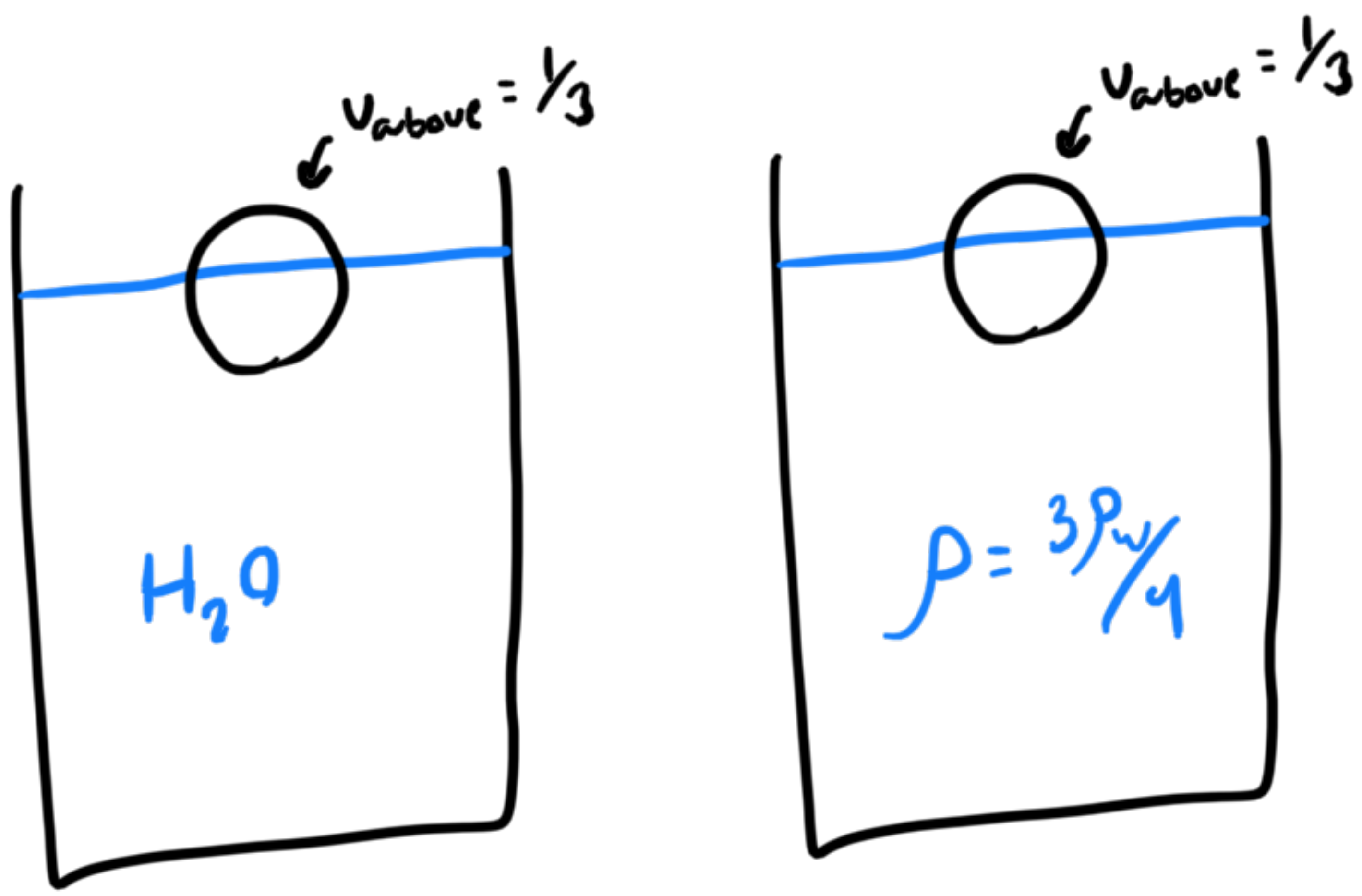
2015 #11

11, 19, 20



$$m_{sp} g = \boxed{F_B = \rho V_{submerged} g}$$

$$m_{sp} = \frac{\rho_w \frac{2}{3} V_{sph} \cancel{g}}{\cancel{g}}$$

$$= \frac{2}{3} \rho_w V_{sph}$$

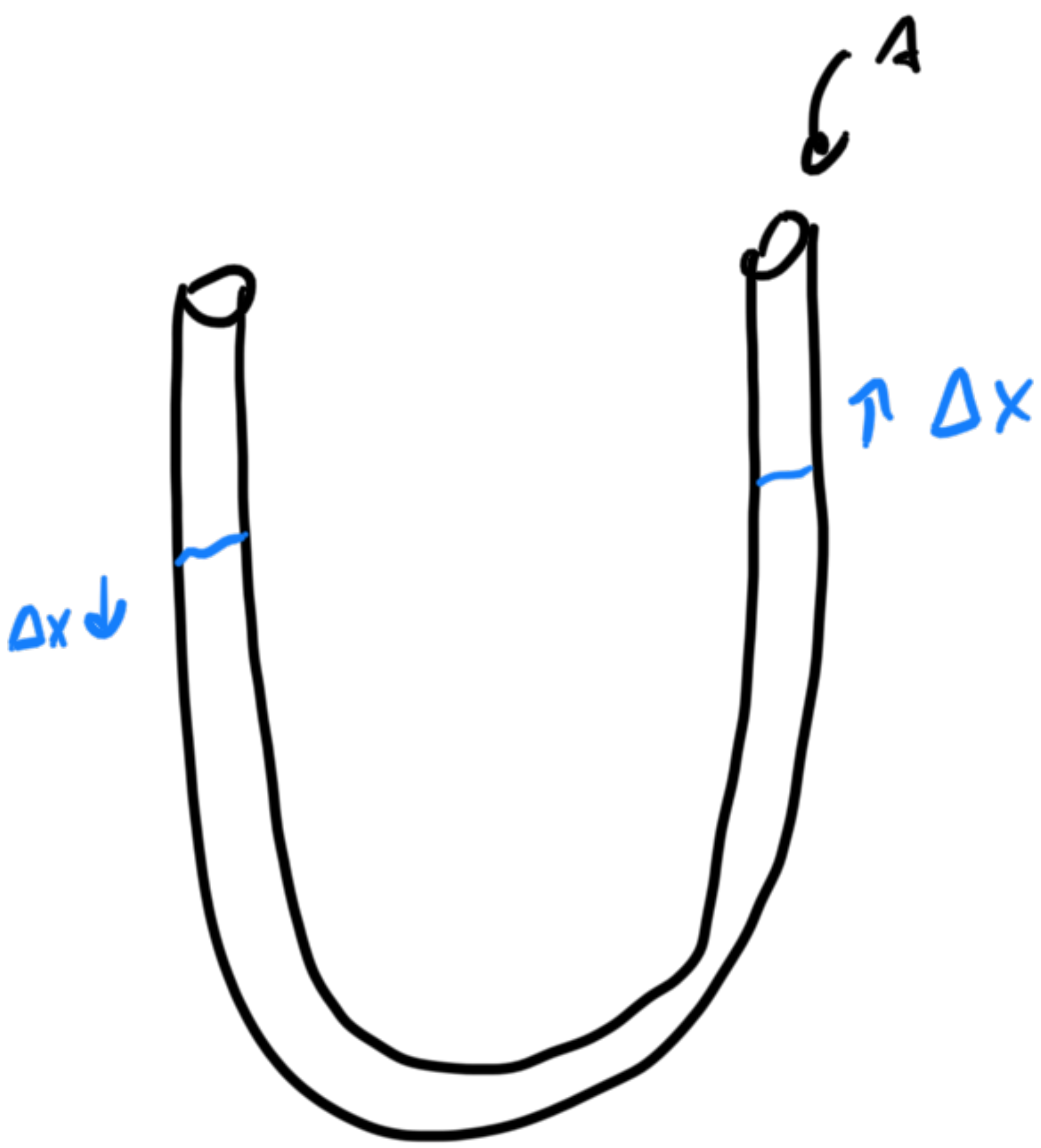
$$F_{b_{oil}} = \rho_{oil} V_{sub_{oil}} g = \frac{3}{4} \rho_w V_{sub_{oil}} g = m_{sp} g$$

$$\frac{2}{3} \cancel{\rho_w} V_{sph} \cancel{g} = \frac{3}{4} \cancel{\rho_w} V_{sub_{oil}} \cancel{g}$$

$$\frac{V_{sub_{oil}}}{V_{sph}} = \frac{2/3}{3/4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$$

