2018B #6

$$50\ \frac{km}{h} \times \left(1 = \frac{1000\,m}{1\,km}\right) \times \left(1 = \frac{1\,h}{3600\,s}\right) = \ldots\ m/s$$

What's the period of this pendulum?

A. $\sqrt{g/L}$ $= \left[\frac{m}{s^2}\cdot\frac{1}{m}\right]^{1/2} = \frac{1}{s}$

$\boxed{\text{B.}}$ $\sqrt{L/g}$ $= \left[m\cdot\frac{s^2}{m}\right]^{1/2} = s$

~~C.~~ $\sqrt{mg/L}$

~~D.~~ $\sqrt{L/mg}$

~~E.~~ $\sqrt{gL}$

$[T] = s$ ← goal

$[mg] = N$ → $[m] = kg$, $[g] = m/s^2$

$[L] = m$

2018 B #6 cont.

$[F] = N = \frac{kg \cdot m}{s^2}$

$[x] = m$

$[I] = m^4$

$[E] = N/m^2 = \frac{kg}{m \cdot s^2}$

$[L] = m$

$X = F(I)^{-1} E^{\beta} L^{\gamma}$

($\beta = -1$ to cancel $\frac{kg}{s^2}$ from F.)

$X = \frac{F}{IE} L^{\gamma}$

$\left[\frac{F}{IE}\right] = \frac{\frac{\cancel{kg}\, m}{\cancel{s^2}}}{m^4 \cdot \frac{\cancel{kg}}{m \cancel{s^2}}} = \frac{m}{m^3} = \frac{1}{m^2}$

$[x] = m = \left[\frac{F}{IE}\right] \cdot [L^{\gamma}]$

$\frac{1}{m^2} \cdot m^3 \Rightarrow \gamma = 3$

D

#10

at depth h :

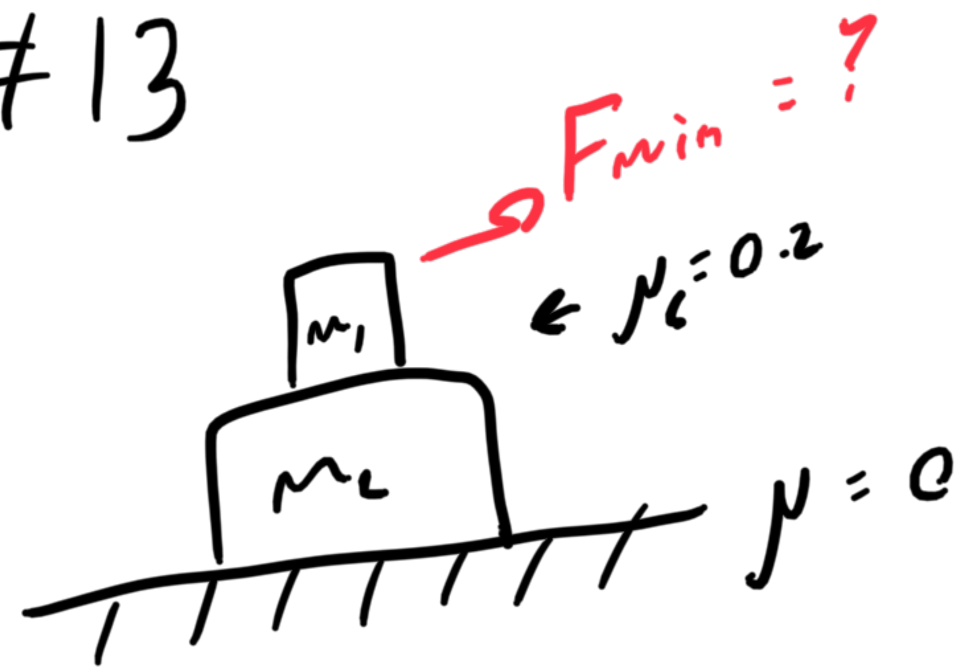
$$PV = \overbrace{nRT}^{\text{const.}} \Rightarrow V \propto P^{-1}$$

$$P \propto h \Rightarrow V \propto h^{-1}$$

$$h \rightarrow 2h \Rightarrow V \rightarrow V/2$$

$$B \propto V \Rightarrow B \rightarrow B_0/2$$

#13



Normal force

$$F_g = \mu N = \mu m g = (0.2)\cdot(2\,kg)\cdot(9.8\ m/s^2) = 4N$$

$F \geq F_g$ to slide $\Rightarrow$ A B?

