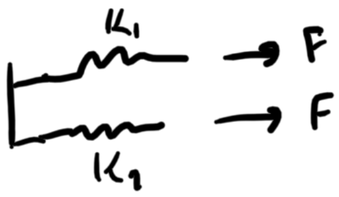
2019 B#3

Ratio of $\frac{U_1}{U_2} = ?$



$$F_1 = F_2 = F$$

$$K_1 x_1 = K_2 x_2 = F \qquad x_2 = \frac{K_1 x_1}{K_2}$$

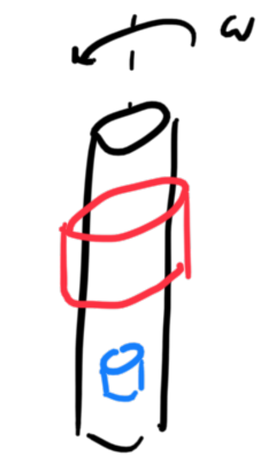
$$U = \frac{1}{2} K x^2$$

$$U_1 = \frac{1}{2} K_1 x_1^2$$

$$U_2 = \frac{1}{2} K_2 x_2^2 = \frac{1}{2} K_2 \left(\frac{K_1 x_1}{K_2}\right)^2$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2} K_1 x_1^2}{\frac{1}{2} K_2 \left(\frac{K_1 x_1}{K_2}\right)^2} = \frac{K_1}{\frac{K_1^2}{K_2}} = \frac{1}{\left(\frac{K_1}{K_2}\right)} = \frac{K_2}{K_1}$$

#6



$$\oiint g \cdot dA = 4\pi G M_{enc}$$ Gauss's Law for grav.

See: Newton's shell thm.

$$2\pi r \ell g = 4\pi G M_{enc}$$

$$g = \frac{2GM}{r\ell} = \frac{2G\lambda}{r} \qquad \lambda = \frac{M}{\ell}$$

How does ρ_{dust} depend on r?

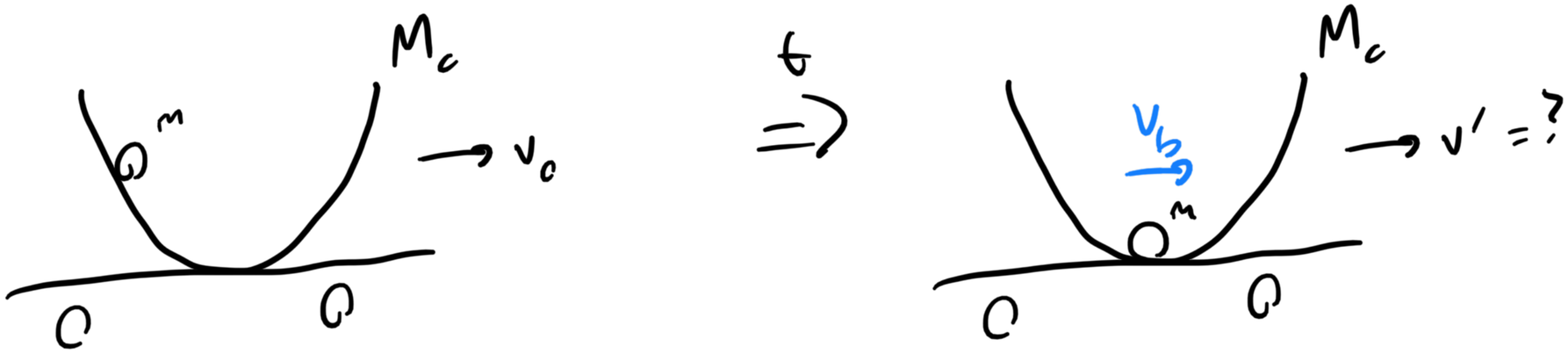
$$g = a_{centripetal} = \omega^2 r$$

$$\omega^2 r = \frac{2G\lambda}{r} \Rightarrow \lambda = \frac{\omega^2}{2G} r^2 = \frac{\omega^2}{2\pi G} A$$

A = cross-section

$$\rho = \frac{\lambda}{A} \Rightarrow \rho = \frac{\omega^2 A}{2\pi G} \Big/ A = \frac{\omega^2}{2\pi G}$$

#10



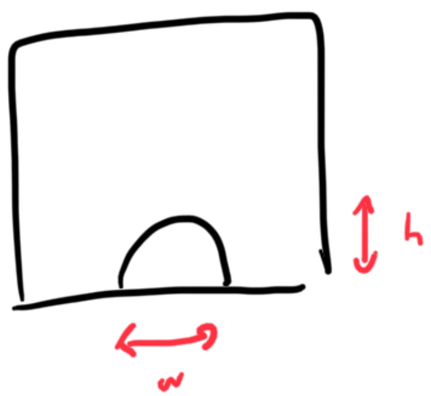
$$P_i = m v_0 + M_c v_0$$

$$P_f = m(v' + v_b) + M_c v'$$

$$v' = v_0 - \frac{m}{M_c + m} v_b = \frac{(M_c + m) v_0 - m v_b}{M_c + m}$$

(D)

#13



h', w' = ?

