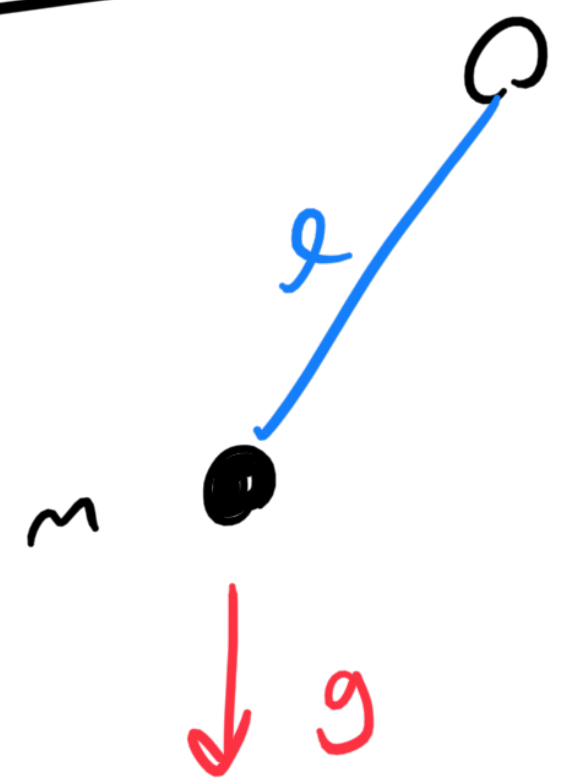
Dimensional Analysis



$T = ?$

$[T] = s$

$[g] = m/s^2$

$[\ell] = m$

A. $\sqrt{g/\ell}$

B. $\sqrt{\ell/g}$

~~C.~~ $\sqrt{mg/\ell}$

~~D.~~ $\sqrt{\ell/mg}$

~~E.~~ $\sqrt{g\ell}$

$$\left[\sqrt{\frac{g}{\ell}}\right] = \sqrt{\frac{m/s^2}{m}} = s^{-1}$$

$$\left[\sqrt{\frac{\ell}{g}}\right] = \sqrt{\frac{m}{m/s^2}} = s$$

2018 B #6

$$X = \frac{F}{I} E^{\alpha} L^{\beta} = \frac{F}{IE} L^{\beta}$$

$[F] = N = \cancel{kg\,m\,s^{-2}}$

$[I] = m^4$

$[E] = N m^{-2} = \cancel{kg\,m^{-1}\,s^{-2}}$ $\Rightarrow$ Newtons must cancel, $\alpha = -1$

$[L] = m$

$$\left[\frac{F}{IE}\right] = \frac{N}{m^4 N m^{-2}} = \frac{1}{m^2}$$

$$X = \left[\frac{1}{m^2}\right] L^{\beta} \Rightarrow \beta = 3 \Rightarrow$$ D

Simple Harmonic Oscillators

$$\vec{F}(x) = -kx$$

$$F(x) = -kx = ma$$

$$-kx = m\ddot{x} = m\frac{d^2x}{dt^2}$$

$$x = A\cos(\omega t + \phi)$$

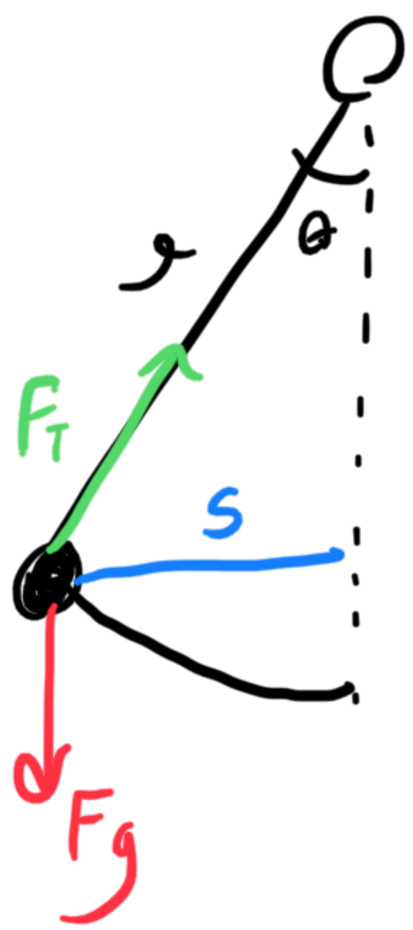
A = max displacement.

$$\omega = \sqrt{\frac{k}{m}}$$

$$E_{osc} = \frac{1}{2}kA^2$$

If $F=-kx$ is from a spring, this looks like the familiar result.

