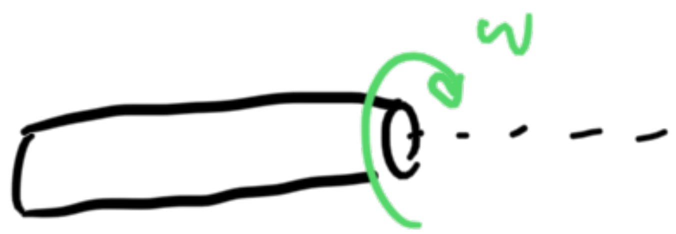
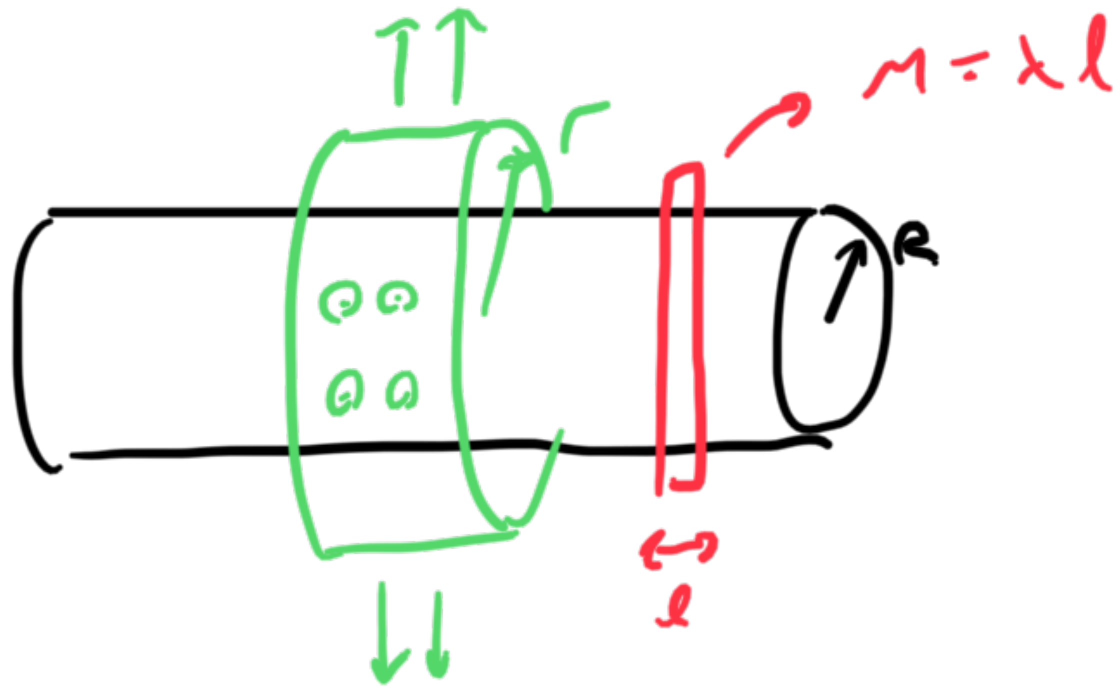
2019 B #6