C

2015 #19



$[f] = \frac{1}{[T]} \Rightarrow$ A or B

$f \sim \frac{1}{2\pi} \Rightarrow$ (A)

$\Rightarrow$

$$T = 2\pi\sqrt{\frac{k}{m}} \quad \text{or} \quad \omega = \sqrt{\frac{m}{k}}$$

$T = 2\pi\sqrt{\frac{\Delta x}{a}}$

for accel. a given displacement Δx

$F = -k\Delta x$

$k = \frac{F}{\Delta x}$

$\Delta m = 2\rho_w A \Delta x$

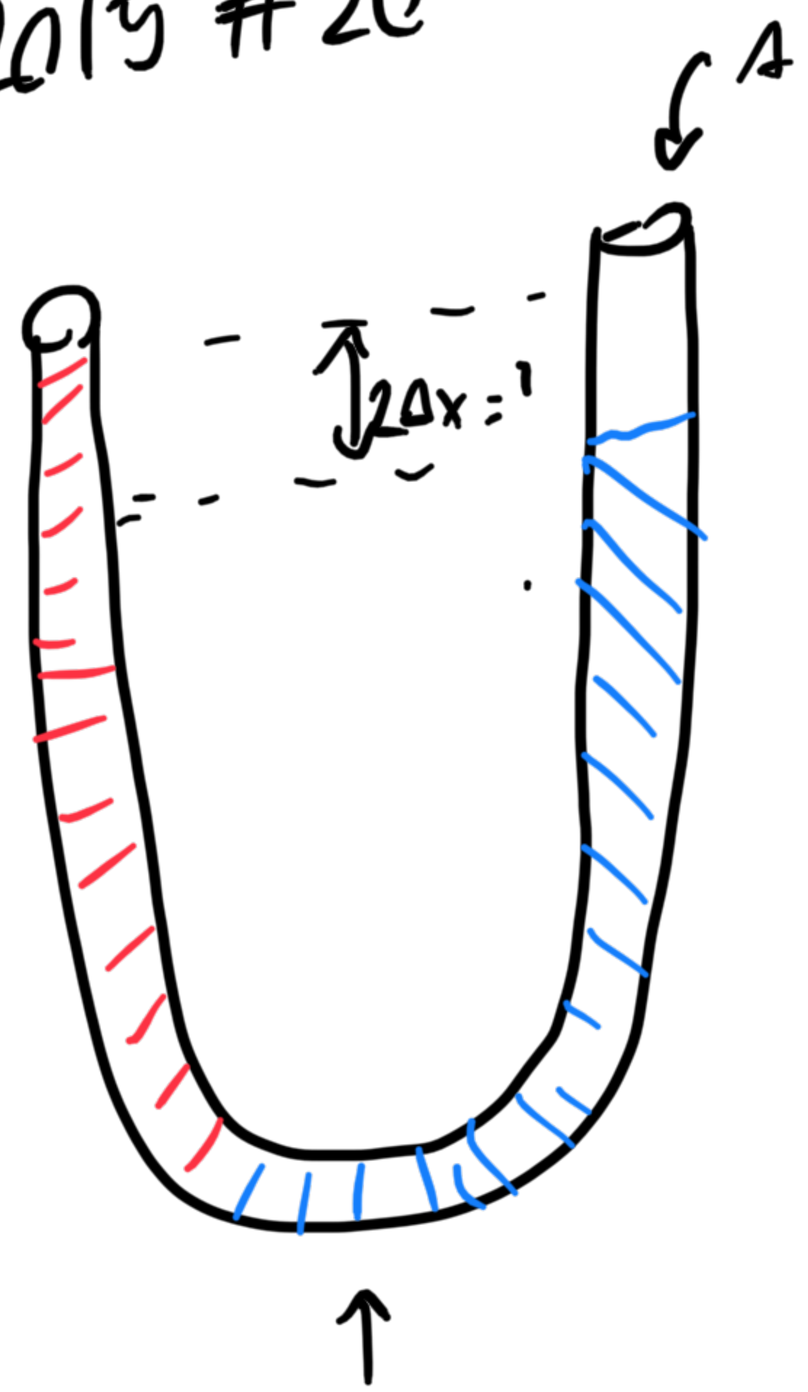
$$k = \frac{F}{\Delta x} = \frac{\Delta m g}{\Delta x} = \frac{2\rho_w A \Delta x g}{\Delta x}$$

