Rotational Dynamics

$F \to N\ (\tau)$: $N = r \times F = rF\sin\phi = rF$ iq $r \perp F$

$x \to \theta$: $\theta = x/R$

$v \to \omega$: $\omega = v/R$

$a \to \alpha$: $\alpha = a/R$

$m \to I$: $I = cmR^2$

$K \to K$: $K = \frac{1}{2} I \omega^2$

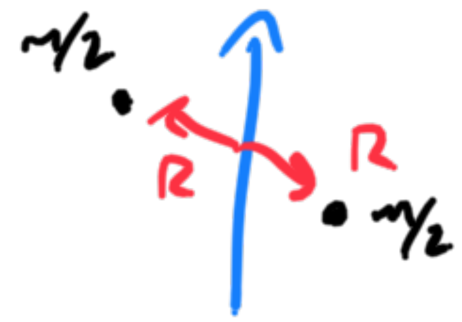
$F = ma$

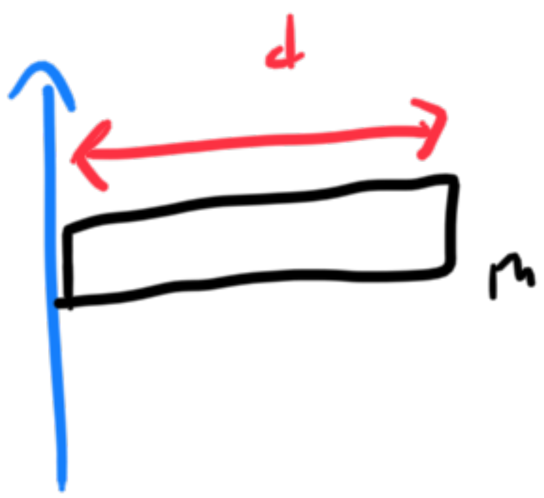
$N = I\alpha$

$x = x_0 + v_0 t + \frac{1}{2} a t^2 \Rightarrow \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

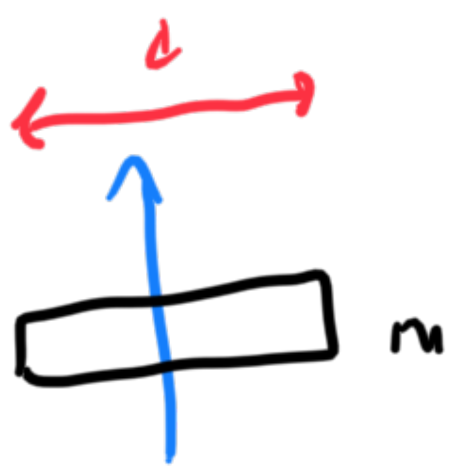
Moments of Inertia

$I_{point} = 1\, mR^2 = I_{ring}$





$I_{end} = \frac{1}{3} md^2$

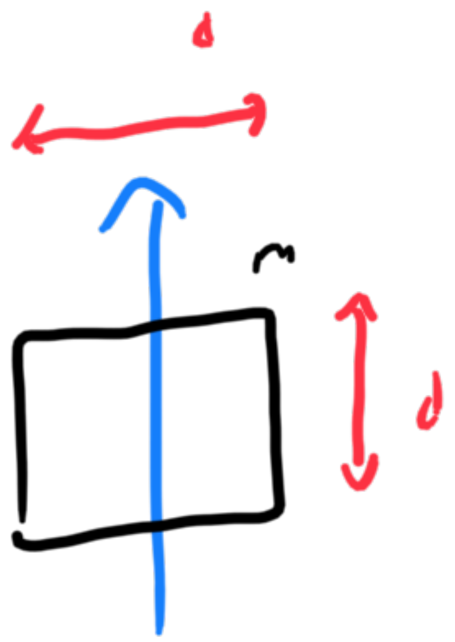


$I_{ctr} = \frac{1}{12} md^2$

$I_{disk} = \frac{1}{2} mR^2$

$I_{hollow} = \frac{2}{3} mR^2$

$I_{solid} = \frac{2}{5} mR^2$



$$I_{sq} = \frac{1}{12} m d^2$$

$$I = \iiint r^2 \, dm$$

Ex.

