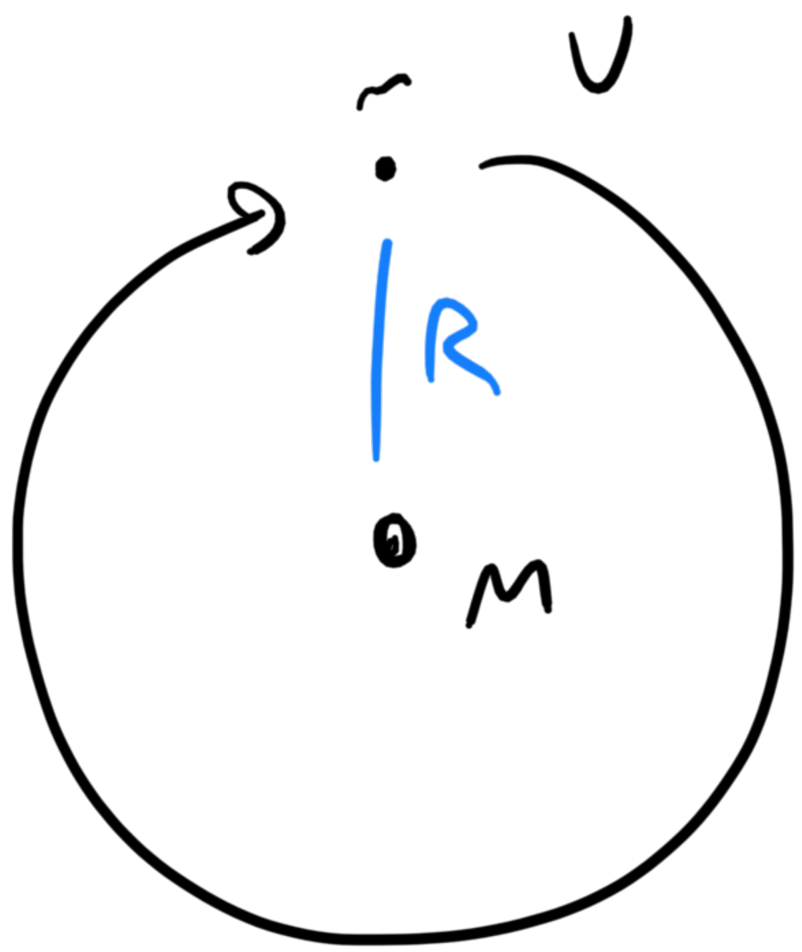
Orbits



$$a_{cent} = v^2/R$$

$$= a_{grav} = GM/R^2$$

for a given $R \Rightarrow v = \sqrt{GM/R}$

what's the period of this orbit?

each orbit is $2\pi R$,

$$T = 2\pi R/v = 2\pi\sqrt{R^3/GM}$$

(K3L: $T^2 \propto R^3$)

Other forces?

Stable when $F \propto R^{-2}$

$F = qR^{-2}$ as a general form.

$qR^{-2} = F = mv^2/R \Rightarrow \boxed{v = \sqrt{q/mR}}$

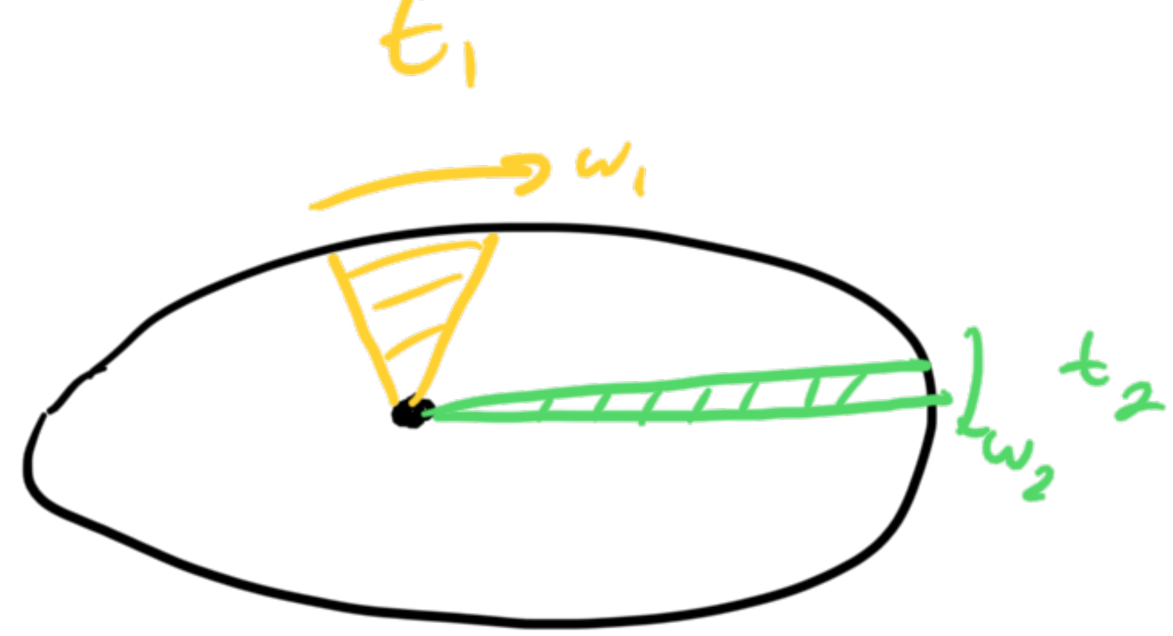
Gravity, $q = GMm \Rightarrow v = \sqrt{GM/R}$

Kepler's Laws & Non-circular orbits.

