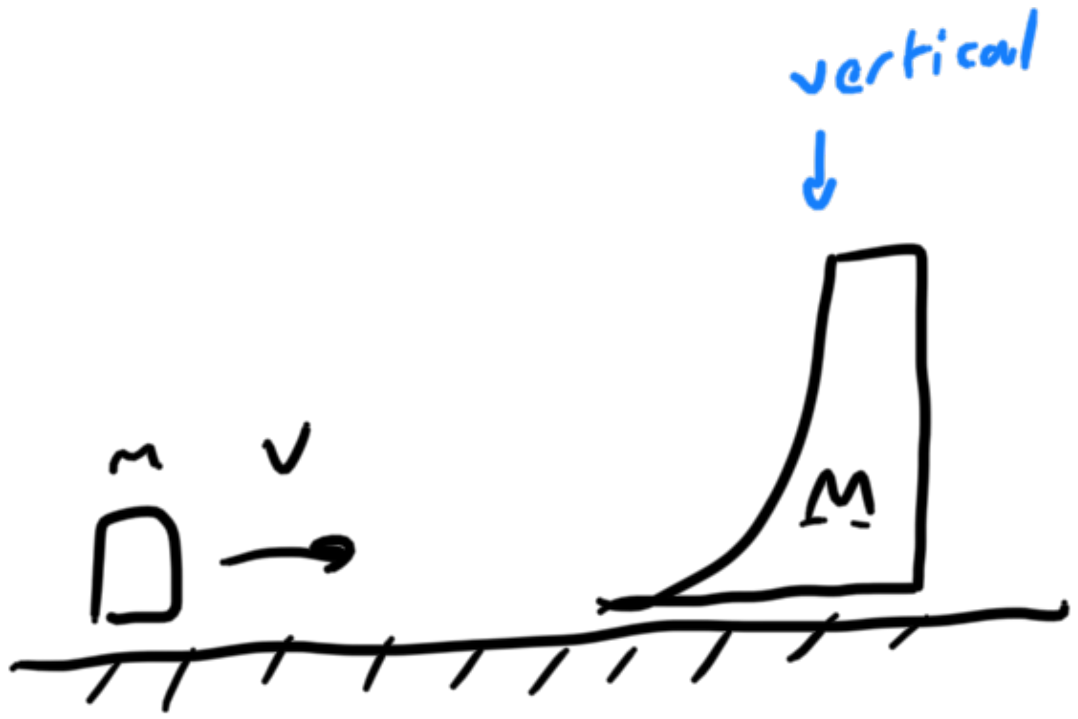
# 2019A #3




$$\frac{mv^2}{2} = mgh + \frac{mv_r^2}{2} + \frac{Mv_r^2}{2}$$ Cons. energy

$$mv = mv_r + Mv_r$$ Cons. horizontal momentum

$$v_r = mv/(m+M)$$

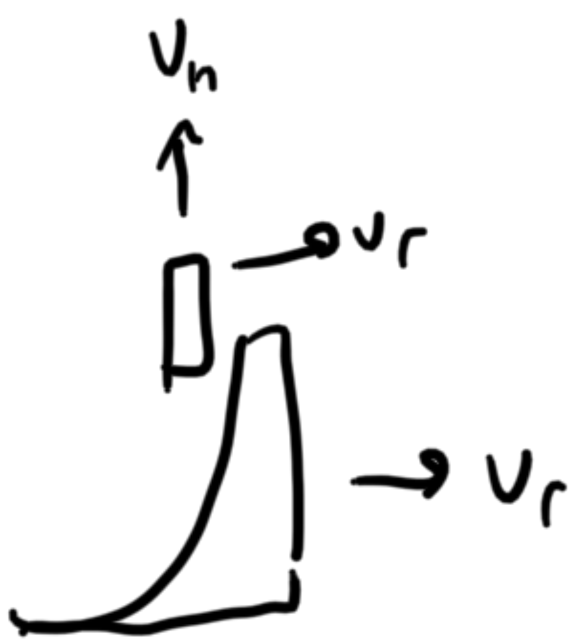
$N = 0$

$$h = \frac{M}{m+M}\frac{v^2}{2g}$$

$h = ?$

$\Downarrow t$

↑ familiar?




HW:

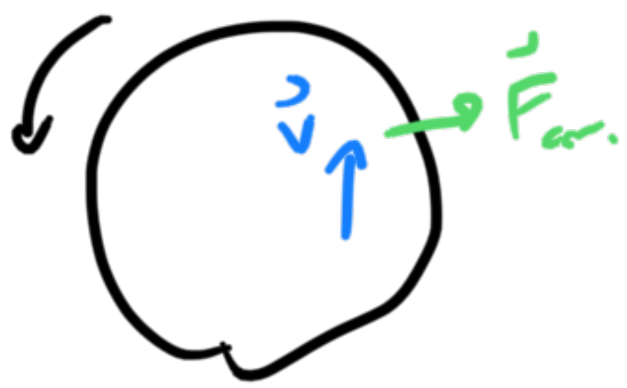
also check limiting cases, $M = 0$, $M \gg m$

# #14

$\vec{F}_{cor.} = -2m\vec{w}\times\vec{v}$

e.g.