$\Delta U = 0$

A ball (m, R) gently dropped on a surface, while spinning with ω_0. Once rolling w/o slipping how fast is it going?

$v = \omega R$

HW: $I = cmR^2$

OR $I_{hol} = \frac{2}{3} mR^2$?

$K_f = K_0$

No energy is lost to the surface; $I_{sp} = \frac{2}{5} mR^2$.

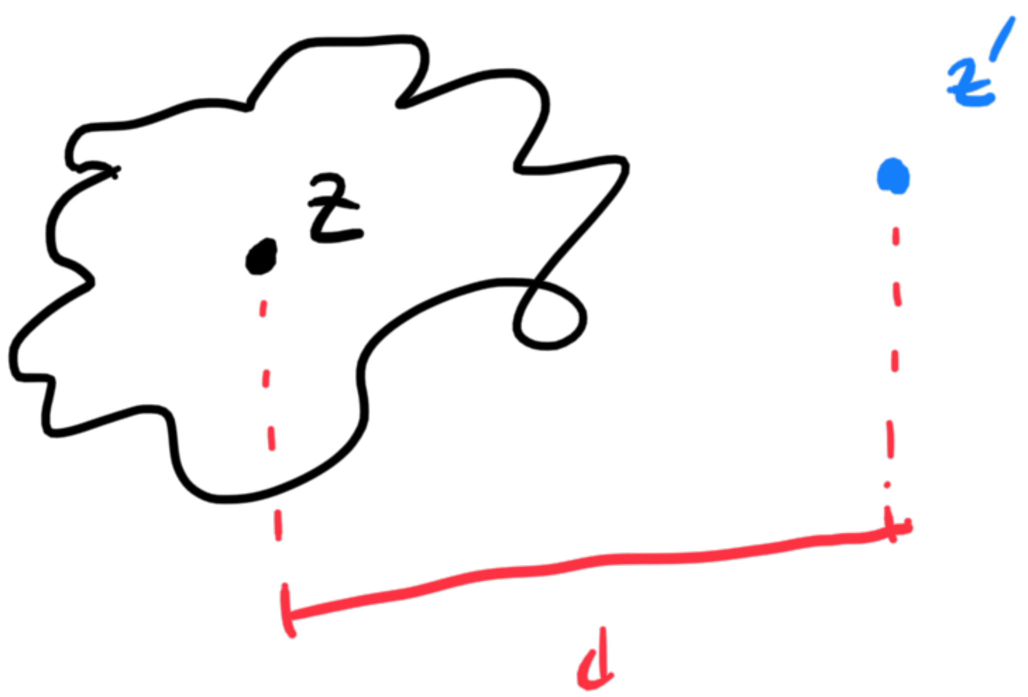
$$K_0 = \frac{1}{2} I \omega_0^2 = \frac{1}{2}\left(\overset{2/3}{\frac{2}{5}} mR^2\right)\omega_0^2 = \frac{\omega_0^2 mR^2}{5} \qquad \left(\frac{\omega_0^2 mR^2}{3}\right)$$

$$\frac{\omega_0^2}{3} = \frac{5}{6}\omega_0^2 \qquad \frac{5}{6}$$

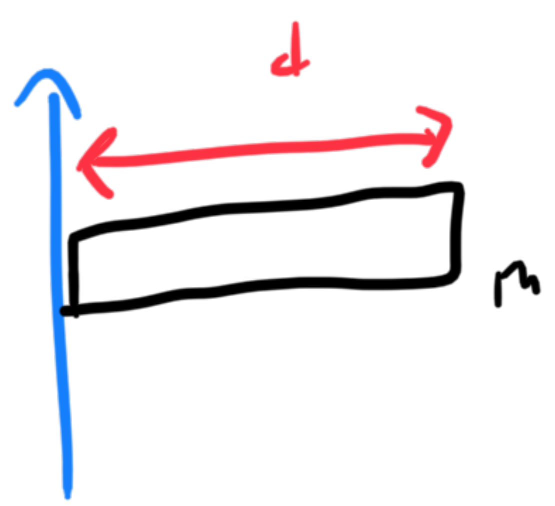
$$= K_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \omega_f^2 \frac{mR^2}{5} + \omega_f^2 \frac{mR^2}{2} = \frac{7}{10}\omega_f^2 mR^2 \Rightarrow \frac{\omega_0^2}{\overset{1/3}{5}} = \frac{7}{10}\omega_f^2$$

$$\omega_f^2 = \overset{2/5}{\frac{2}{7}}\omega_0^2 \Rightarrow \omega_f = \sqrt{\frac{2}{7}}\,\omega_0 \Rightarrow v_f = \sqrt{\frac{2}{7}}\,\omega_0 R. \quad \ldots \quad v_f = \sqrt{\frac{2}{5}}\,\omega_0 R$$