$$m = \rho_w V = \rho_w A L$$

$K? \rightarrow k = 2\rho_w A g$

$$T = 2\pi\sqrt{\frac{2\rho_w A g}{\rho_w A L}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{2g}{L}} \quad \text{(A)}$$

2015 #20



$\rho_0 = \frac{1}{2}\rho_w$

$$\rho_0 A L g + \rho_w A \Delta x g = \rho_w A (L - \Delta x) g$$

$$= \rho_w A L g - \rho_w A \Delta x g$$

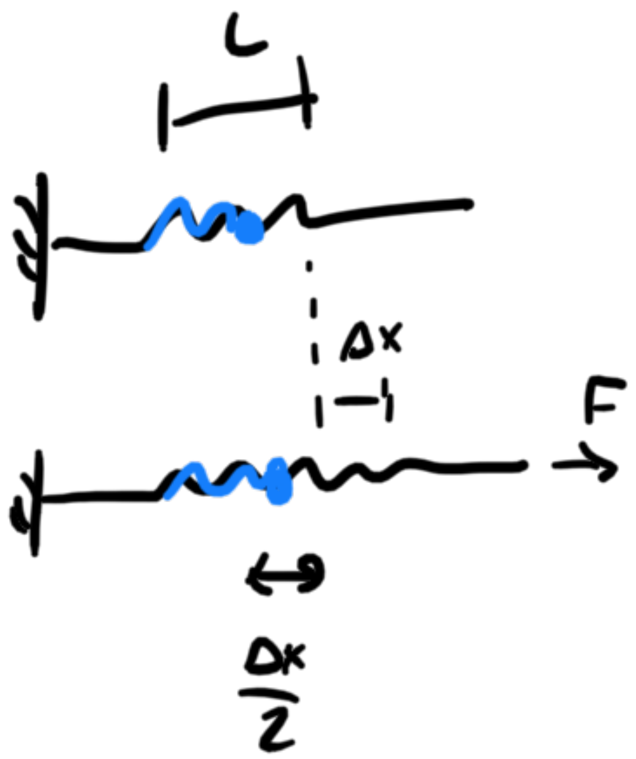
$$-\rho_0 \cancel{A} L \cancel{g} + \rho_w \cancel{A} L \cancel{g} = 2\rho_w \cancel{A} \Delta x \cancel{g}$$