$\vec{w}\ \odot$




$-\vec{w}\times\vec{v}$

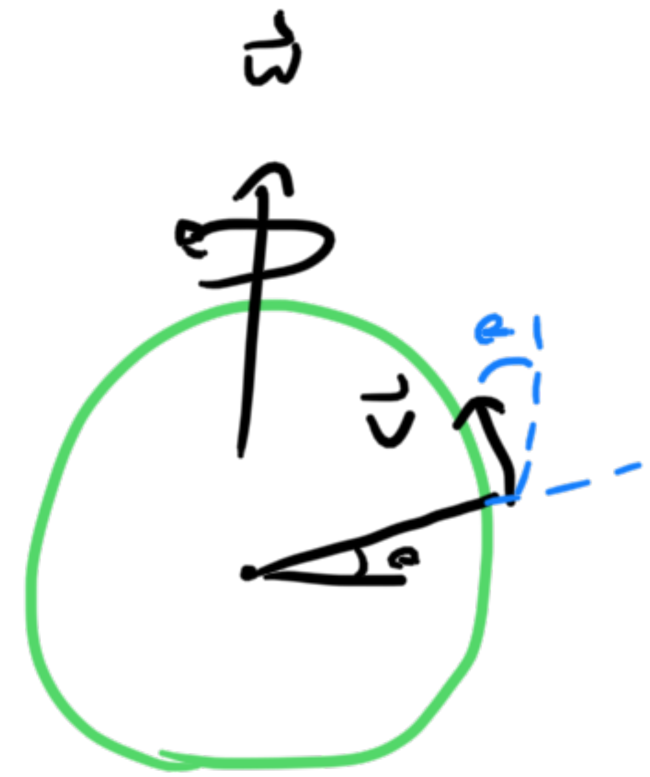
$\vec{a}_{cor} = -2\vec{w}\times\vec{v}$

$d(t) = \frac{a_{cor}t^2}{2}$

$t = L/v$

$|a_{cor}| = 2wv\sin\theta$

$d = \frac{2wv\sin\theta\,(L/v)^2}{2} = L^2 w\sin\theta/v = 1.8\,mm$




$\vec{w}\times\vec{v}\ \odot$

$\vec{F}_{cor}\ \otimes$ East

# #16

$$d = gt^2/2$$

$$\Delta t = d/c$$

$$t + \Delta t$$

$$d + \Delta d = \frac{g}{2}(t + \Delta t)^2$$

$$= \frac{g}{2}\left(t^2 + 2t\Delta t + (\Delta t)^2\right)$$

$$d + \Delta d = d + \frac{g}{2}\left(2t\Delta t + (\Delta t)^2\right)$$

$$\Delta d = g t \Delta t + \cancelto{0}{\frac{g}{2}(\Delta t)^2}$$

$$\varepsilon = \frac{\Delta d}{d} \leq 0.05$$

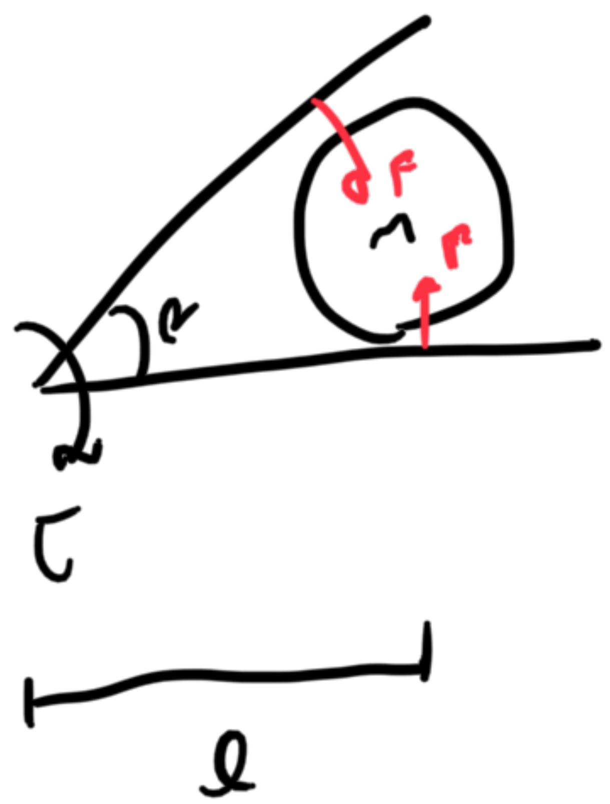
$$= \frac{\cancel{g}\cancel{t}\Delta t}{\cancel{g}t^{\cancel{2}}/2} = 2\Delta t/t$$