I. all bound orbits are elliptical w/ orbited body at a focus.

Corollary: circular orbits are elliptical w/ 2 foci at same point in space.

II. a line joining orbiter and orbited sweep equal area in equal time.



$T \cdot \frac{r^2}{2} \, d\theta/dt = \pi a b$ (if you know some calculus)

III. $T^2 \propto R^3$ for a circular orbit.

Generalizes to elliptical orbits w/o difficulty (HW).

Non-circular orbits

$\neq \frac{\sqrt{GM}}{R}$

Range of v:

$0 \to \sqrt{GM/R}$ elliptical orbit (R = apogee, max distance)

$\sqrt{GM/R}$ circular orbit

$\sqrt{GM/R} \to \sqrt{2GM/R}$ elliptical orbit (R = perigree, min distance)

$\sqrt{2GM/R}$ parabolic orbit (escape speed)

$\sqrt{2GM/R} \to$ "∞" hyperbolic orbit

2017 #1

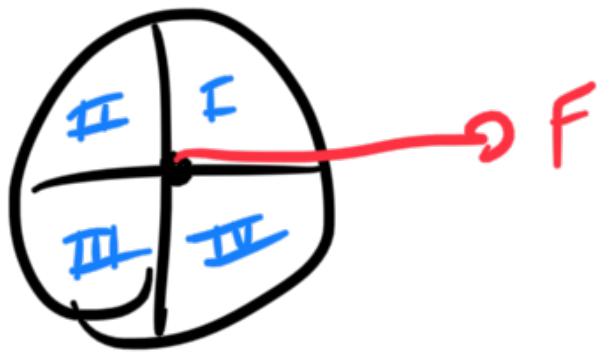
$$N = F_{cent} = \frac{ms^2}{R} \propto s^2$$

$$F_\mu = \mu N = mg \propto s^2$$

$$\mu \propto s^{-2} \quad \text{since} \quad mg = \text{cst.}$$

D

2017 #18



where is there a point with zero acceleration?

$\vec{a} = -(\vec{a}_c + \vec{a}_t)$

a_t = tangential accel.

a_c = centripetal

a = translational

~~II~~ ~~I~~

~~III~~ IV

Rolling w/o slipping, $\|\vec{a}\| = \alpha R$

since $r \in [0, R] \Rightarrow$ there is some

r s.t. $\|\vec{a}_c + \vec{a}_t\| = \alpha R$

$\Rightarrow a_{net} = 0.$ (!)

D

Step 1: equal magnitudes.

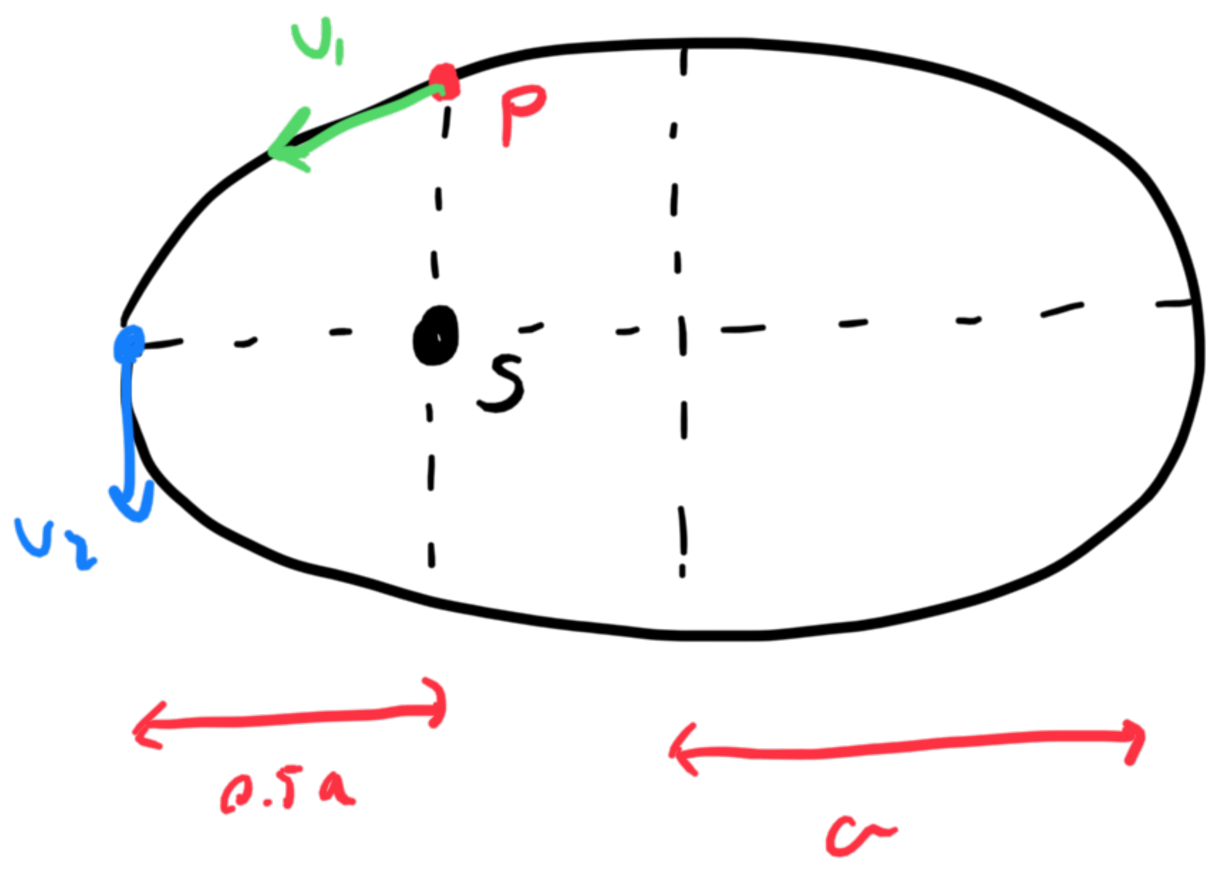
$\|a_c + a_t\| = \sqrt{\|a_c\|^2 + \|a_t\|^2} = r\sqrt{\omega^4 + \alpha^2}$