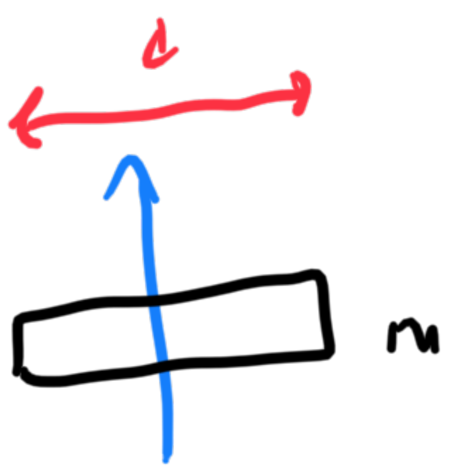
Parallel axis theorem.



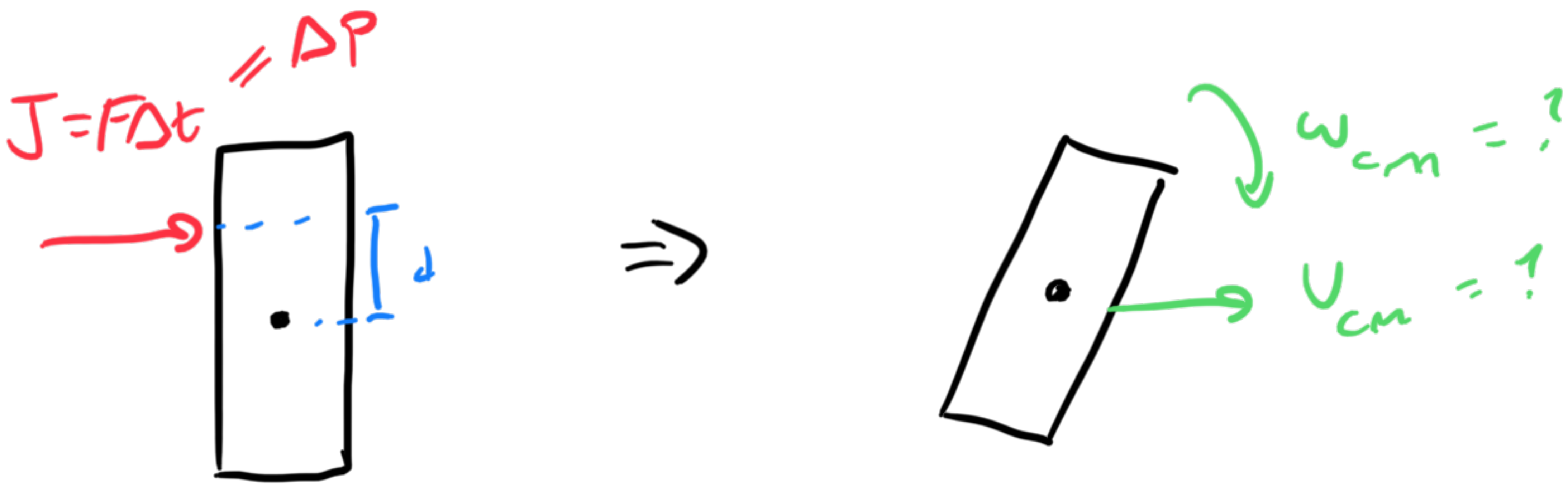
$$I_{z'} = I_z + md^2$$



$$I_{end} = \frac{1}{3} md^2 = \frac{1}{12} md^2 + m\left(\frac{d}{2}\right)^2 = \frac{1}{3} md^2$$



$$I_{ctr} = \frac{1}{12} md^2$$



M, L

$g = 0, \mu = 0$

Linear:

$$J = F\Delta t = Ma\Delta t = Mv_{cm}$$

$$v_{cm} = J/M$$

independent of d!