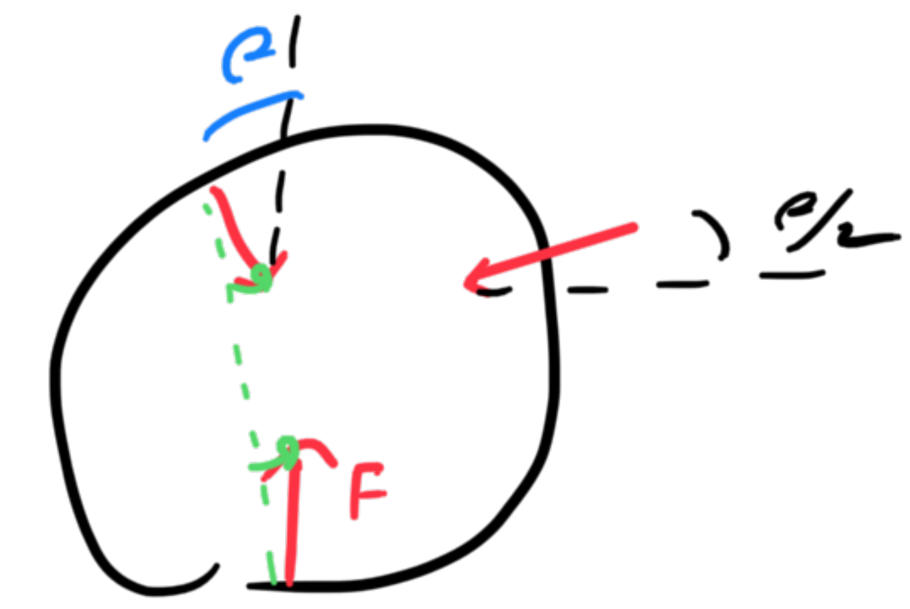
$$\frac{\sqrt{2gd}}{c} \leq 0.05$$

$$d \leq \frac{(0.05)^2 c^2}{2g} = 13.6\ \text{m} \simeq 14\ \text{m}$$

(C)

# #17



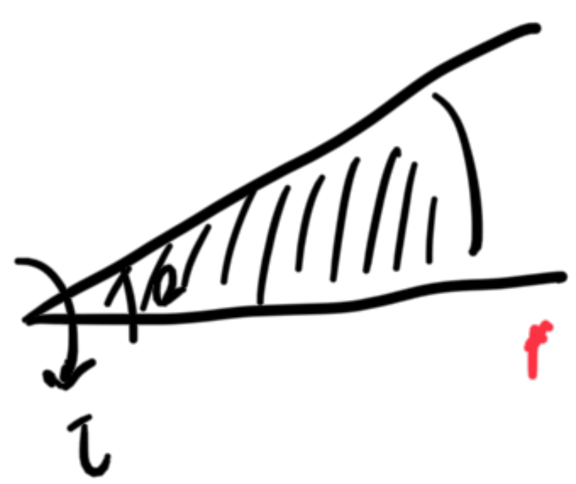

$$F = \tau / l$$

$$F_\perp = 2F\sin(\theta/2)$$

$$= 2\,\tfrac{\tau}{l}\sin(\theta/2)$$

(A)

# #18




$$W = \int F \cdot dx$$

OR

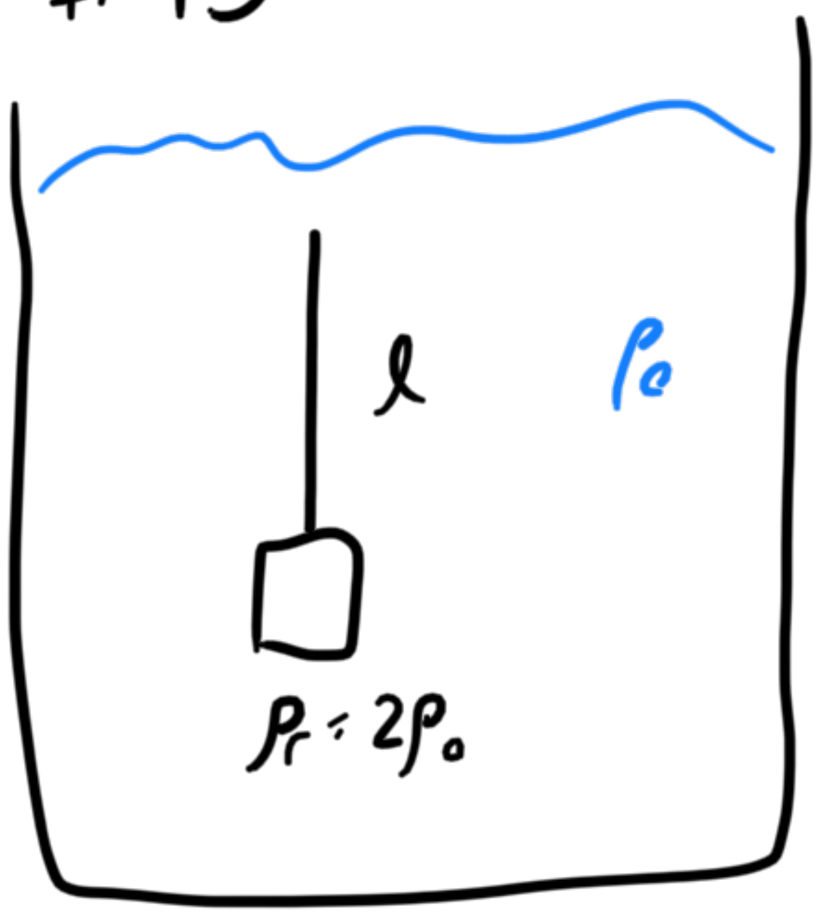
$$W = \int \tau \cdot d\theta$$

$$W = \tau\theta = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\tau\theta}{m}}$$

(D)