$$(\rho_w - \rho_0) L = 2\rho_w \Delta x$$

$$= \frac{1}{2}\rho_w L = 2\rho_w \Delta x \Rightarrow \Delta x = L/4$$

$$\Rightarrow \text{distance} = L/2$$

2015 #25



$F_2 = F$

$\Delta x_2 = \frac{\Delta x}{2}$

$k_2 = \frac{F_2}{\Delta x_2} = 2\frac{F}{\Delta x} = 2k$

1. middle spring does not stretch

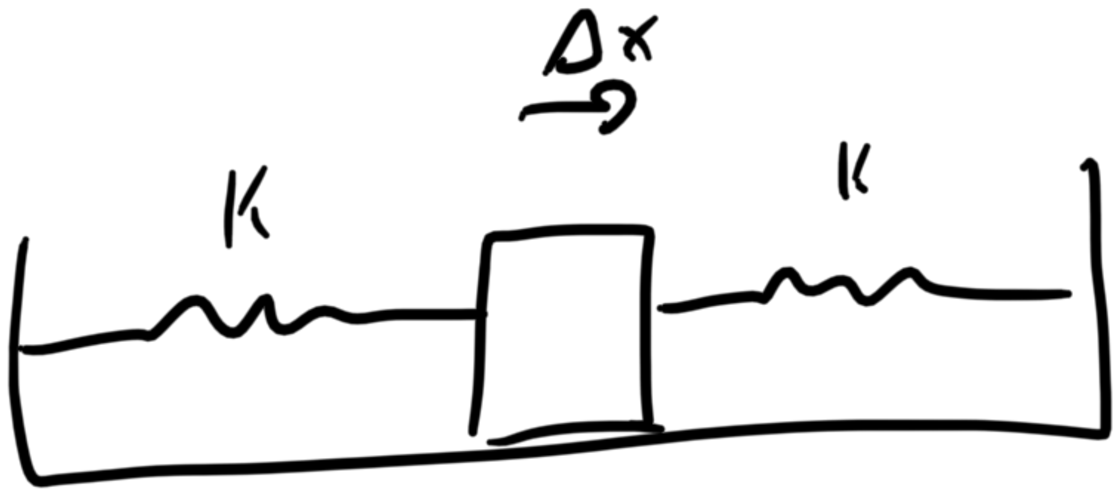
$\omega = \sqrt{k/m}$

2. middle of middle spring is stationary

$k' = k + 2k = 3k \Rightarrow \omega' = \sqrt{3k/m}$

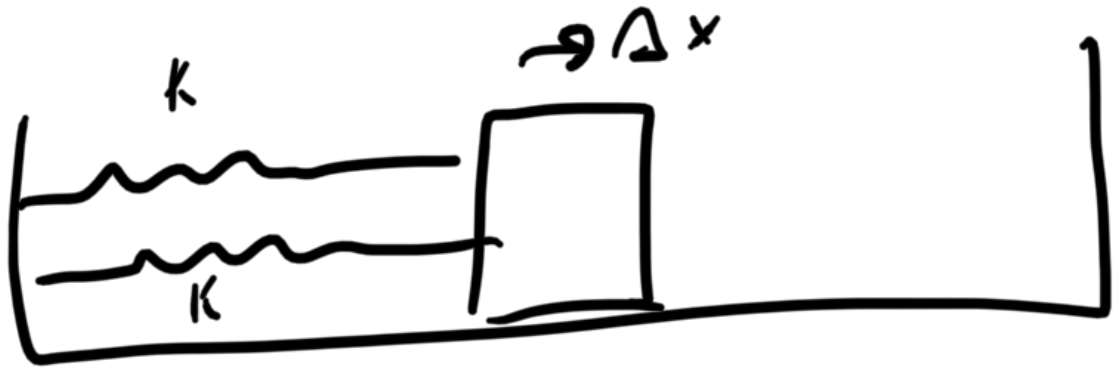
$\frac{\omega'}{\omega} = \sqrt{3}$ A

(1)



$$F = k\Delta x - (k(-\Delta x)) = 2k\Delta x$$

(2)



$$F = k\Delta x + k\Delta x = 2k\Delta x$$

(3)