Only gravity,

$\rho \propto r^{\alpha}$

$\alpha = ?$



Gauss's Law

$$\boxed{(2\pi r l)\cancel{m} g = 4\pi G M_{enc} \cancel{m}}$$

$$2\pi r l g = 4\pi G M_{enc}$$

$$g = \frac{2GM}{rl} = \frac{2G\lambda}{r}$$

$$g = \omega^2 r$$

$$\omega^2 r = \frac{2G\lambda}{r}$$

$A = \pi r^2$

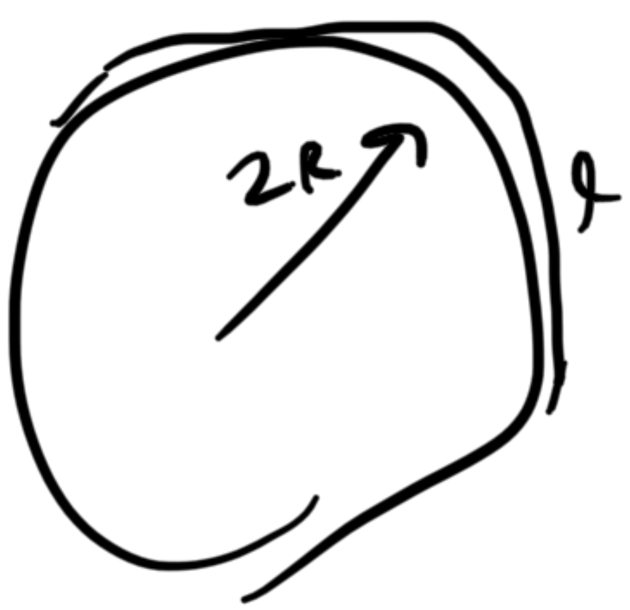
$$\lambda = \frac{\omega^2 r^2}{2G} = \frac{\omega^2}{2\pi G} A$$

Independent of r!

