Rotational Dynamics

$F = ma \Rightarrow N = I\alpha$

$F \to \tau, N$ $\qquad \vec{N} = \vec{r} \times \vec{F}$ (or $N = rF$ if $r \perp F$)

$a \to \alpha$ $\qquad \alpha = a/r$

$v \to \omega$ $\qquad \omega = v/r$

$x \to \theta$ $\qquad \theta = x/r$

$m \to I$ $\qquad I \propto mr^2$

$K \to K$ $\qquad K = \frac{1}{2} I\omega^2$

Conversions

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \Rightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

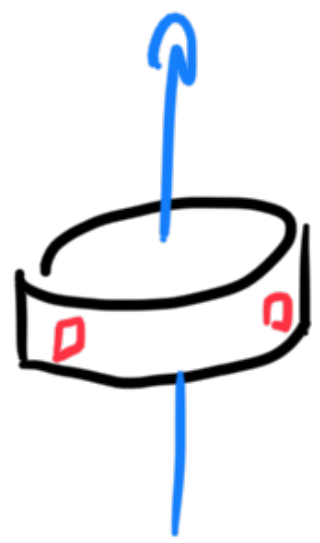
(a, α constant)

Moment of Inertia

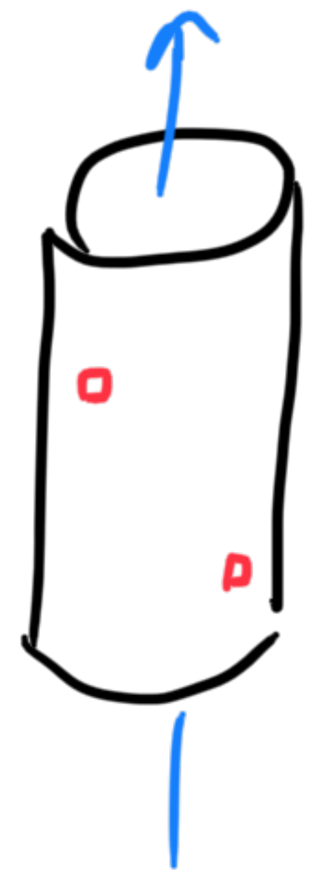
m, R

m, R

m, R, l



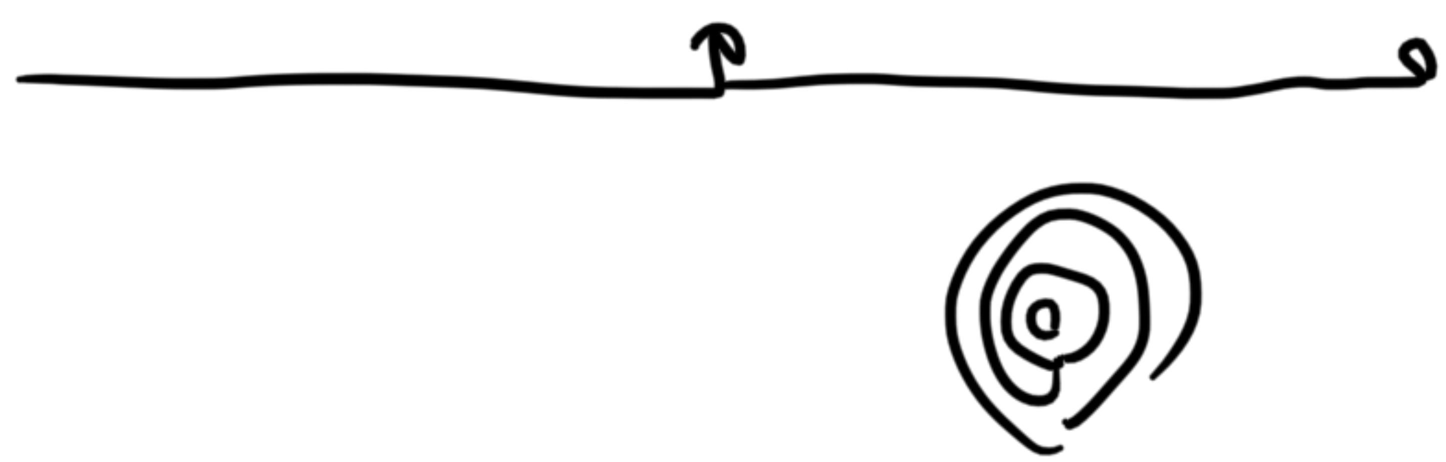
vs.