HW

$\alpha R = r\sqrt{\omega^4 + \alpha^2}$

$r = \frac{\alpha R}{\sqrt{\omega^4 + \alpha^2}} = \frac{R}{\sqrt{\alpha^2 t^4 + 1}}$

2017 #25



$v_2 = ?$

$$SP + \sqrt{SP^2 + a^2} = 2a$$

$$\Rightarrow SP = 3a/4$$

v_3 = speed at apogee

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_3^2 - \frac{GMm}{3a/2}$$

$$\frac{m v_2 a}{2} = \frac{3 m v_3 a}{2} \Rightarrow v_3 = \frac{1}{3} v_2$$

$$\frac{4}{9} m v_2^2 = \frac{GMm}{a/2} - \frac{GMm}{3a/2} = \frac{4GMm}{3a}$$

$$\Rightarrow v_2^2 = \frac{GMm}{a}$$

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_1^2 - \frac{GMm}{3a/4}$$

$$v_1^2 = v_2^2 - \frac{4GM}{a} + \frac{8GM}{3a}$$

$$= GM\left(\frac{3}{a} - \frac{4}{a} + \frac{8}{3a}\right) = \frac{5GM}{3a}$$

$$\frac{v_2^2}{v_1^2} = \frac{3GM/a}{\frac{5}{3} GM/a} = \frac{9}{5} \Rightarrow \boxed{v_2 = \frac{3}{\sqrt{5}} v_1}$$

A

α = angle btwn helix and horizontal

2016 #4

$a_w = g \sin\alpha$

$a_c = v_h^2/r$

$a_{w_v} = a_w \sin\alpha = g \sin^2\alpha$

$a_{w_h} = a_w \cos\alpha = g \sin\alpha \cos\alpha$

~~A~~

~~B~~

~~C~~

D

E

$a = \sqrt{a_w^2 + a_c^2}$

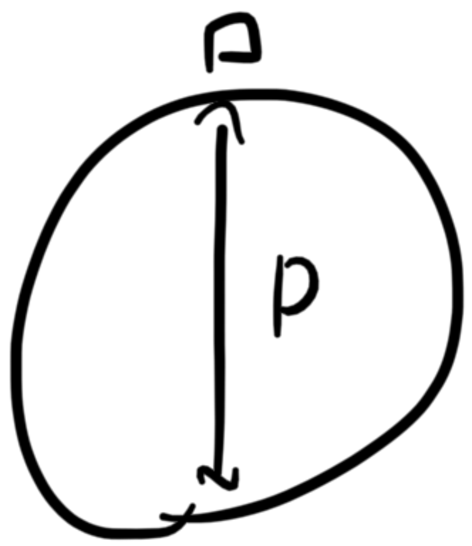
$a_c = \frac{v_h^2}{r} = \frac{(a_h t)^2}{r} = \frac{(g \sin\alpha \cos\alpha)^2 t^2}{r}$

$a = \sqrt{P + Qt^4} \Rightarrow$ non-linear in t

2016 #8

II = cons. of angular momentum

2016 #9



$mgh = \frac{mv^2}{2}$ cons. energy

$h = R(1-\cos\theta)$ geometry

$mg\cos\theta \geq \frac{mv^2}{R}$ contact forces

$\cos\theta = v^2/gR \Rightarrow v = \sqrt{\frac{2gR}{3}} = \sqrt{\frac{gD}{3}}$ E