$M = \omega^2/2\pi G (Al)$

$$\lambda = \frac{\omega^2}{2\pi G}(A)$$

1x stuff

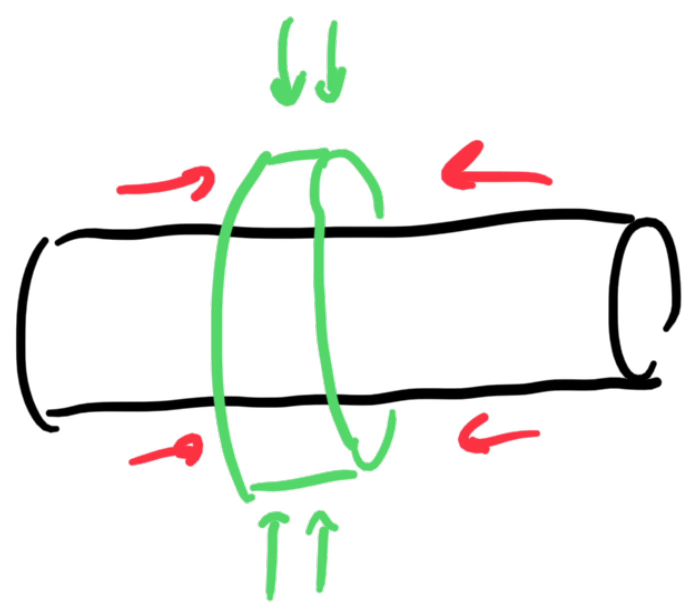


$M = \omega^2/2\pi G (4Al)$

$$\lambda = \frac{\omega^2}{2\pi G}(4A)$$

4x stuff

Gauss's Law for gravity



$$A_{gauss} \cdot \cancel{m}g_{thru\ gauss} = 4\pi G M_{enc} \cancel{m}$$

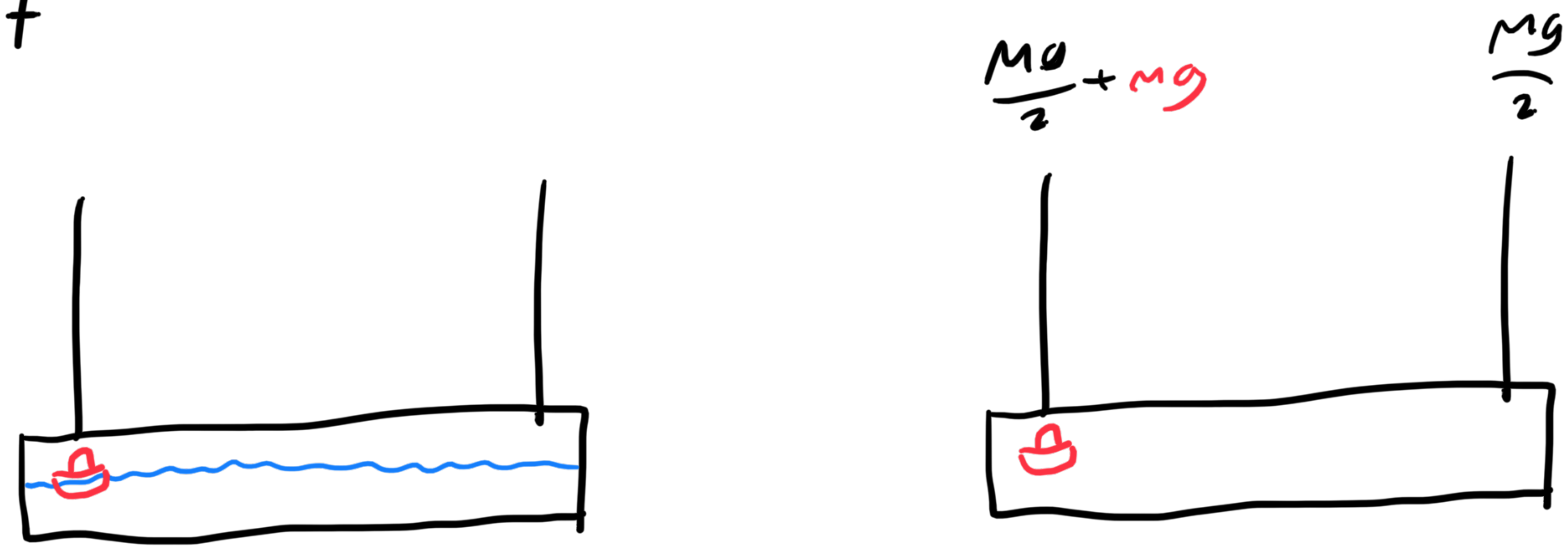
surface area of gaussian surface

dot product

force thru surface

enclosed mass.