$$F = k\frac{\Delta x}{2} + k\frac{\Delta x}{2} = k\Delta x$$

2015 #21

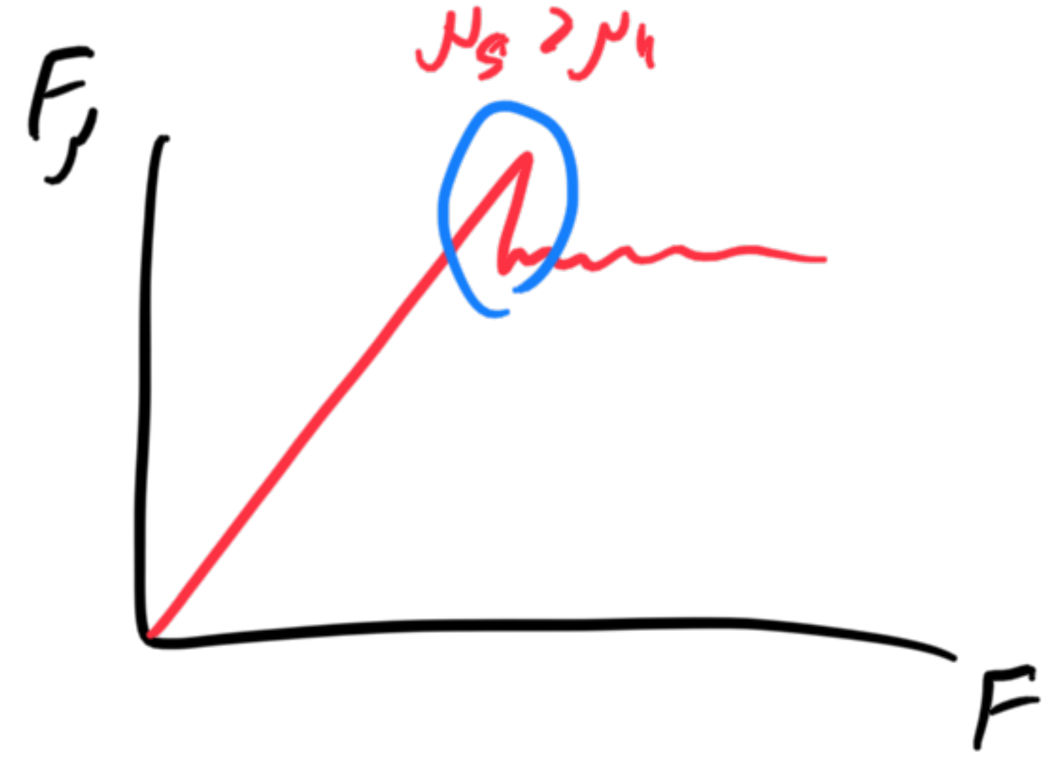
$C_R = 0.9 \Rightarrow V_{n+1} = 0.9 V_n$

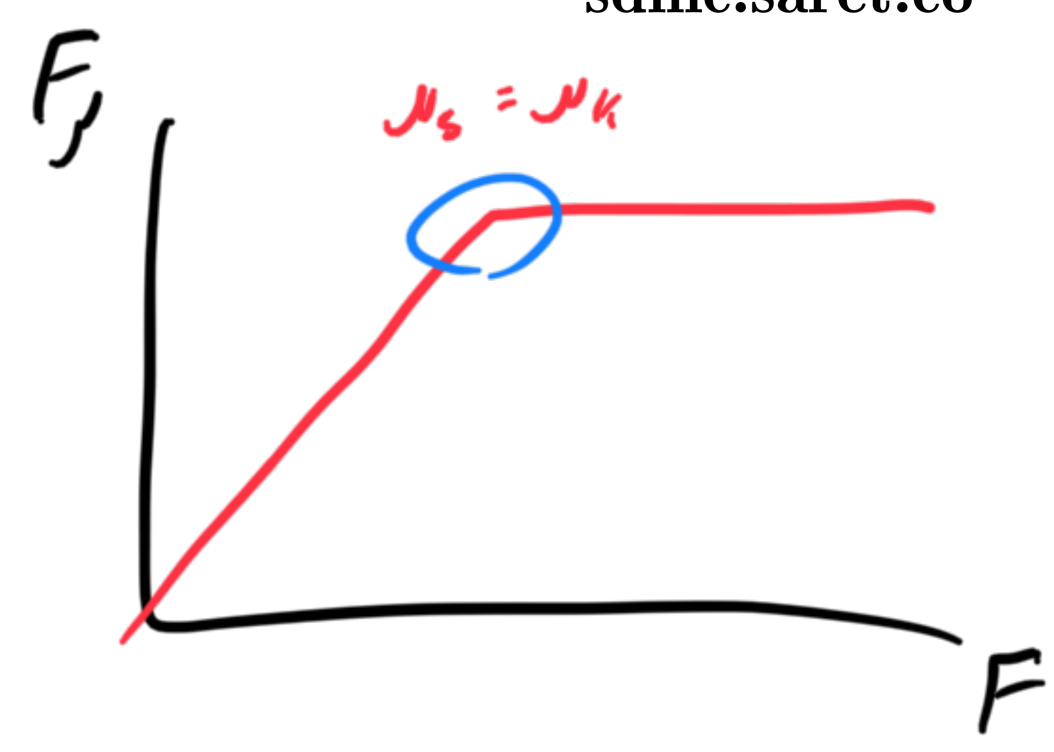
$T_n = \frac{2v_n}{g}$ $\quad V_n = C_R^n V_0$