$v \to 2v, \quad t \to 2t \Rightarrow w \to 4w, \; h \to 4h$

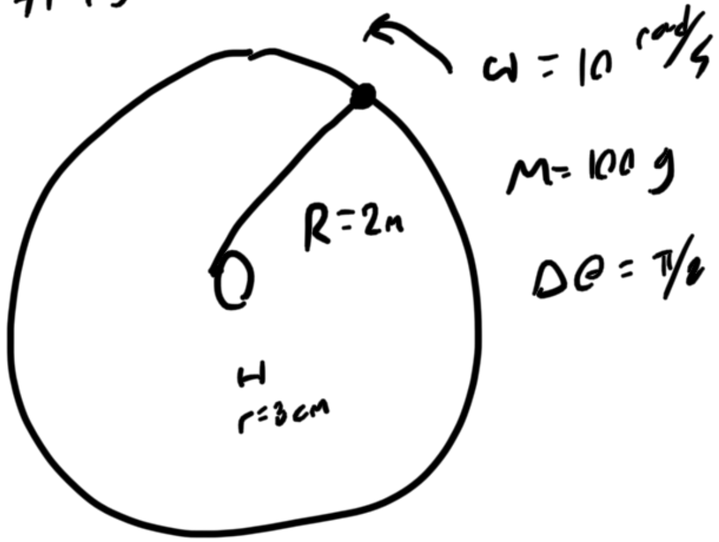
$[v] = [L]/[T] \Rightarrow [L] = v^2/g \Rightarrow$ all lengths scale with v^2

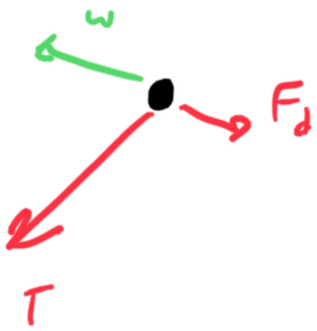
$[g] = [L]/[T]^2$

$h = \frac{1}{2}vt = \frac{v^2}{2g} \propto v^2/g$

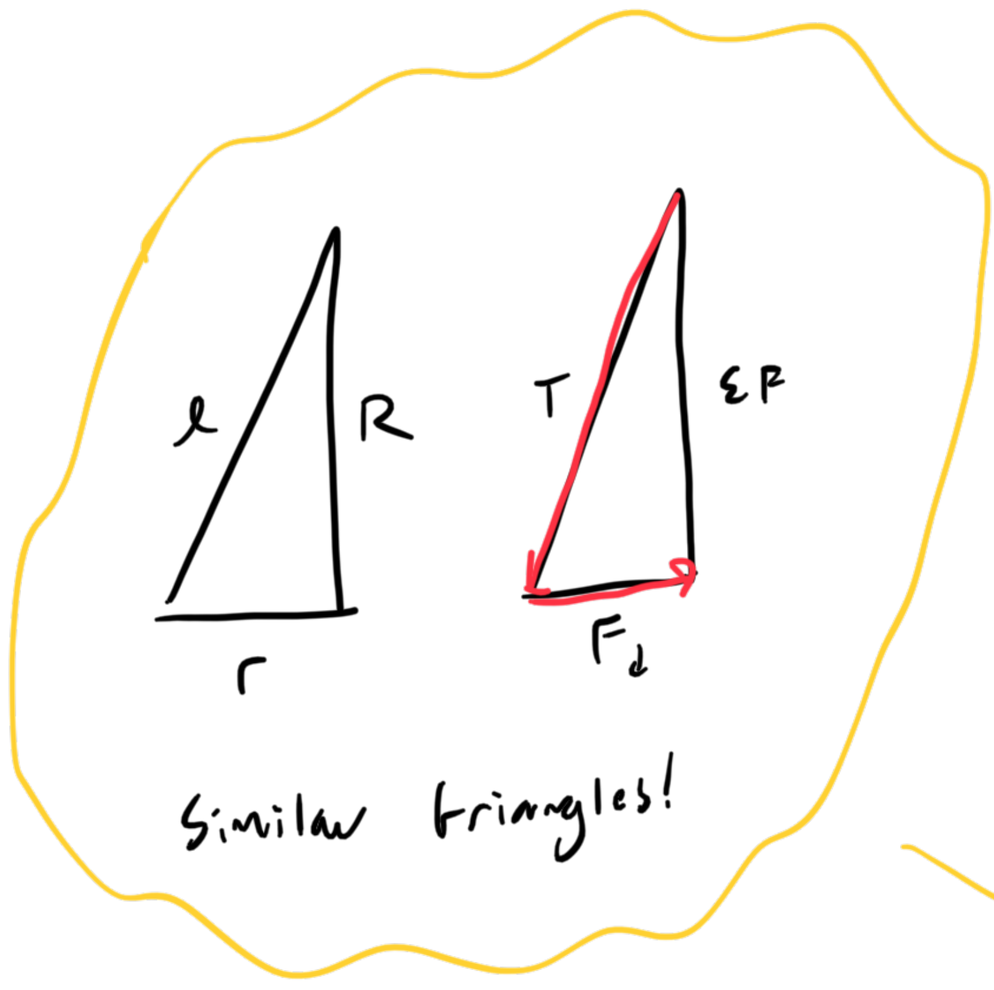
$d = \underbrace{v\sin\theta}_{v_h} \cdot \underbrace{\frac{2v\cos\theta}{g}}_{\Delta t} = \frac{v^2 \sin 2\theta}{g} \propto v^2/g$