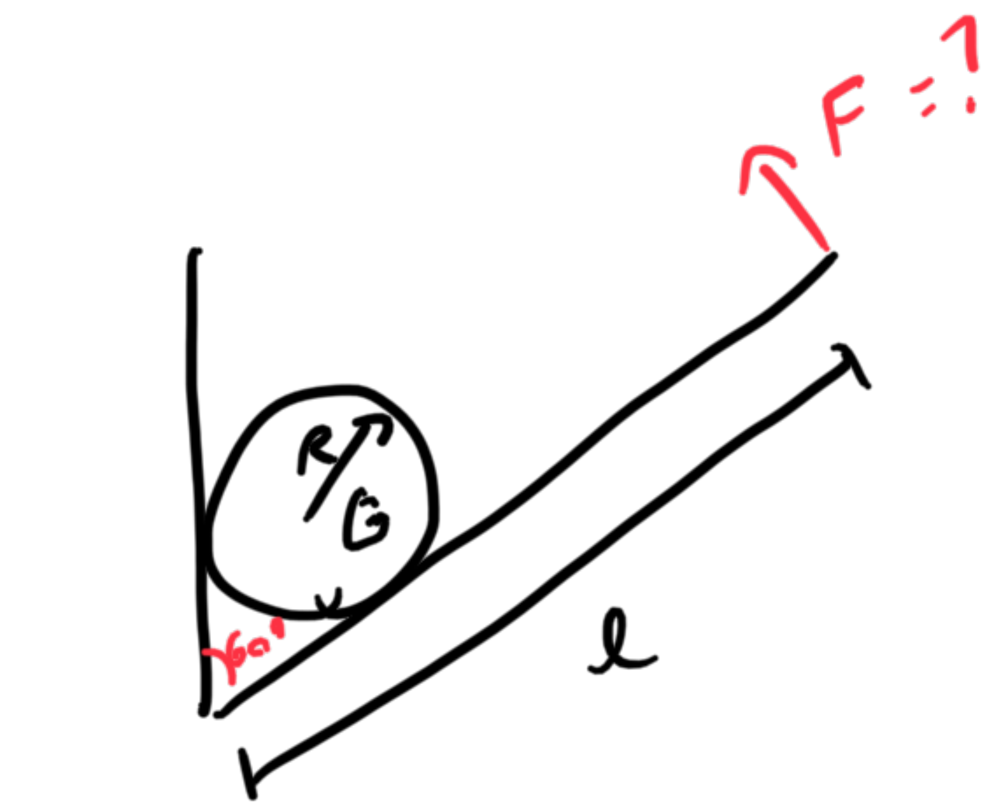
#7



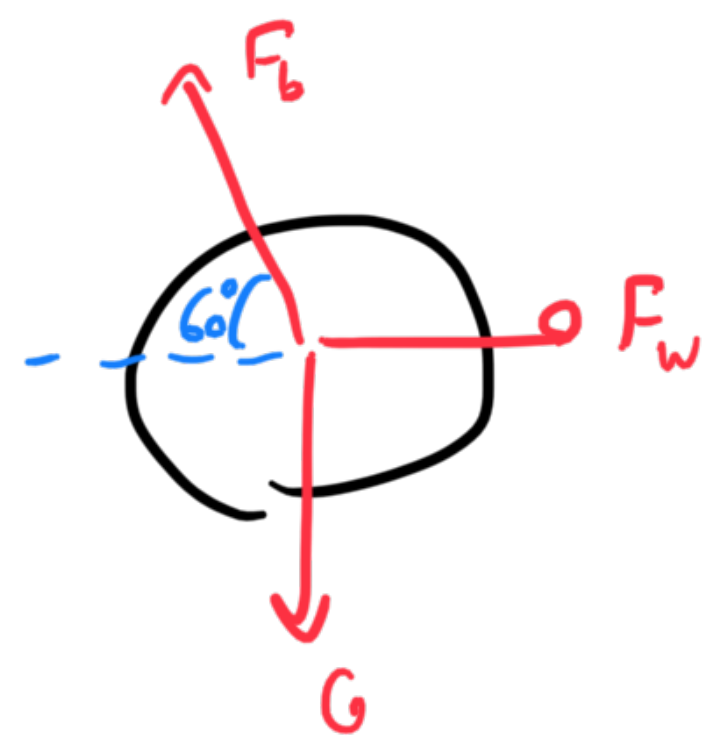
Pascal's principle → equal pressure on trough