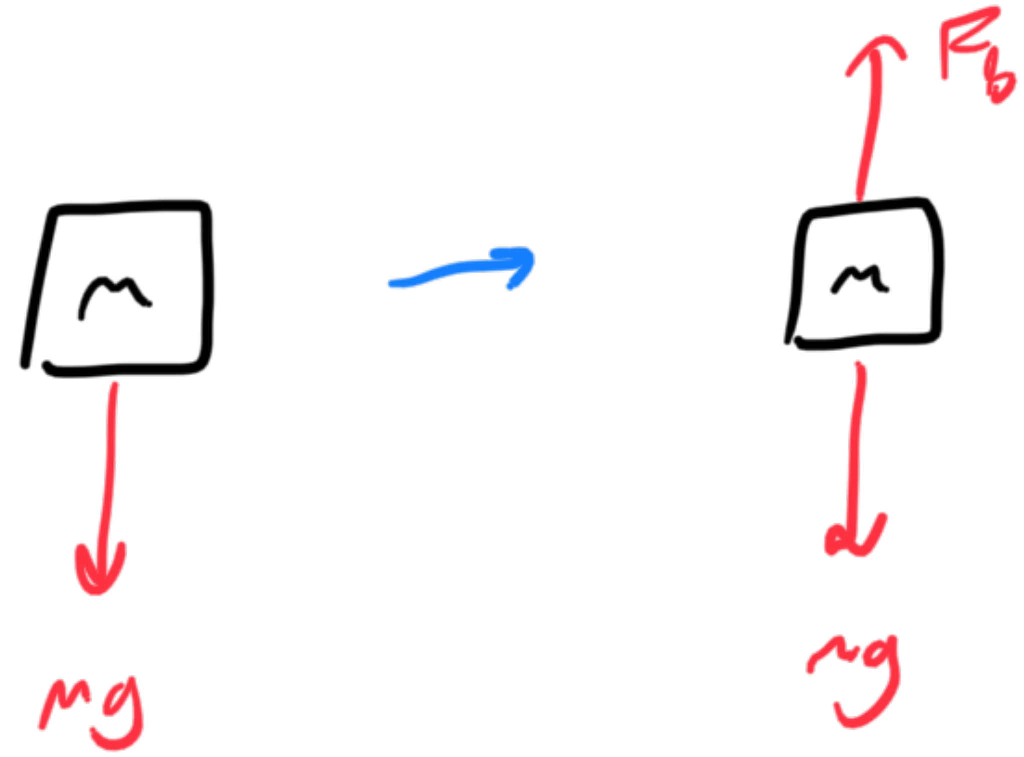
# #19




$$\omega = \sqrt{g/\ell}$$

$$\omega' = \sqrt{g'/\ell}$$

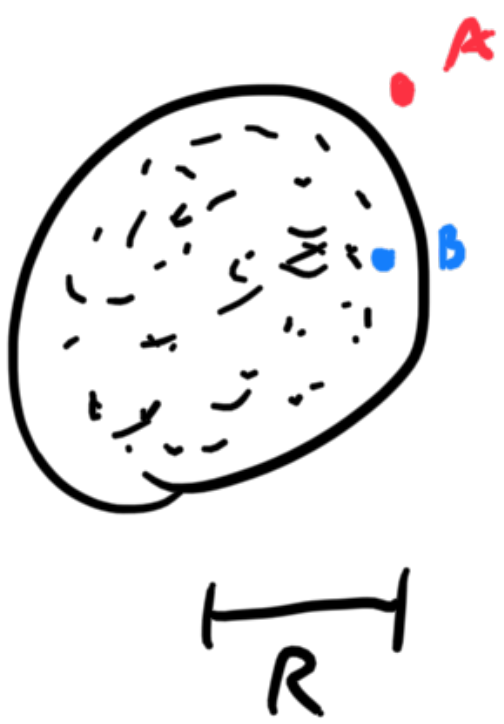
$$= \sqrt{g/2\ell}$$




$$F_b = (\rho_r - \rho_0)Vg$$

$$F_{net} = \frac{mg}{2}$$

# #21




$v_A, T_A$

$v_B, T_B$

$$m\omega^2 r = G\frac{mM}{r^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi}{GM}$$

