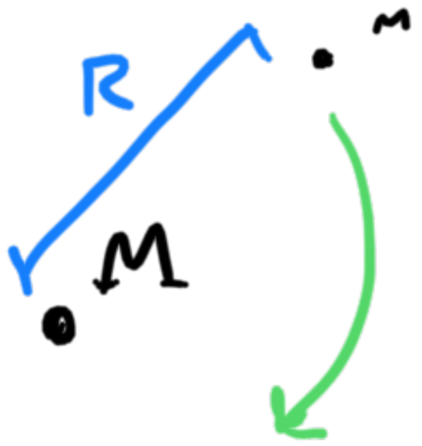
Orbits



M is stationary.

$$a_{cent} = \frac{v^2}{R} = a_{grav} = \frac{GM}{R^2}$$

So, for given R,

$$v = \sqrt{\frac{GM}{R}}$$

Period $T = ?$

$$\omega = \frac{v}{R} = \frac{\text{rad}}{\text{s}}$$

$$T = \frac{R}{v} = \frac{1}{\omega} \cdot 2\pi$$

$$T = \frac{d}{v} = \frac{2\pi R}{\sqrt{GM/R}}$$

$$= 2\pi \sqrt{\frac{R^3}{GM}}$$

Orbits are stable when

$F \propto R^{-2} \Rightarrow$ all have the same form

$F = q R^{-2}$ $\quad v = ?$

$$\frac{v^2}{R} = \frac{q R^{-2}}{m} \Rightarrow v = \sqrt{q/mR}$$

Gravity:

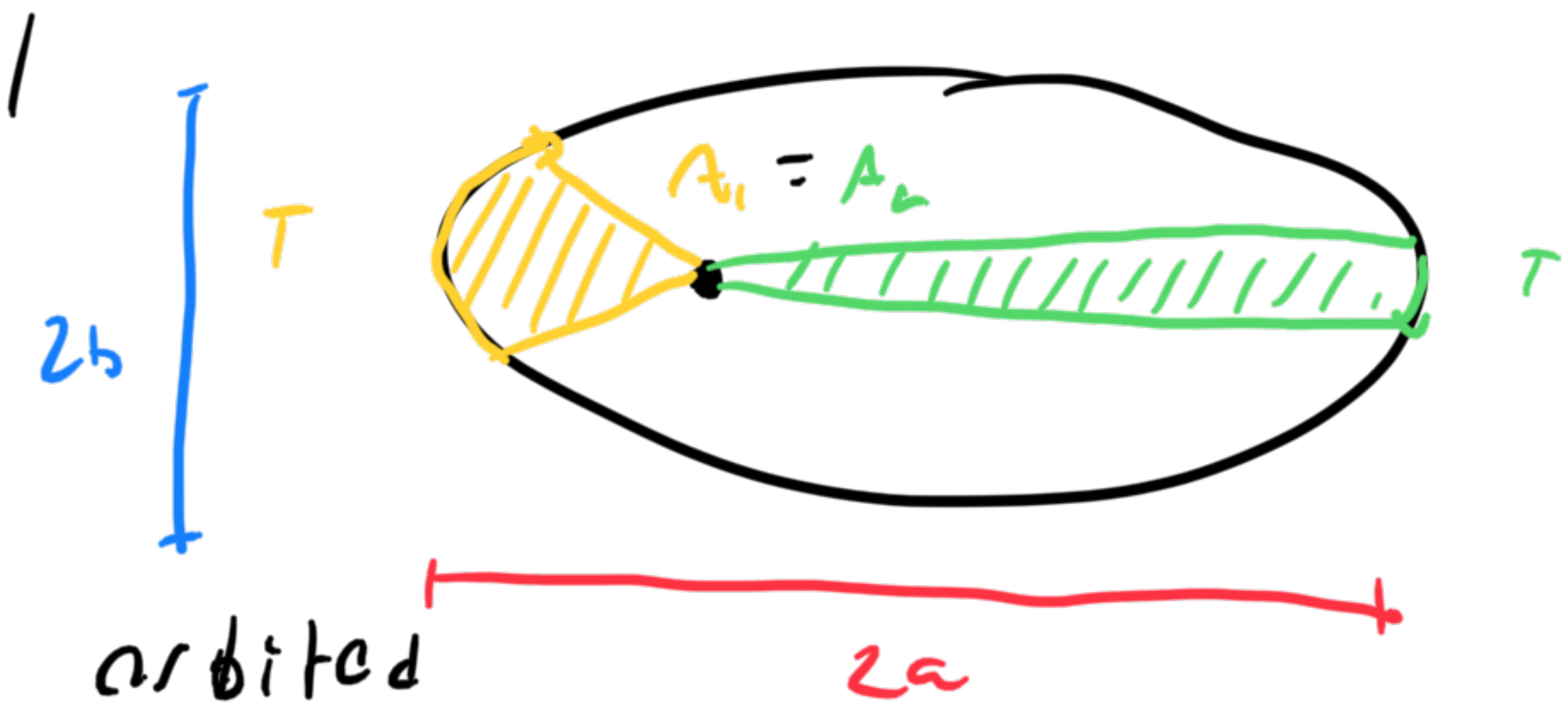
$$F = \frac{GMm}{R^2} \Rightarrow q = GMm$$

$$v = \sqrt{GM/R} \quad \checkmark$$

Elliptical Orbits

Kepler's Laws.

