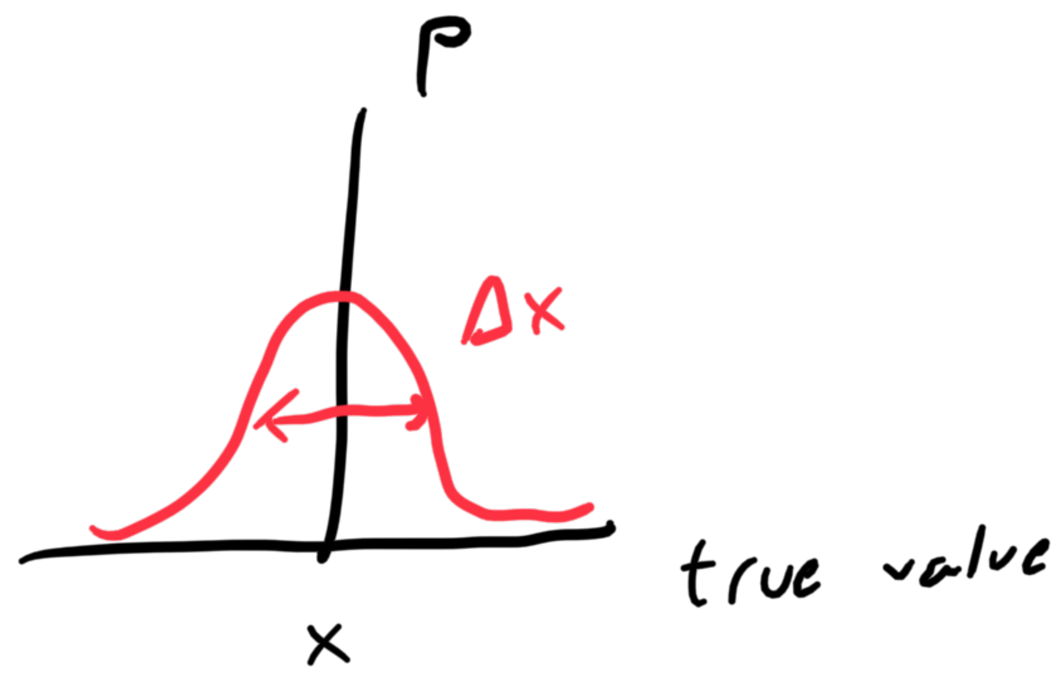
$[P] = J/s = \frac{kg\, m^2}{s^2} \cdot \frac{1}{s}$

$=$

$[F \cdot v] = N \cdot \frac{m}{s} = \frac{kg\, m}{s^2} \cdot \frac{m}{s}$

Error Propagation

$x \pm \Delta x \Rightarrow$



$x \pm \Delta x$

$y \pm \Delta y$

$x + y \pm \boxed{\Delta(x+y)}$

$(\Delta x)^2 \neq \Delta(x^2) \neq \boxed{\Delta x^2}$

Don't!

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$

$$\Delta(x^a) = |a|\, x^{a-1}\, \Delta x$$

#19

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$

$$\Delta(x^a) = |a|\, x^{a-1}\, \Delta x$$

$$v = \frac{s}{t}$$

$$\Delta v = \Delta\left(\frac{s}{t}\right)$$

$$\Delta\left(\frac{1}{t}\right) = \Delta(t^{-1}) = |-1|\, t^{-2}\, \Delta t$$

$$= \frac{\Delta t}{t^2}$$

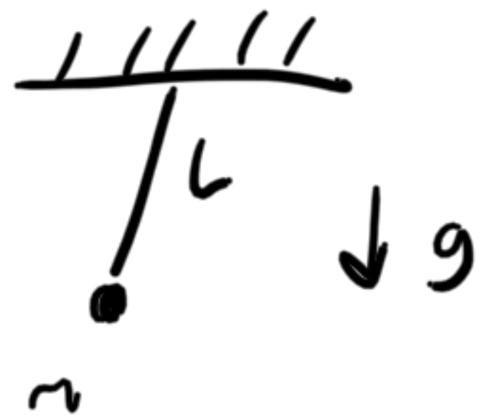
$$\Delta v = \Delta\left(s \cdot \frac{1}{t}\right) = \sqrt{\left(s\,\Delta\left(\frac{1}{t}\right)\right)^2 + \left(\frac{1}{t}\Delta s\right)^2}$$

$$= \sqrt{\left(s\,\frac{\Delta t}{t^2}\right)^2 + \left(\frac{1}{t}\Delta s\right)^2}$$

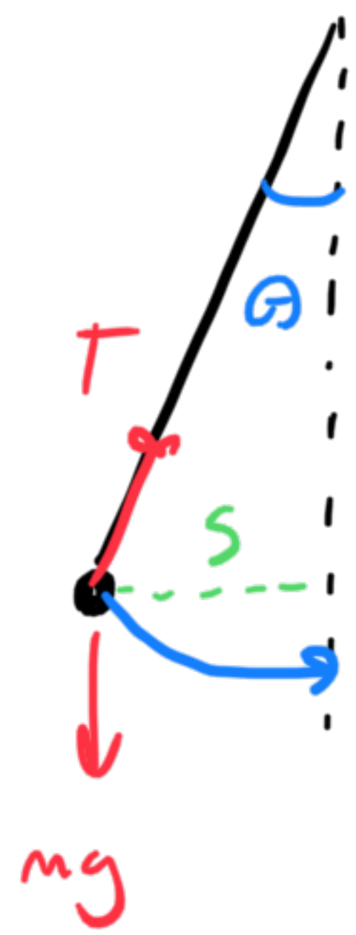
Oscillations

$w = \sqrt{K/m}$

$F = -Kx \Rightarrow w = \sqrt{K/m}$



$w = \sqrt{g/L}$



$$\Sigma F \simeq T\sin\theta = T\left(\theta - \underbrace{\boxed{\frac{\theta^3}{3!}}}_{\sim 0.1\%} + \frac{\theta^5}{5!} - \dots\right)$$

$$\simeq T\theta$$

$$\simeq -mg\theta = -mg\,S/L$$

$$F \simeq -mg\,S/L$$

$$w = \sqrt{\frac{mg}{mL}} = \sqrt{g/L}$$

#21

$$\boxed{T = 2\pi\sqrt{I/mgx}}$$ HW? ← use prev. slide

$$I = I_c + Mx^2 = \frac{1}{12}ML^2 + Mx^2$$

minimize $T \Rightarrow$ minimize I/Mx

$$I/Mx = x + \left(\frac{L^2}{12}\right)x^{-1}$$

$$\frac{1}{y} = c\left(y + 1/y\right)$$

min $y = 1$

$$c = \frac{L}{2\sqrt{3}}$$

$$\boxed{x = cy = c = L/2\sqrt{3}}$$