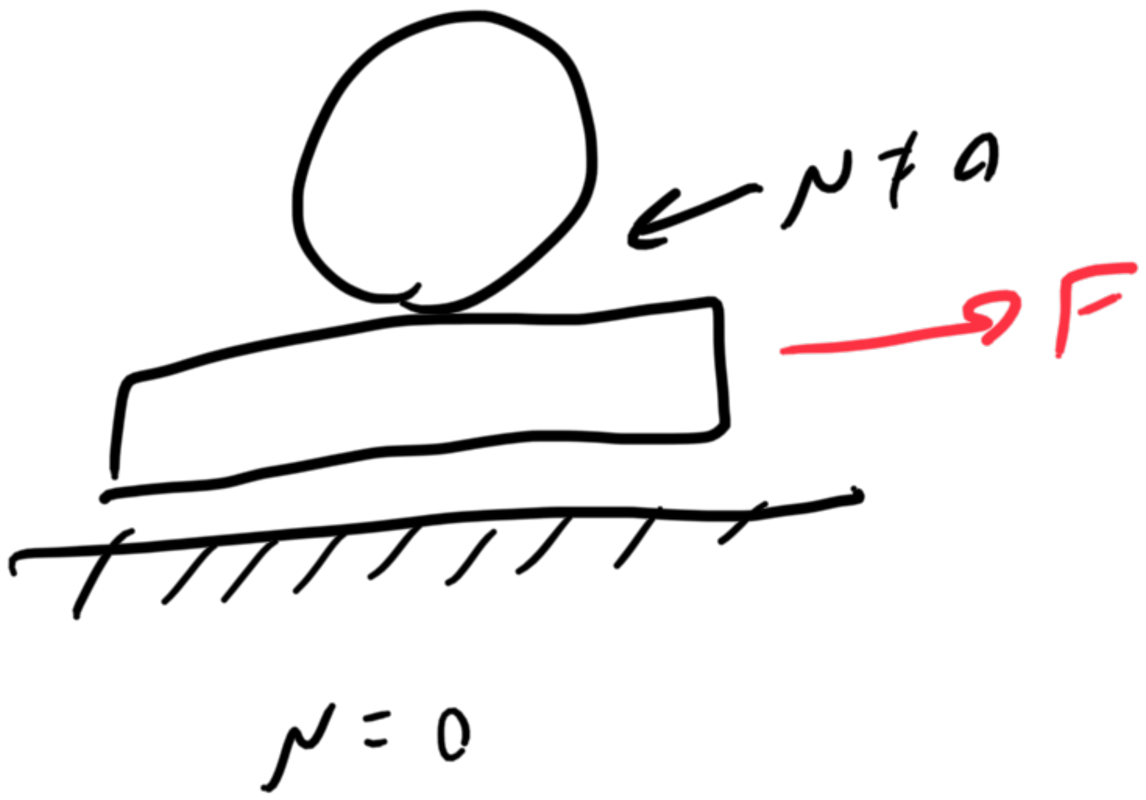
$$T^2 = \frac{4\pi r^3}{GM} = \frac{4\pi}{\rho G}$$

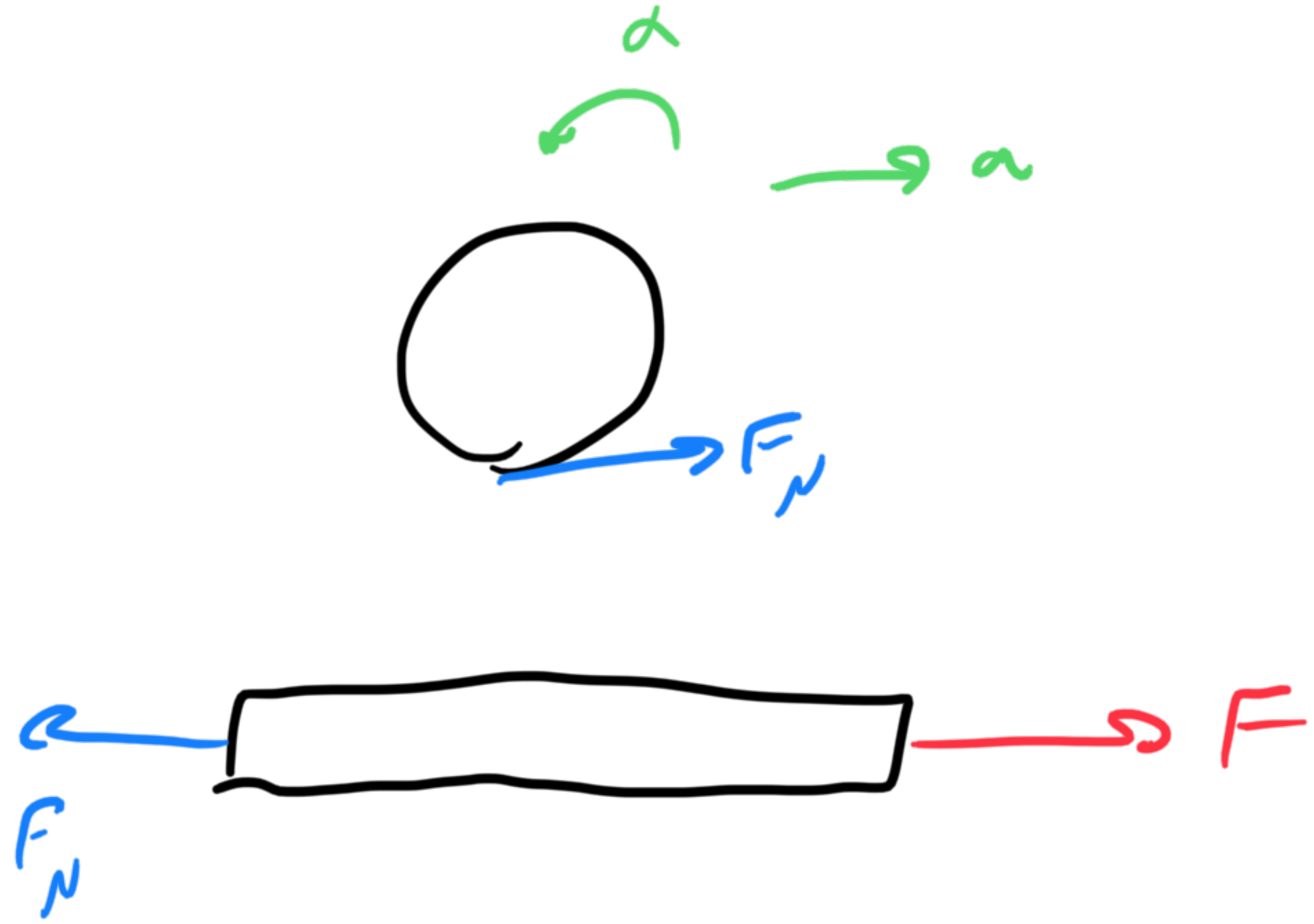
$$\boxed{T_A = T_B} \Rightarrow \omega_A = \omega_B$$