$$T = \sum_{n=0}^{\infty} T_n = \sum_{n=0}^{\infty} \frac{2v_0}{g} C^n = \frac{2v_0}{g} \sum_{n=0}^{\infty} C^n = \frac{2v_0}{g} \frac{1}{1-C} = \frac{2v_0}{g} \cdot 10 = 100s \qquad B$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

2015 #22



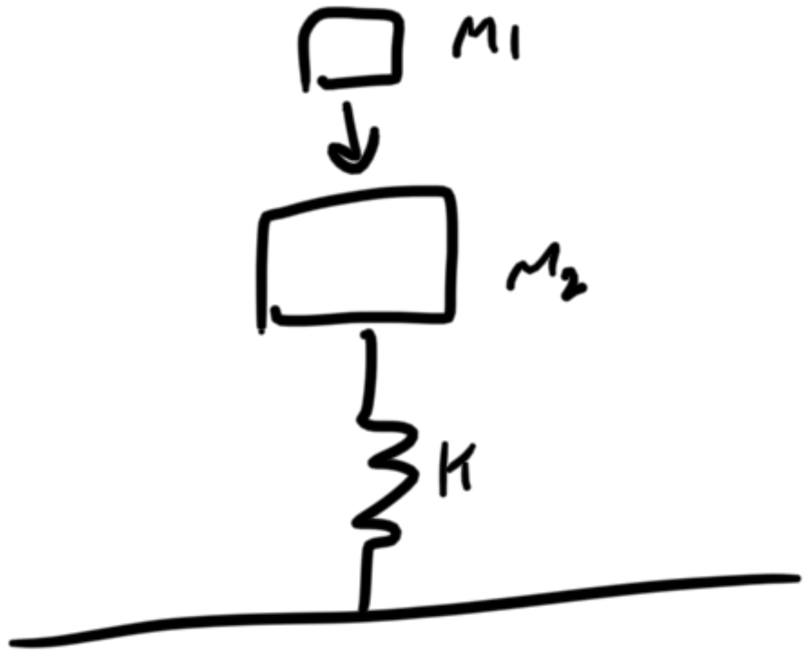


$\theta < \theta_c$, $\Delta E = 0$ $\Rightarrow$ C, D, E

$\theta \to 90^\circ$, $\Delta E = 0$ $\Rightarrow$ C, D

Since $\mu_k = \mu_s = \mu$, no sudden loss $\Rightarrow$ C

2015 #23



already compressed!

(by h)

Compression from h to $x+h$

instead of 0 to x

!

$$\Delta E = \frac{1}{2}k(h+x)^2 - \frac{1}{2}kh^2 = \frac{1}{2}kx^2 + khx$$

$$h = \frac{-m_2 g}{k}$$

$$v_1 = \sqrt{2gy}$$

$$v_f = \frac{m_1 v_1}{m_1+m_2} = \frac{m_1\sqrt{2gy}}{m_1+m_2}$$

after col.

$$E = \frac{(m_1+m_2)v_f^2}{2} + \frac{1}{2}kh^2 + mg0$$

at lowest pt. $(x<0)$

$$E = 0 + \frac{1}{2}k(h+x)^2 + (m_1+m_2)gx$$

$$\frac{(m_1+m_2)v_f^2}{2} = \frac{1}{2}kx^2 + m_1 gx$$

B