Rotational:

$$J_N = F\Delta t\, d = \boxed{N\Delta t} = I\alpha\Delta t$$

$$= I\omega_{cm} = \frac{1}{12}ML^2\,\omega_{cm}$$

$$\Rightarrow \omega_{cm} = 12\,Jd/ML^2$$

"Does d affect the energy imparted on the rod by this impulse?"

$$\Delta E = F \Delta y = J \, {\Delta y}/{\Delta t}$$

$$\Delta y = d\Delta\theta + \Delta x$$

(small Δt)

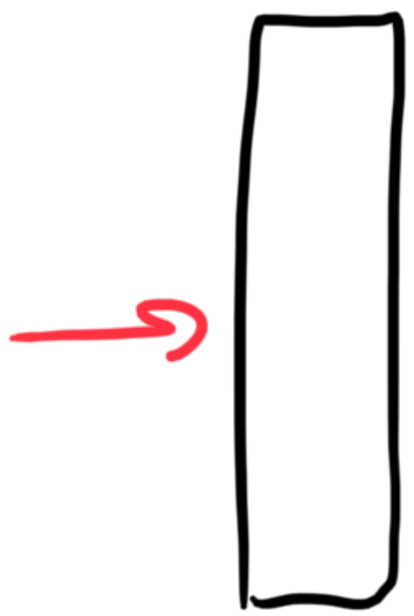
$$\alpha = {Jd}/{I\Delta t} \quad \text{and} \quad a = {J}/{M\Delta t}$$

$$\Rightarrow \Delta\theta = \frac{1}{2}\alpha\Delta t^2 \qquad \Delta x = \frac{1}{2}a\Delta t^2$$

$$= \frac{Jd\Delta t}{2I} \qquad = \frac{J\Delta t}{2M}$$

$$\Rightarrow \Delta y = \frac{J\Delta t}{2M}\left(1 + \frac{d^2}{L^2}\right)$$

$$\Rightarrow \Delta E = \frac{J^2}{2M}\left(1 + \frac{d^2}{L^2}\right) \quad ???$$

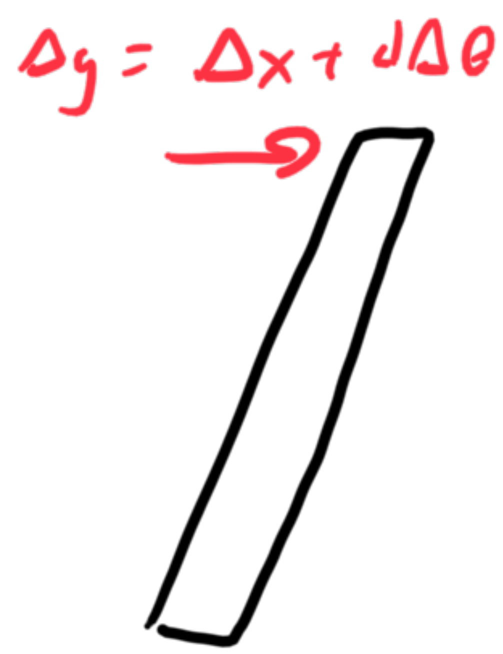


Δt
$\Longrightarrow$

$$W_c = F\Delta x = J\,{}^{\Delta x}/_{\Delta t}$$

$J,\ \Delta t$ are fixed

Δt
$\Longrightarrow$



$$W = F\Delta y = J\frac{\Delta y}{\Delta t}$$

$$\frac{W}{W_c} = \frac{\Delta y}{\Delta x} = 1 + \frac{d\Delta\theta}{\Delta x}$$

2017 #14

$$I = cmR^2$$

$$I_{pc} = (c+1)mR^2 \quad \text{(parallel axis thm)}$$

$$a = \alpha R = \frac{\tau R}{I} = \frac{mgR^2 \sin\theta}{(c+1)mR^2} = \frac{g\sin\theta}{c+1}$$