I. all bound orbits are elliptical with orbited at a focus



II. a line joining orbiter and orbited sweep equal areas in equal time

$$T \cdot \frac{r^2}{2} \cdot d\theta/dt = \pi a b$$

III $T^2 \propto R^3$ $\Rightarrow$ generalize to ellipse $\boxed{HW}$

Non-circular orbit speeds

$\left[0, \sqrt{GM/R}\right)$ elliptical (R = apogee/furthest distance)

$\sqrt{GM/R}$ circular

$\left(\sqrt{GM/R}, \sqrt{2GM/R}\right)$ elliptical (R = perigree/closest approach)

$\sqrt{2GM/R}$ parabolic (escape speed)

$\left(\sqrt{2GM/R}, \infty\right)$ hyperbolic

2017 #1

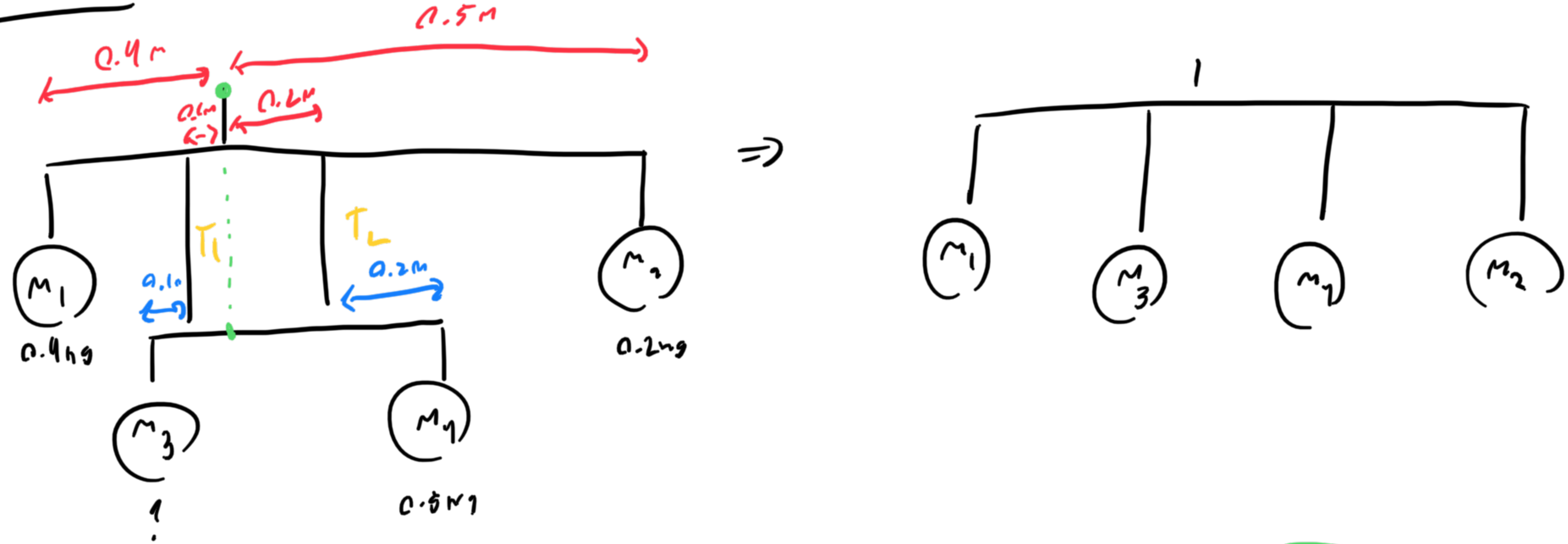
$N = ma_{cent} = \frac{mv^2}{R}$

$F_f = \mu N$ cancels out mg

so $\mu N = mg = \mu mv^2/R$

$\mu \propto v^{-2}$ (D)

2017 #6



$$\tau_t = (0.4\,m)\, m_1 g + (0.1\,m)\, T_1 - (0.2\,m)\, T_2 - (0.5\,m)\, M_2 g = 0$$