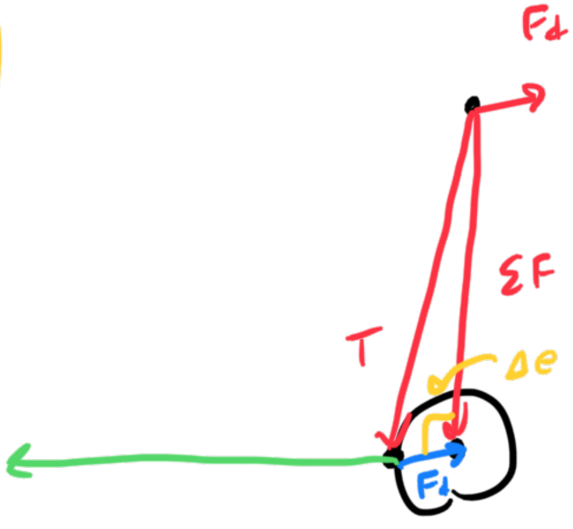
#15





ΣF is exactly radial!



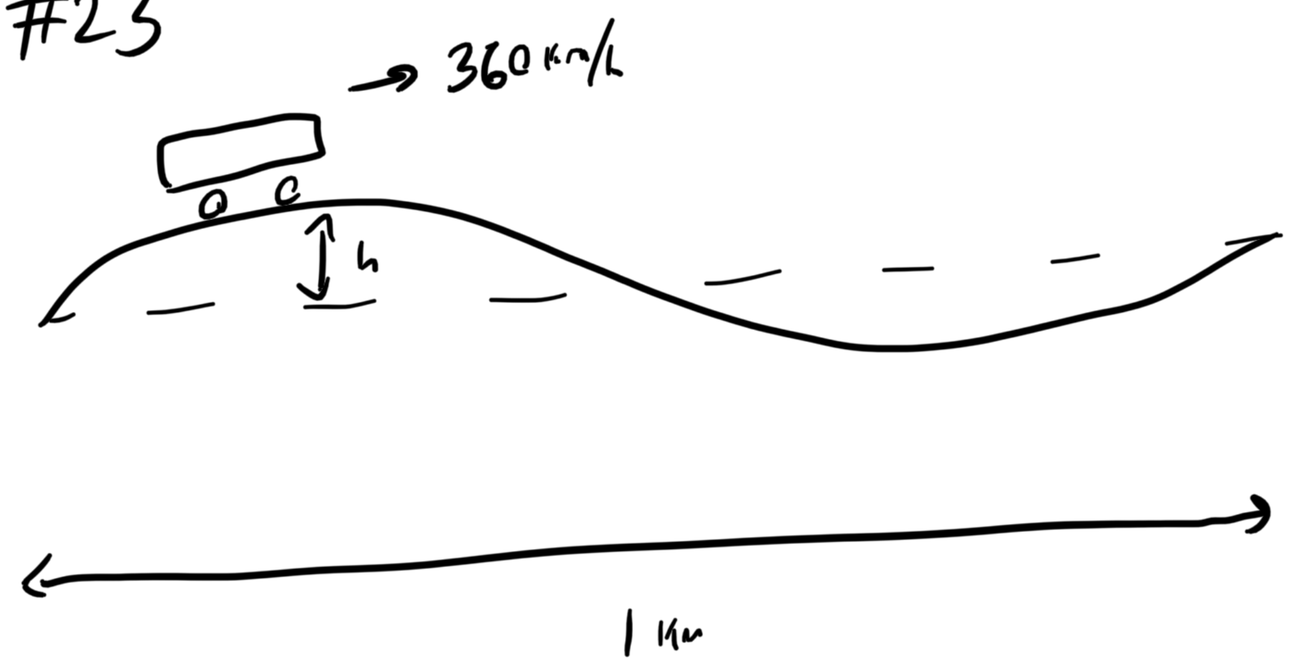


$|\Sigma F| \sim |T|$

$= MR\omega^2 = 20N$

$$\frac{F_d}{T} = \frac{r}{R} = \frac{3cm}{2m} = \frac{0.3N}{20N}$$

#23



$a_{max} = 0.1\ m/s^2 \Rightarrow h = ?$

$A = h$

$T = \frac{1\ km}{360\ rev/hr} = 10\ s$

$\Rightarrow$ spring system

$T = 2\pi\sqrt{m/k} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$a = \frac{F}{m} = \frac{kA}{m} = \frac{4\pi^2 h}{T^2}$

$h \leq \frac{T^2}{4\pi^2} a_{max}$

$= 0.25\ m$

(B)