m, R, L

$$I_{ring} = mR^2$$

$$I_{disk} = \frac{1}{2} m R^2$$



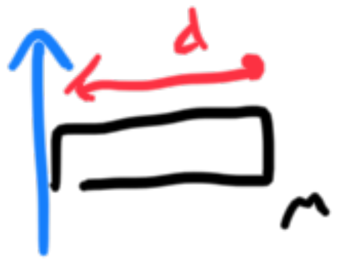
List of Moments

$$I = \int r^2 \, dm$$

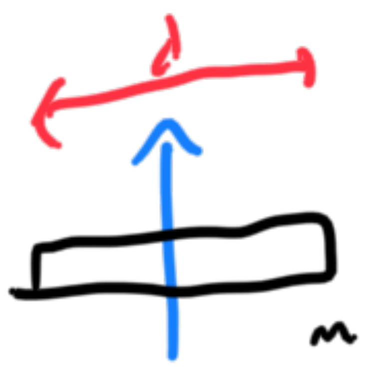
$$dm = \frac{m}{d}\,dx \qquad I = \int x^2 \, dm = \frac{m}{d}\int_0^d x^2 \, dx$$

$$= \frac{md^3}{3d} = \frac{1}{3} m$$

$$I = 1mr^2 = I_{ring}$$



$$I_{rod_{end}} = \frac{1}{3} m d^2$$



$$I_{rod_{center}} = \frac{1}{12} m d^2$$

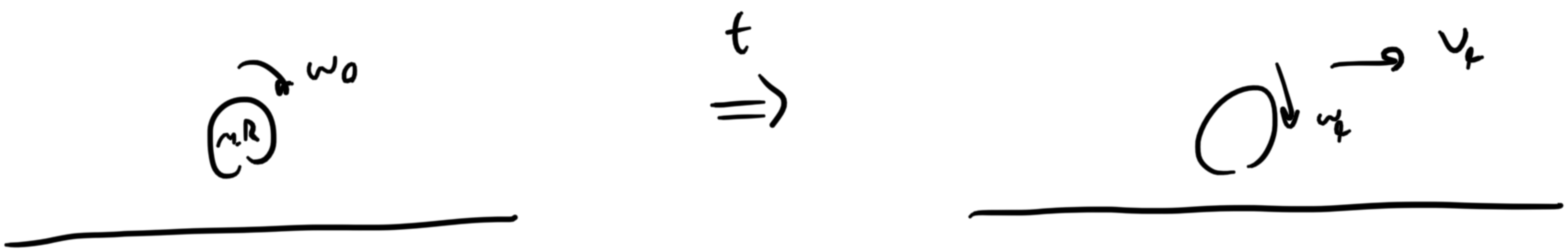
$$I_{disk} = \frac{1}{2} m r^2$$

$$I_{hollow} = \frac{2}{3} m r^2$$

$$I_{solid} = \frac{2}{5} m r^2$$

Example

A ball (solid sphere, $I = \frac{2}{5}mR^2$) with mass m, radius R, gently dropped on a surface. $\omega_0 = \omega_0$. $v_f = ?$ Energy is conserved.



$$v_f = \omega_f R$$

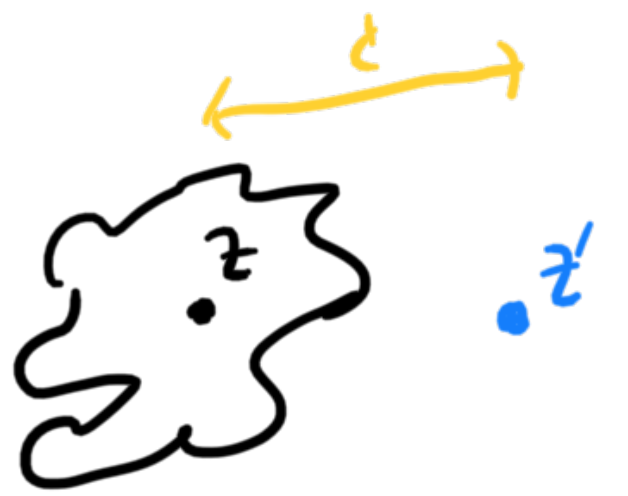
$$E_0 = \frac{1}{2} I \omega_0^2$$

$$E_f = \frac{1}{2} I \omega_f^2 + \frac{1}{2} m v_f^2 = \frac{1}{2} I \left(\frac{v_f}{R}\right)^2 + \frac{1}{2} m v_f^2 = \frac{1}{2} I \omega_0^2$$

$$\frac{1}{5} m \omega_0^2 R^2 = \frac{1}{5} m v_f^2 + \frac{1}{2} m v_f^2 = \frac{7}{10} m v_f^2$$