minimize $c \Rightarrow$ A

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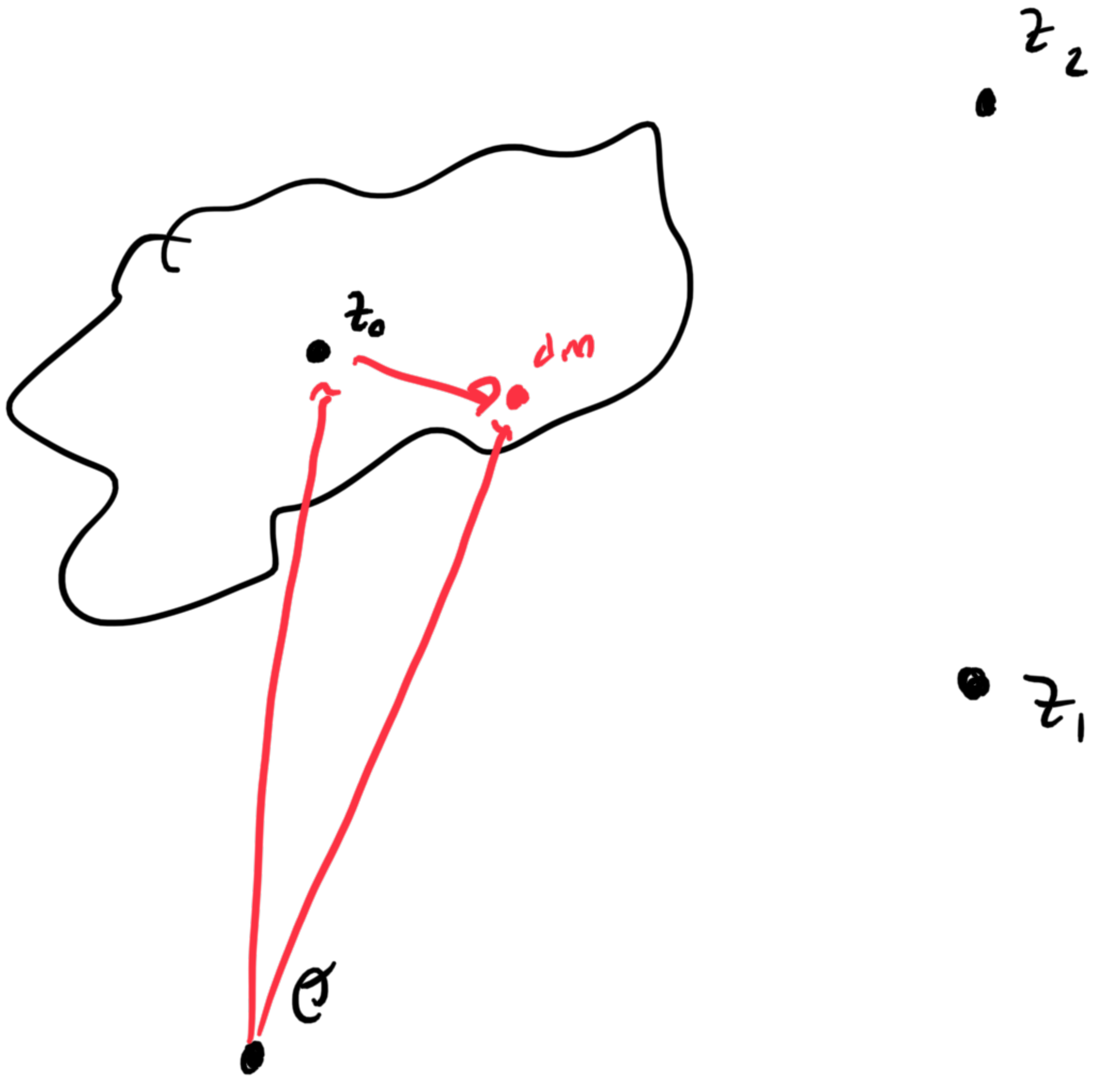
$c = 2/5$