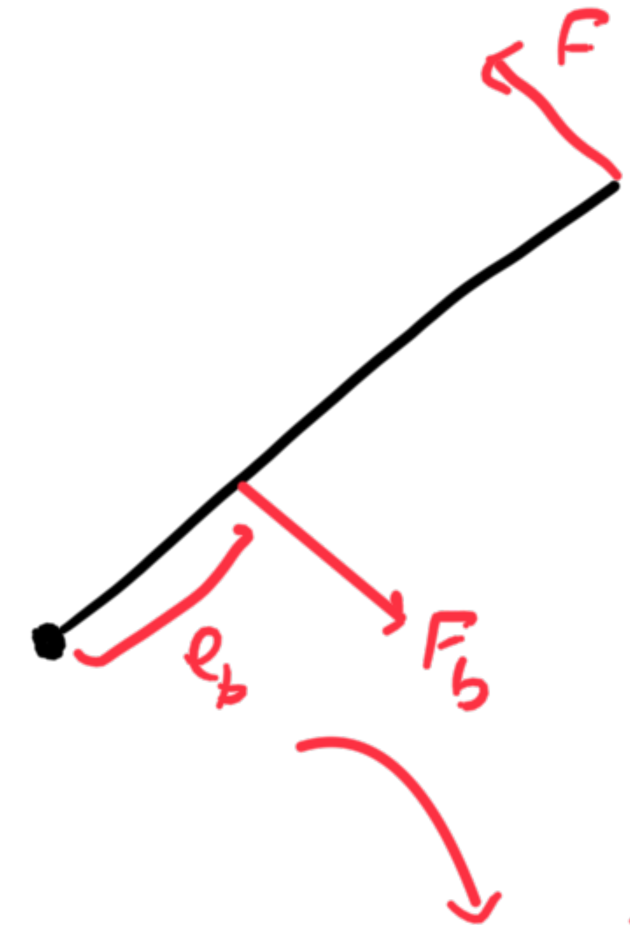
definitely increases B

#9



$$F_b \sin\theta = G$$





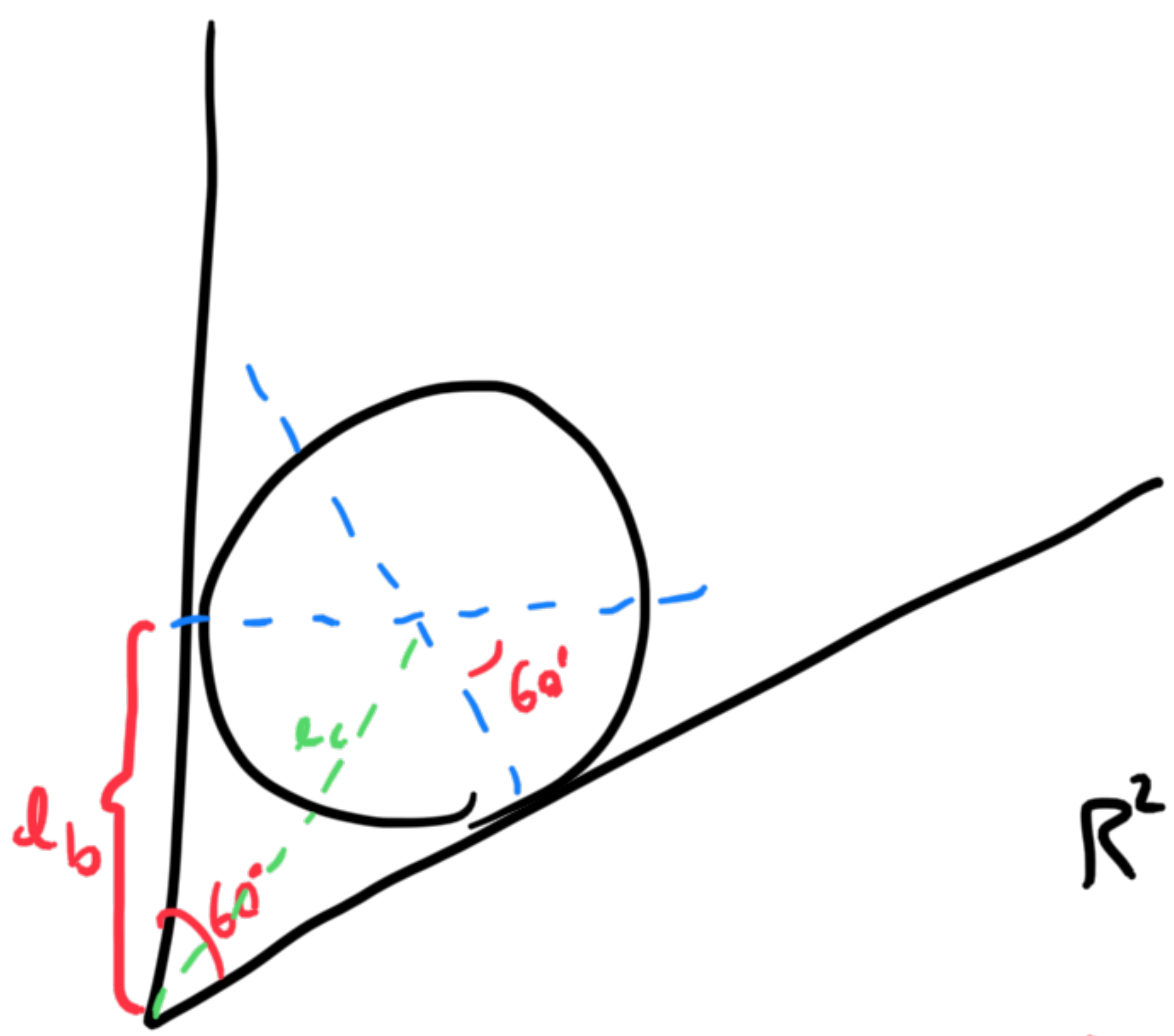
$$0 = Fl - F_b \overbrace{\sqrt{3}R}^{l_b}$$