Correct for :

\# 13

$$a = \frac{F}{m_1+m_2}$$

$$F_2 = m_2 a = \frac{m_2}{m_1+m_2} F = \frac{F}{3}$$

$$N = m_1 g$$

$$F_2 < \mu_s m_1 g \Rightarrow \frac{F_{min}}{3} = \mu_s m_1 g$$

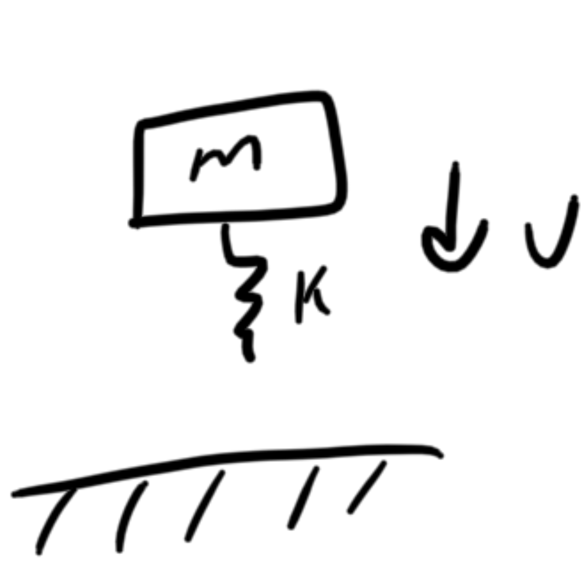
$\cancel{\#}$

F

$$F_{min} = 3\mu_s m_1 g = 12\ N$$

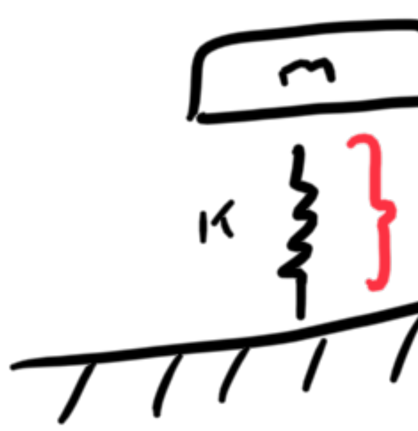
D

#17



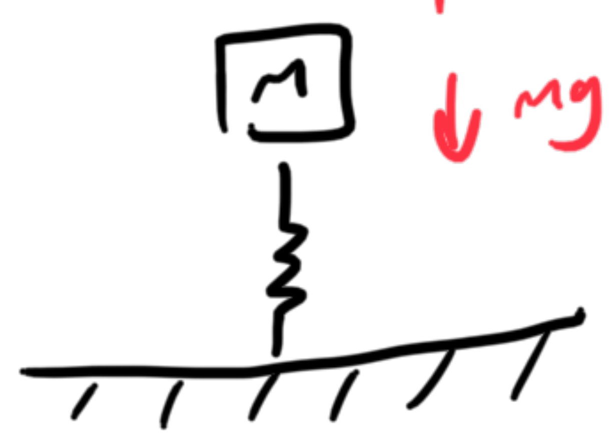
$v_{max} = ?$

initial state:



ℓ = original equil. length

equilibrium state:



$\uparrow F_s = mg = -k\Delta x = -k(x-\ell)$

$\downarrow mg$

$x = \ell - mg/k$

grav.

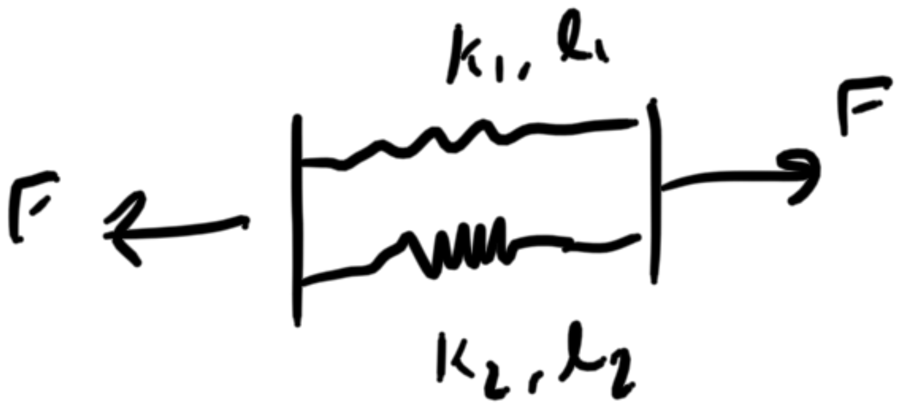
$E_i = \frac{1}{2}mv^2 + \frac{1}{2}k(mg/k)^2 + \cancel{\underbrace{mg\,\frac{mg}{k}}_{?}}$