$$\omega = v/R \qquad v_A = \omega_A R_A \qquad \boxed{v_A > v_B}$$

$$v_B = \omega_B R_B$$

# #23



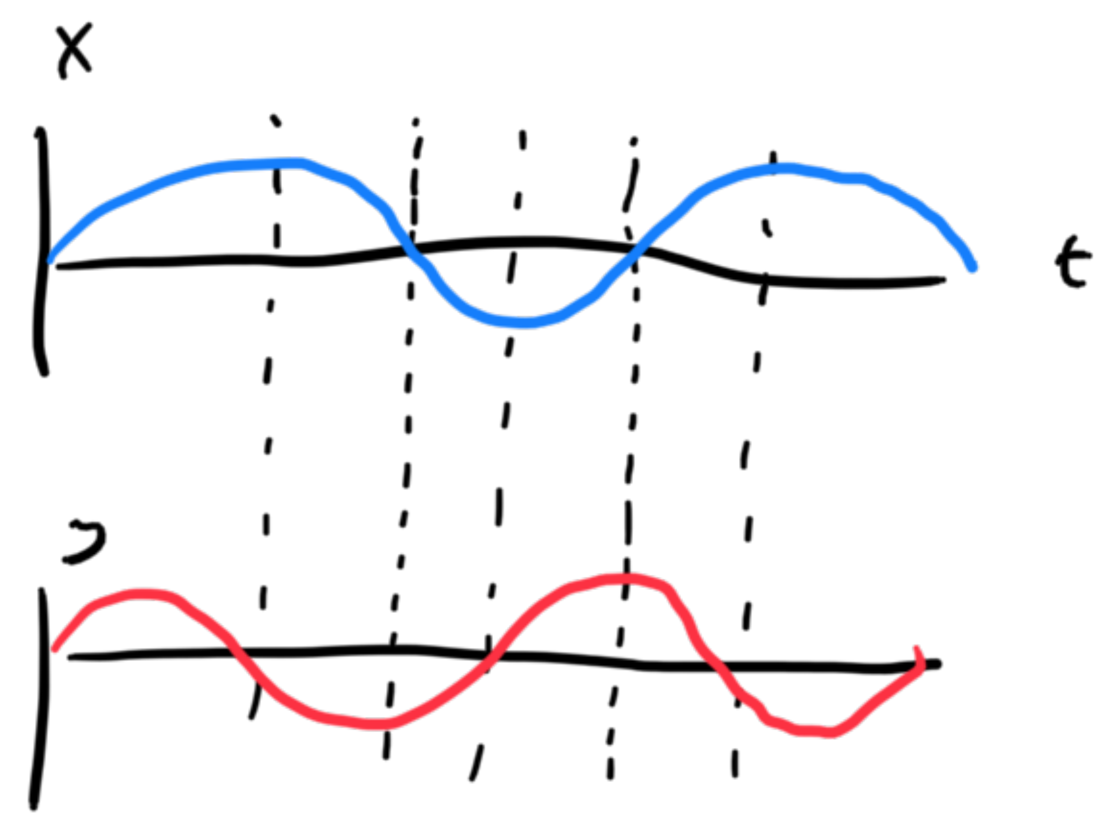
# #24

$$U(x,y) = 9kx^2 + 16ky^2 = U(x) + U(y) = 9kx^2 + 16ky^2$$

oscillator: $U(z) = \frac{1}{2}kz^2$

$$U(x,y) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 \qquad k_x = 18k \quad k_y = 32k$$

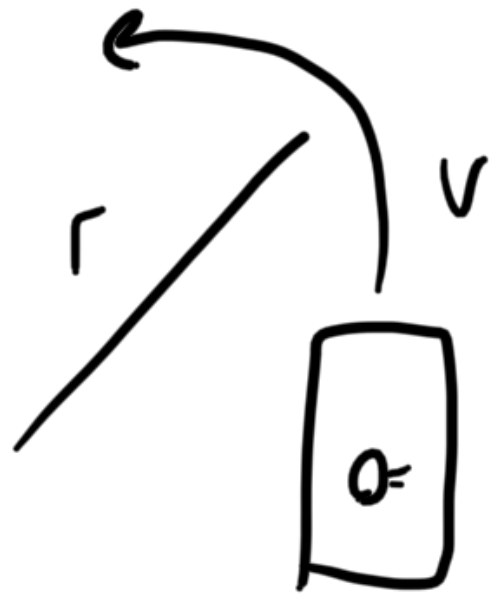
$$T_x = 2\pi\sqrt{\frac{m}{18k}} \qquad T_y = 2\pi\sqrt{\frac{m}{32k}} = \boxed{\frac{3}{4}T_x}$$ lowest period