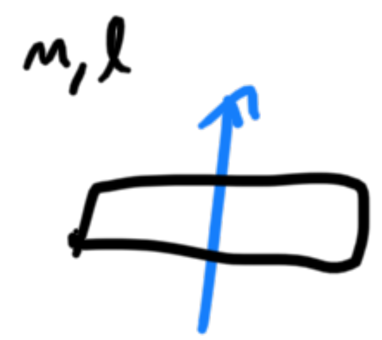
$$v_f = \sqrt{2/7}\, \omega_0 R$$

Parallel axis thm.



$$I'_z = I_z + md^2$$

Ex.



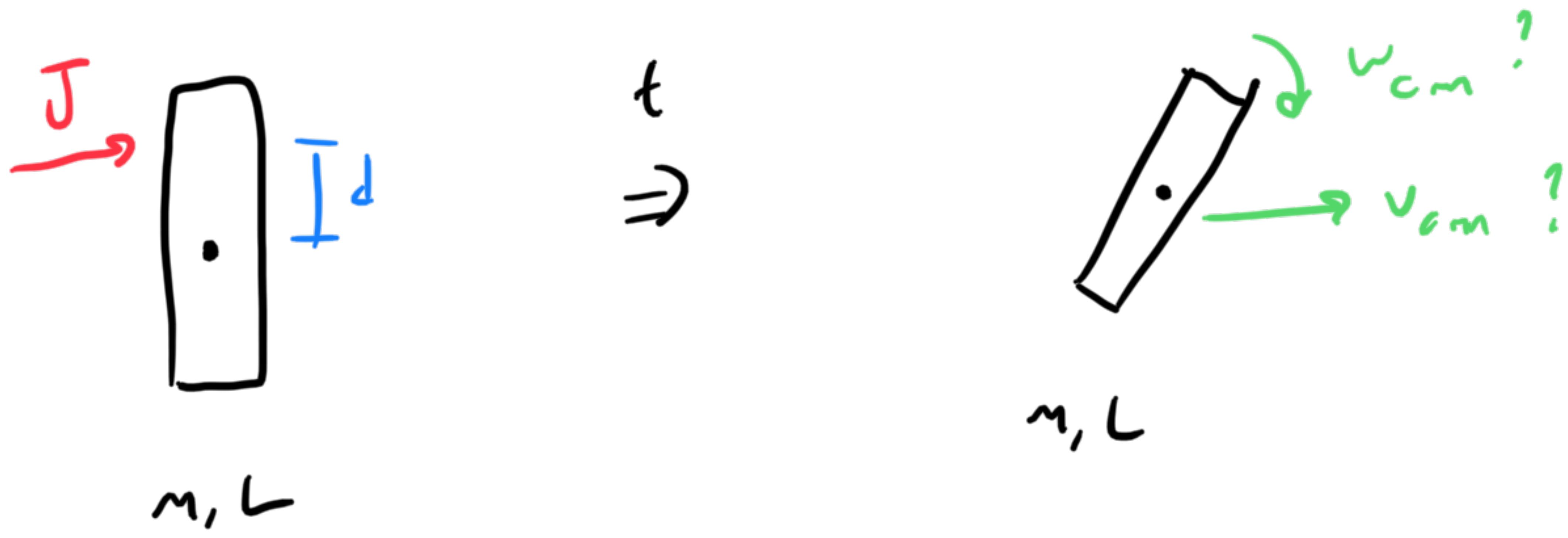
$$I_c = \frac{1}{12} m l^2$$

vs.