$$I_1 =$$

$$I_0 = \iiint (\vec{r} - \vec{z}_0)^2 \, dm$$

Bonus: parallel axis thm proof

$x_i = x_{i0} + a \qquad y_i = y_{i0} + b$

$$I_\mathcal{O} = \sum m_i r_{i\mathcal{O}}^2 = \sum m_i (x_{i0}^2 + y_{i0}^2)$$

$$\boxed{r_{i\mathcal{O}}^2 = (x_i - a)^2 + (y_i - b)^2}$$

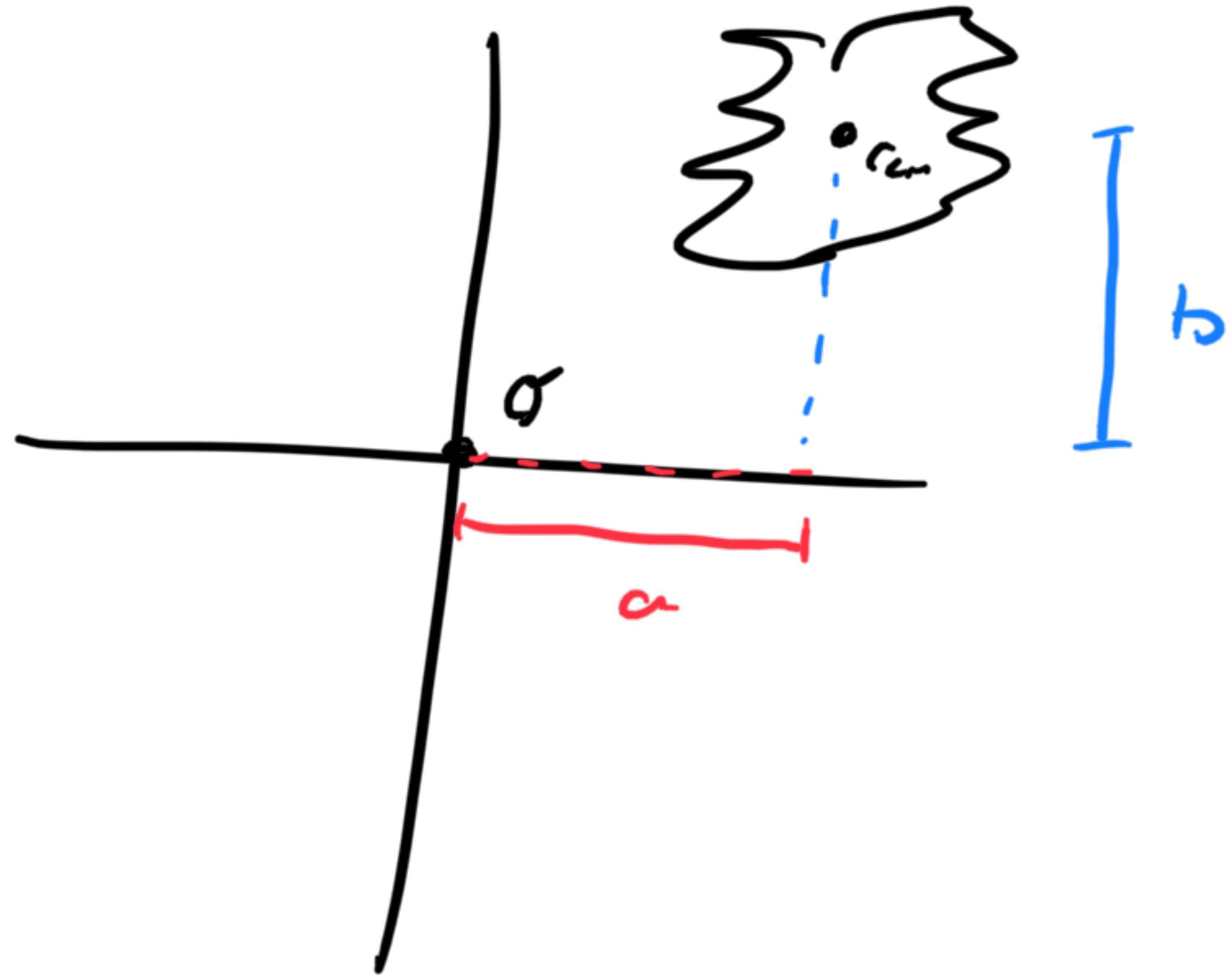
see next slide

$r_{cm} = (a, b)$

$$I_\mathcal{O} = \sum m_i x_{i0}^2 + \sum m_i y_{i0}^2$$