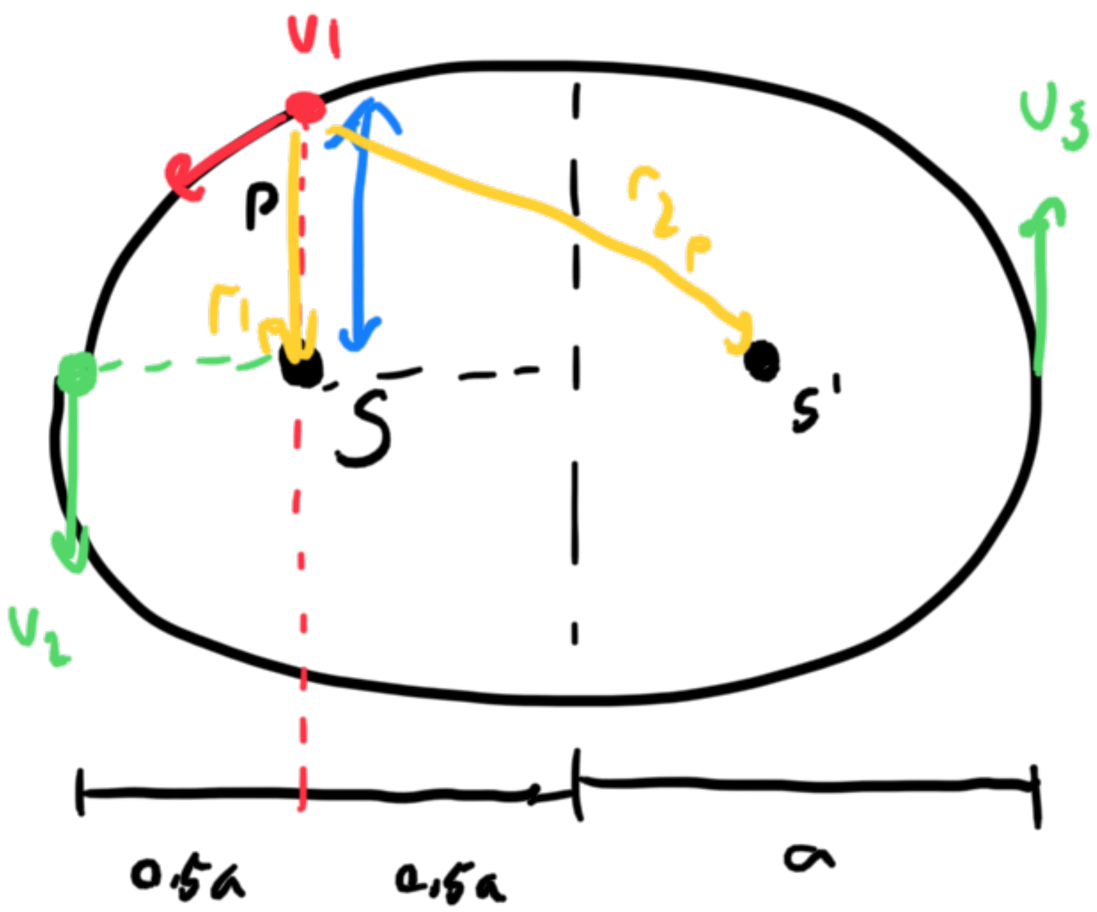
$$\tau_b = (0.2\,m)\, M_3 g - (0.1\,m)\, T_1 + (0.2\,m)\, T_2 - (0.4\,m)\, M_4 g = 0$$

$$\tau_\Sigma = g\left[(0.4\,m)\, m_1 + (0.2\,m)\, m_3 - (0.5\,m)\, m_2 - (0.4\,m)\, m_4\right] = 0$$

Optional HW

Can you treat this as one beam as long as strings are vertical?

2017 #25



$r_1 + r_2$

$r_{1_p} + r_{2_p} = \frac{a}{2} + \frac{3a}{2}$

$SP + \sqrt{SP^2 + a^2} = 2a \Rightarrow SP = \frac{3a}{4}$

$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_3^2 - \frac{GMm}{3a/2}$

$\frac{m v_2 a}{2} = \frac{3 m v_3 a}{2} \Rightarrow v_3 = \frac{1}{3} v_2$

$\Rightarrow \frac{4}{9} m v_2^2 = \frac{GMm}{a/2} - \frac{GMm}{3a/2} \Rightarrow v_2^2 = \frac{3GM}{a}$

$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_1^2 - \frac{GMm}{3a/4}$

$v_1^2 = v_2^2 - \frac{4GM}{a} + \frac{8GM}{3a}$

$= GM\left(\frac{3}{a} - \frac{4}{a} + \frac{8}{3a}\right) = \frac{5GM}{3a}$

$\frac{v_2^2}{v_1^2} = \frac{3GM/a}{5/3\, GM/a} = \frac{9}{5}$

$v_2/v_1 = 3/\sqrt{5}$

2016 #4

$a_c \sim v^2 \sim t^2$

$v \sim t$

$a \sim \sqrt{P + Qt^4}$

2016 #8

II. cons of L

III. $T^2 \propto R^3$ uses $F \propto R^{-2}$

$\Rightarrow$ B

2016 #9

Cons of energy: $mgh = \frac{mv^2}{2}$

geometry: $h = R(1-\cos\theta)$

contact forces: $mg\cos\theta \geq \frac{mv^2}{R}$

$\cos\theta = v^2/gR \Rightarrow v = \sqrt{\frac{2gR}{3}} = \sqrt{\frac{gD}{3}} = 2$ E