$$F = F_b \sqrt{3}\, R/l$$

$$= G \frac{\sqrt{3}R}{l\sin\theta}$$

$$= G \frac{\sqrt{3}R}{\sqrt{3}l/2}$$

$$= 2RG/l \qquad \text{B}$$

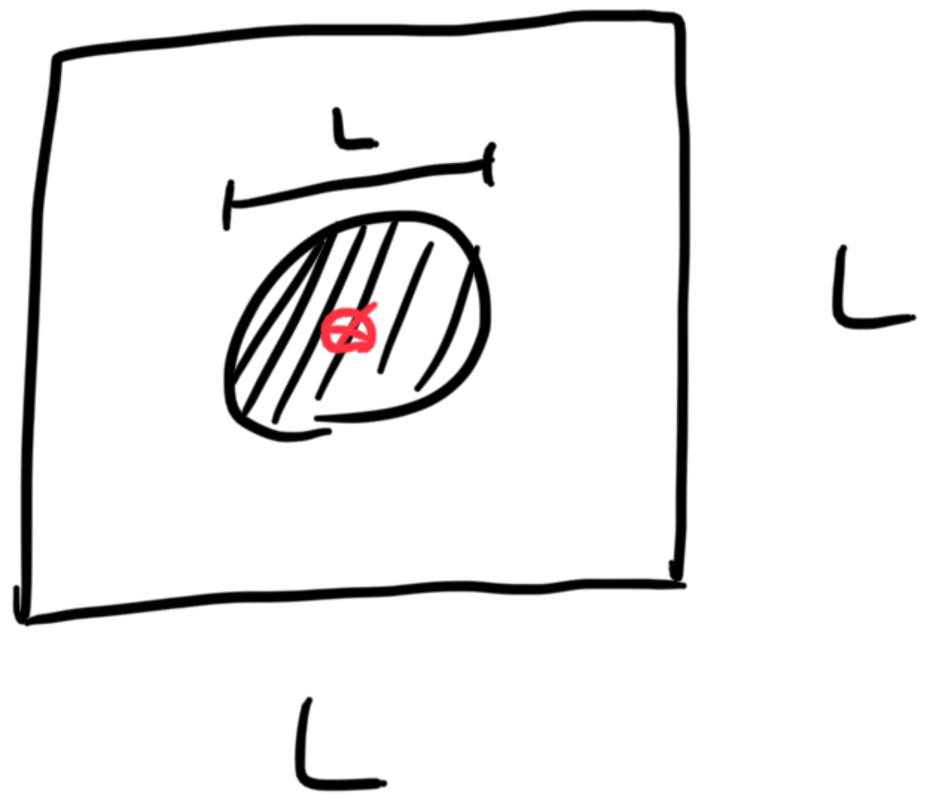


$$R^2 + l_b^2 = l_c^2$$

$$l_b = \sqrt{3}R$$

#11

$I = ?$



Hy: $I_{sq} = \frac{1}{6} M L^2$

$I_c = \frac{1}{2} M R^2$

$I_{cut} = \frac{1}{2} M_{cir} (L/2)^2$

$M_{cir} = M \frac{A_{cir}}{A_{sq}} = M \frac{\pi (L/2)^2}{L^2} = M \frac{\pi}{4}$