$$I_e = I_c + m\left(\frac{l}{2}\right)^2$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) m d^2$$

$$= \frac{1}{3} m d^2$$



$\boxed{J = F\Delta t}$, $J_N = N\Delta t$

$= F d \Delta t = J d$

$J = \Delta p = m \Delta v_{cm}$

$\Rightarrow v_{cm} = J/m$

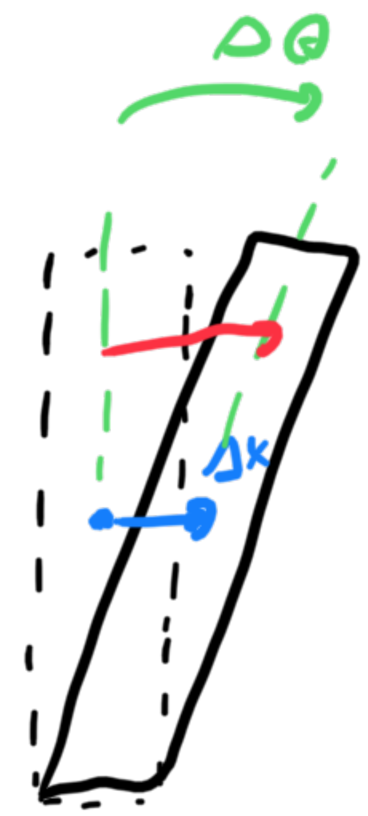
Ind. of d!

$J_N = \Delta L = I \Delta \omega_{cm}$

$= \frac{1}{12} m L^2 \omega_{cm}$

$J d = \frac{1}{12} m L^2 \omega_{cm}$

$\Rightarrow \omega_{cm} = 12 J d / m L^2$



$\Delta y \simeq \Delta x + d\Delta\theta$

$$\Delta E = W = F\Delta y = J\,\Delta y/\Delta t$$

Δt small, $\Delta y \simeq d\Delta\theta + \Delta x$

$$\alpha = \frac{N}{I} = \frac{Jd}{\Delta t}\,\frac{1}{I}$$

$$a = \frac{F}{M} = \frac{J}{\Delta t}\,\frac{1}{M}$$

$$\Delta\theta = \frac{1}{2}\alpha\Delta t^2 = \frac{Jd\Delta t}{2I}$$

$$\Delta x = \frac{1}{2}a\Delta t^2 = \frac{J\Delta t}{2M}$$

$$\Rightarrow$$

$$\Delta y = \frac{J\Delta t}{2M}\left(1 + d^2/L^2\right)$$

$$\Delta E = \frac{J^2}{2M}\left(1 + d^2/L^2\right)$$

2017 #14, 15

14 all M, R which would have the largest a down an incline?

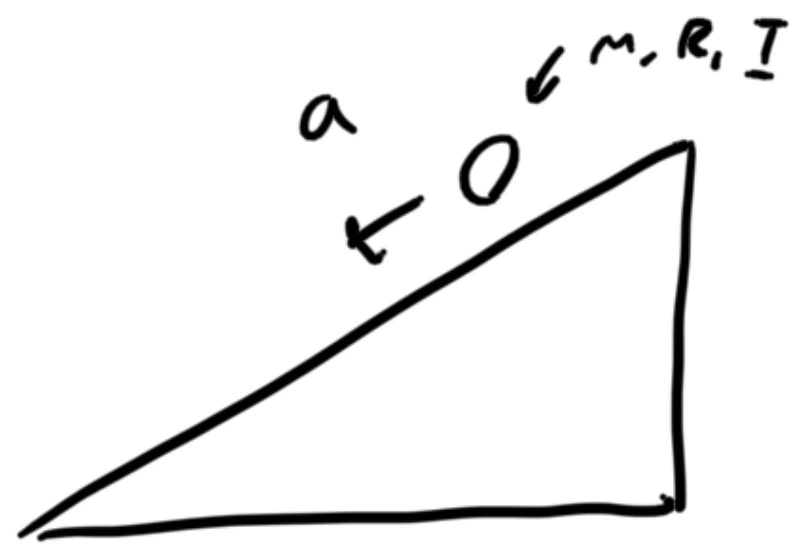
A. solid sphere

B. solid disk

C. hollow sphere

D. ring

E. all same



15 Four spheres same incline, largest speed?

A. M, R

B. $2M$, $R/2$

C. $M/2$, $2R$

D. $3M$, $3R$

E. all same