$$= \sum m_i [x_i^2 - \cancel{2ax_i} + a^2 + y_i^2 - \cancel{2by_i} + b^2]$$

$$\frac{\sum m_i x_i}{M} = 0 \qquad \frac{\sum m_i y_i}{M} = 0$$

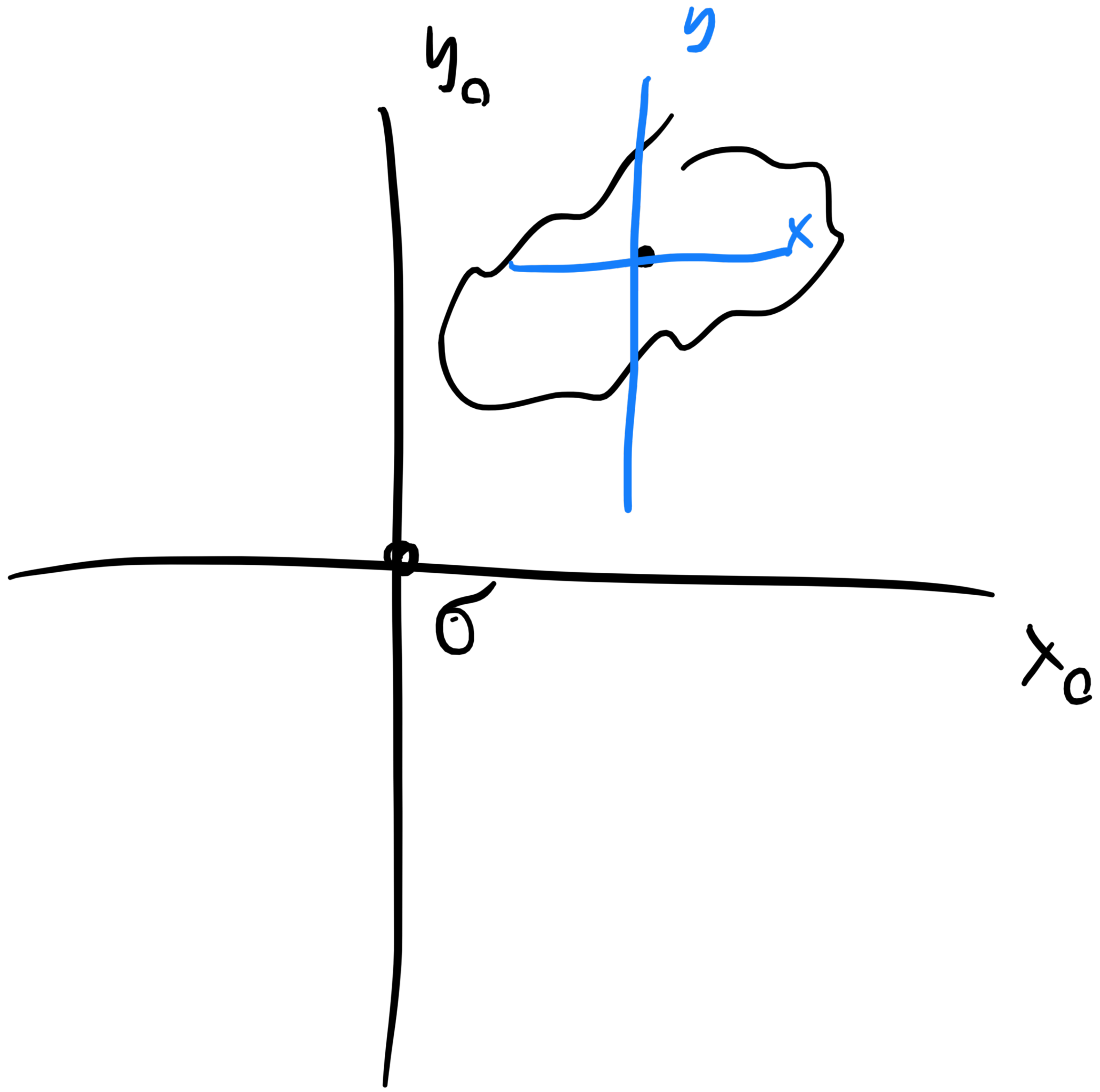


$$I_\mathcal{O} = \sum m_i (x_i^2 + y_i^2) + \sum m_i (a^2 + b^2)$$

$$= \sum m_i r_i^2 + \sum m_i R^2$$

$$= I_{cm} + mR^2$$

R is dist. $\mathcal{O}$ to r_{cm}



y_0
y
x
O
x_0