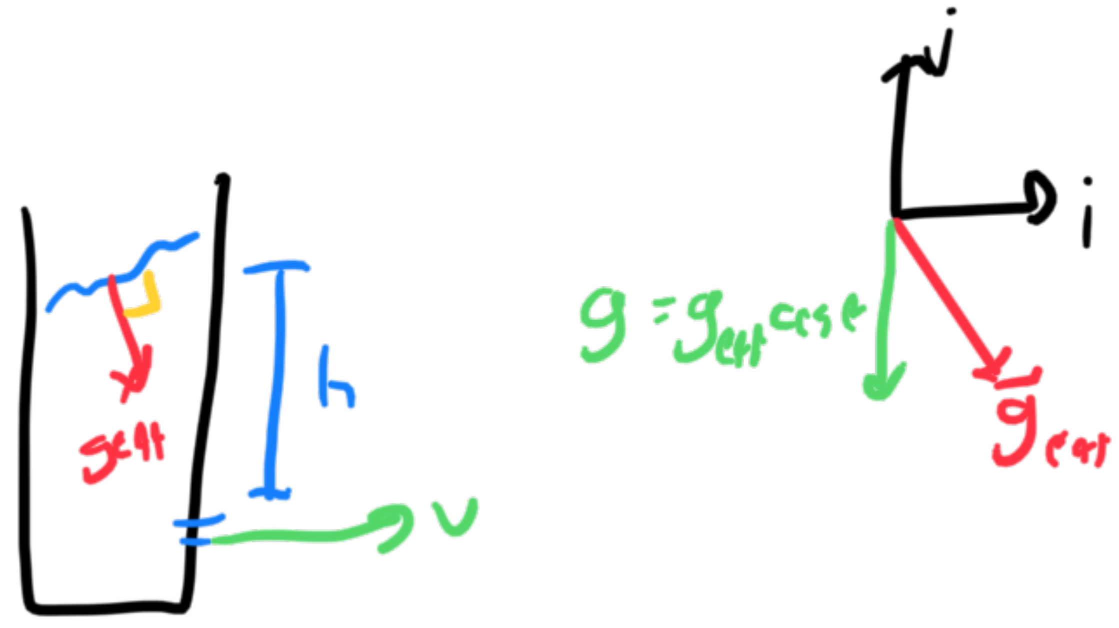



Max period: LCM, $3T_x$ and $4T_y$

Lissajous curves

# #25




$$\vec{g}_{eff} = -g\hat{j} + \vec{a}_c = -g\hat{j} + \frac{v^2}{r}\hat{i}$$

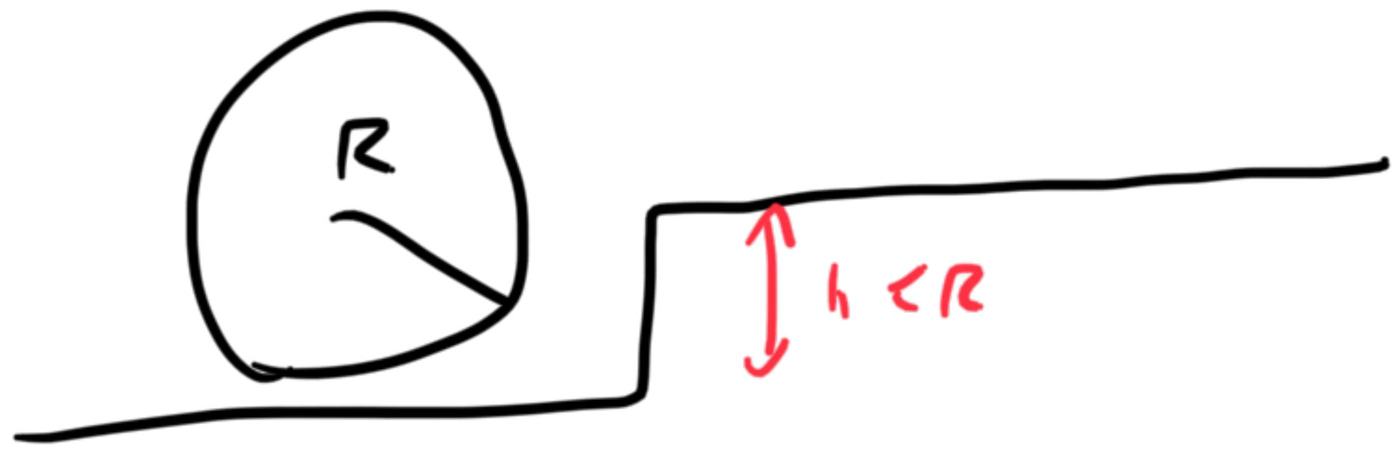
$\vec{g}$ at $\theta$ from vertical

$$h_{eff} = h\cos\theta$$

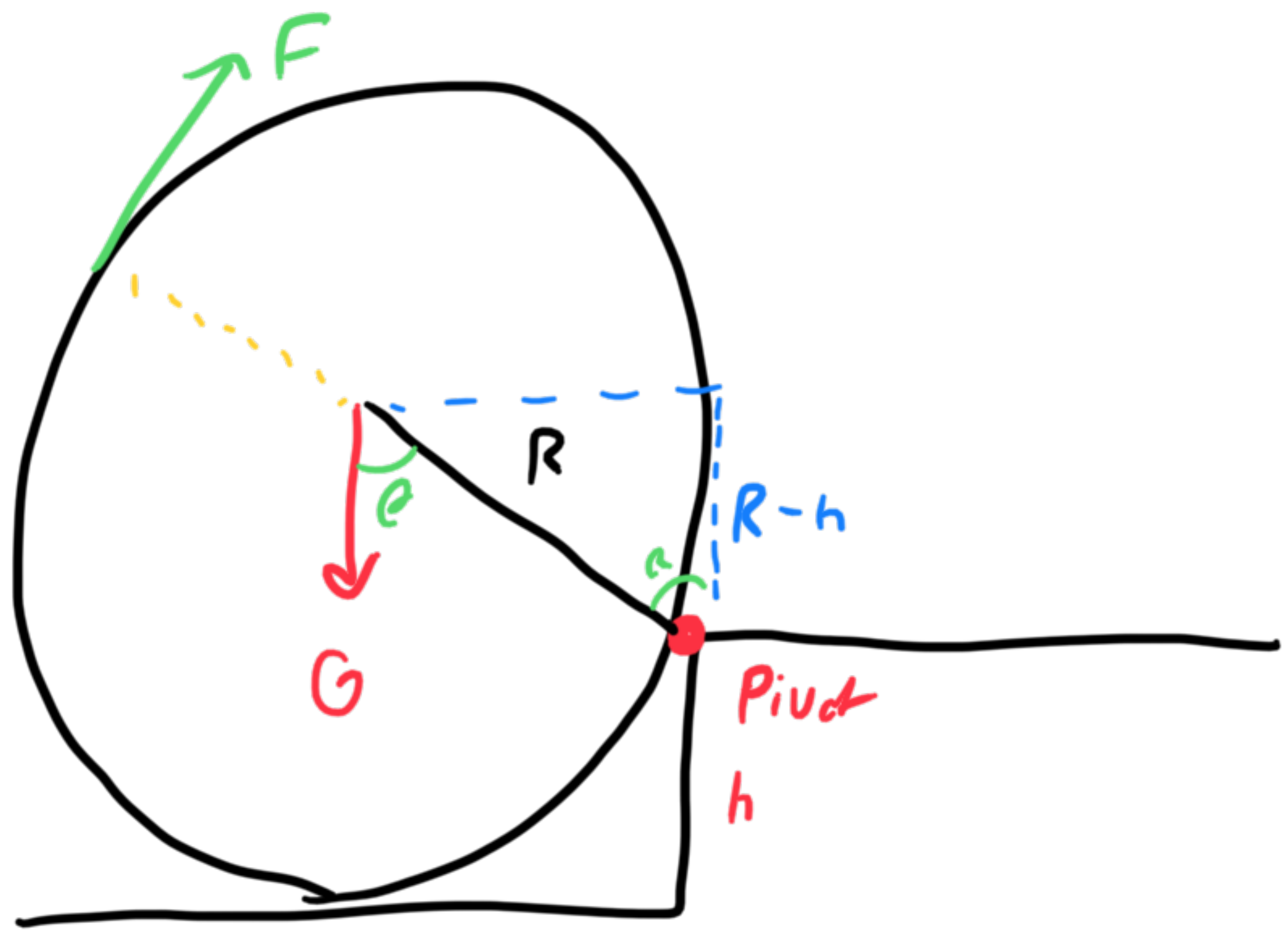
$$P = \rho g_{eff} h_{eff} = \rho g_{eff} h \cos\theta = \rho g h$$