$I_{cut} = \frac{1}{2} \left(M \frac{\pi}{4}\right) \left(\frac{L}{2}\right)^2 = \frac{1}{2} \frac{\pi}{4} \frac{L^2}{4} M$

$= \frac{\pi}{32} M L^2$

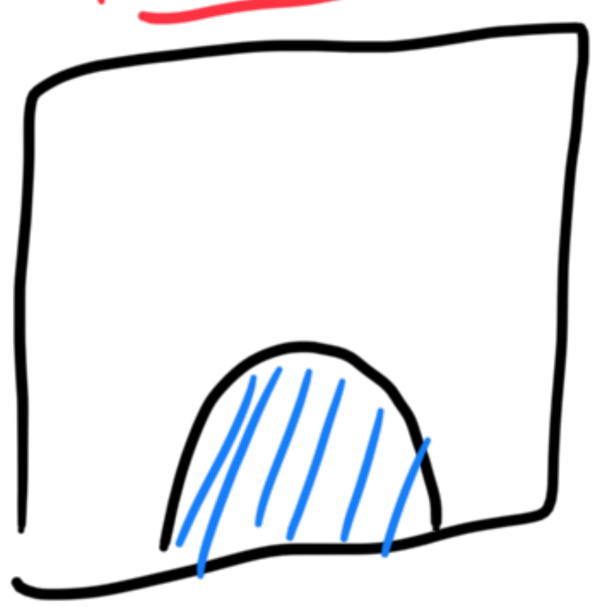
$I_{tot} = I_{sq} - I_{cir}$

$= \left(\frac{1}{6} - \frac{\pi}{32}\right) M L^2$

A

#13

$\boxed{v,\ \ell,\ g}$



$\Rightarrow \quad v \to 2v$

$$[L] = \frac{[v^2]}{[g]} = \frac{[L]^2/[T]^2}{[L]/[T]^2} = [L]$$

$$[L] \propto v^2$$

$\ell \to 4\ell$ when $v \to 2v$

#16-17

μ_k

$T_{\text{return to start}} = ?$

$F \rightarrow \tau = FR$

$m \qquad I = mR^2$

$a \qquad a = F/mR$

$= \alpha/R$

when it stops spinning

$v = -v_0 \, , \; \omega = v_0/r$

$F = -\mu_k mg \Rightarrow a = -\mu_k g$

$\Delta x = v_{avg} t = (v_0 + (-v_0)) \Delta t = 0$

$$t = \frac{v_f - v_0}{a} = \frac{-v_0 - v_0}{-\mu_k g} = \frac{2v_0}{\mu_k g}$$

(B)

#17

$I \rightarrow \frac{1}{2} mR^2 \qquad \alpha \rightarrow 2a/R$

(D)

#12

$V_0 \pm 10\%$

$a \pm 10\%$

$$d = v_0 t + \frac{1}{2} a t^2$$

larger for small t

larger for large t

A

#25

Minimize $\Delta g / g$

$$\frac{\Delta(x^a)}{x^a} = |a| \frac{\Delta x}{x}$$

$$\frac{\Delta(xy)}{xy} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$g = h\, t^{-2}$$

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta h}{h}\right)^2 + \left(\frac{\Delta(t^{-2})}{t^{-2}}\right)^2}$$

$$\frac{\Delta(t^{-2})}{t^{-2}} = 2 \frac{\Delta t}{t}$$

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta h}{h}\right)^2 + \left(2 \frac{\Delta t}{t}\right)^2}$$

$\Delta t = 0$ is best   A