$= \frac{1}{2}m v_{max}^2$

$\Rightarrow v_{max} = \sqrt{v^2 + \frac{mg^2}{k}}$

E

#18



this acts like 1 spring with $k=?$ and $l=?$

$$\Sigma F = 0 = k_1(l-l_1)+k_2(l-l_2)$$

$$\Rightarrow k_1 l_1 + k_2 l_2 = (k_1+k_2)\, l$$

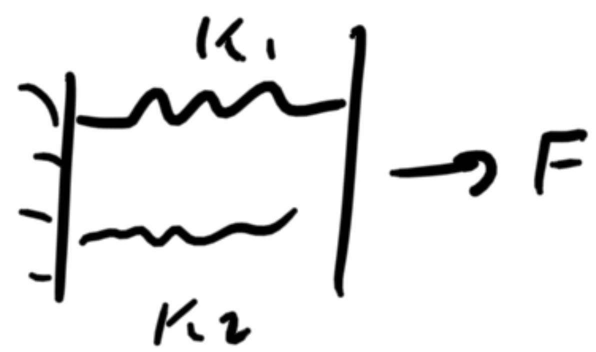
$$l = \frac{k_1 l_1 + k_2 l_2}{k_1+k_2}$$

B

Δx

$\frac{\Delta x}{2}$

$$\frac{k\Delta x}{2} = F \Rightarrow \Delta x = 2F/k$$



$\Delta x = ?$

$$\Delta x = \frac{F_1}{k_1} = \frac{F_2}{k_2} \qquad F_1+F_2=F$$

$$k_1\Delta x + k_2\Delta x = F \Rightarrow \Delta x = \frac{F}{k_1+k_2}$$

#19

Error propagation

Δx Δy aka $x \pm \Delta x$ and $y \pm \Delta y$

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$

$$\Delta(x^a) = |a|\, x^{a-1}\, \Delta x$$

$$\Delta(y^{-1}) = \frac{1}{y^2}\Delta y$$

$$\Delta(x \cdot y^{-1}) = \sqrt{\left(x \cdot \frac{1}{y^2}\Delta y\right)^2 + (y\,\Delta x)^2}$$

$$\Delta\left(\frac{x}{y}\right) = ?$$

#19

$$v = \frac{s}{t}$$

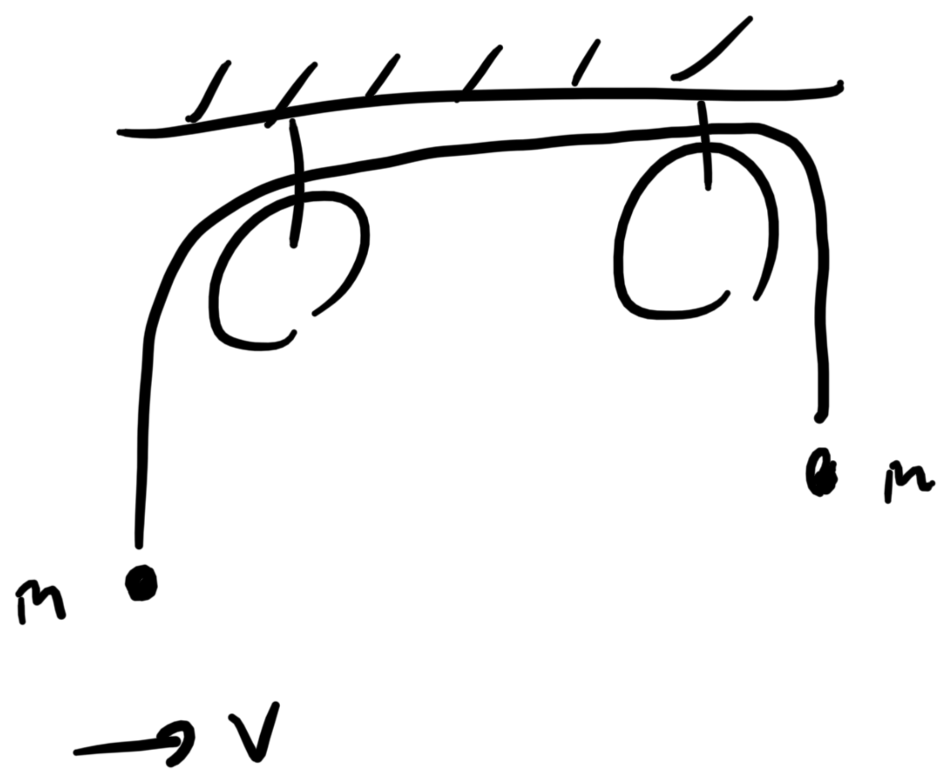
$$\Delta v = \Delta\left(\frac{s}{t}\right)$$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta s}{s}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} = \ldots = 5.4\%$$

$$\Delta v = 5.4\% \cdot v = 19\,\text{m/s}$$

E

#22



B