$$\rho g h = \frac{\rho v_{esc}^2}{2} \Rightarrow v_{esc} = \sqrt{2gh}$$

# #5

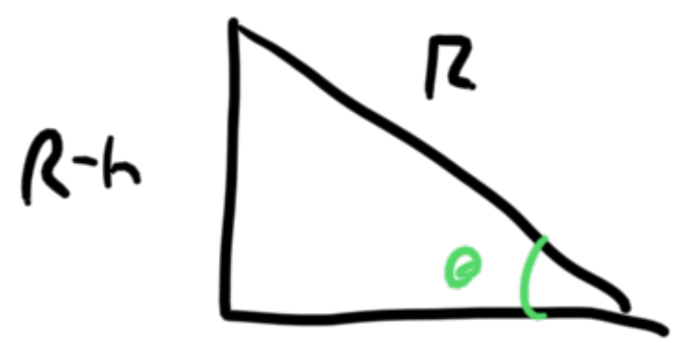



weight $G$




$$\tau_G = G\sqrt{R^2-(R-h)^2}$$

$$= G\sqrt{2Rh-h^2}$$




$$\tau_F = 2RF = G\sqrt{2Rh-h^2}$$

$$F = \frac{G\sqrt{2Rh-h^2}}{2R}$$

# #9


$$v_x = v_c - \omega R\cos\theta$$

$$v_y = -\omega R\sin\theta$$

$$|v| = \sqrt{v_x^2 + v_y^2} = \omega R\sqrt{2(1-\cos\theta)}$$

$$= 2\omega R\sin|\theta/2|$$