Pendulums & Small Angle Approx.



$$F_{T\theta} = F_T \sin\theta$$

Small angle approx: $\sin\theta \approx \theta$

$$F_{T\theta} = F_T\theta = -F_g\theta$$

$$F_{T\theta} \approx -F_g \frac{s}{\ell}$$

$$F_{T\theta}(s) \propto -s$$

Restoring force!

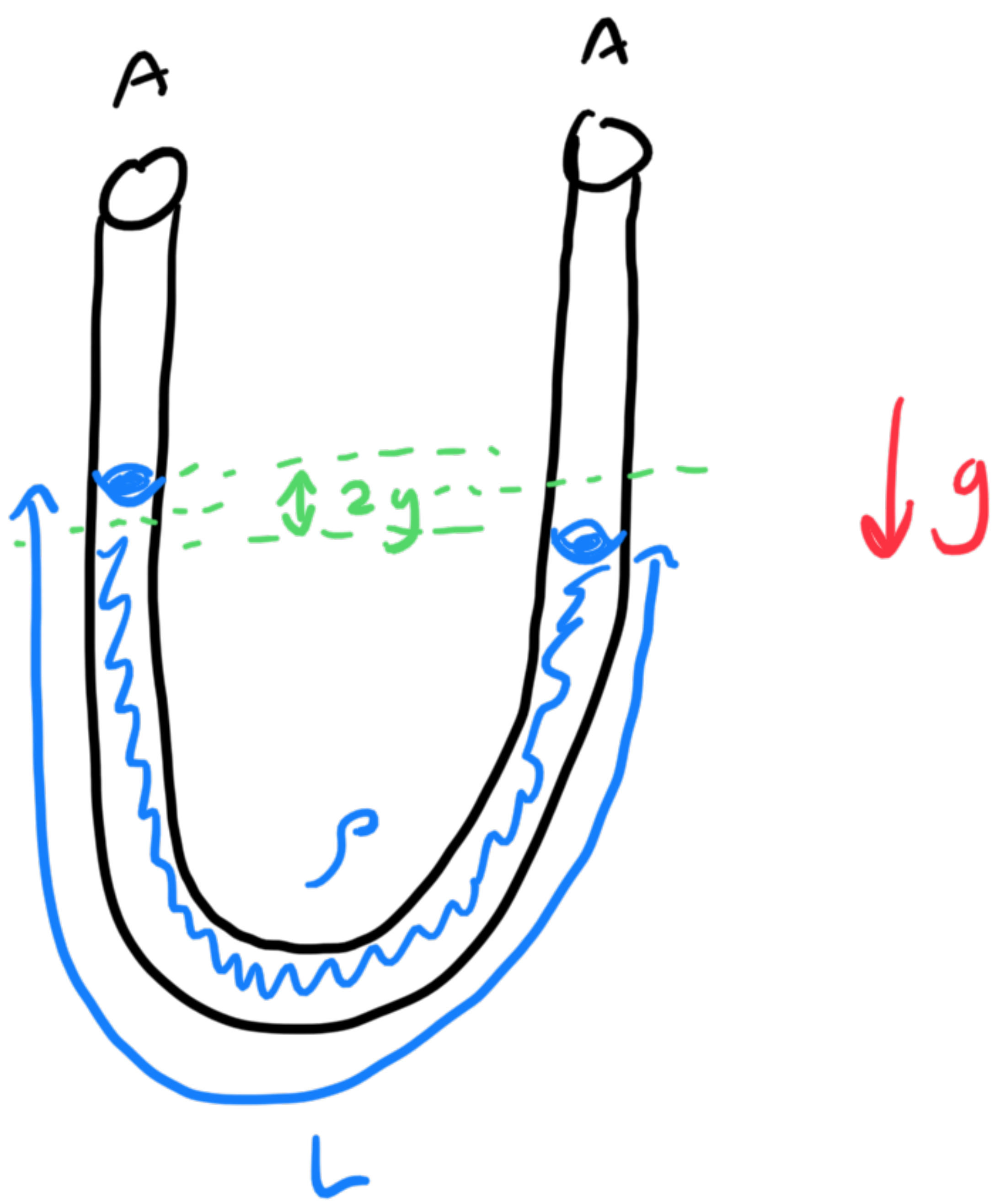
$$\omega = \sqrt{k/m}$$

$$F(s) = -ks$$

$$k = mg/\ell$$

$$m = m$$

$$\omega = \sqrt{\frac{mg/\ell}{m}} = \sqrt{\frac{g}{\ell}}$$



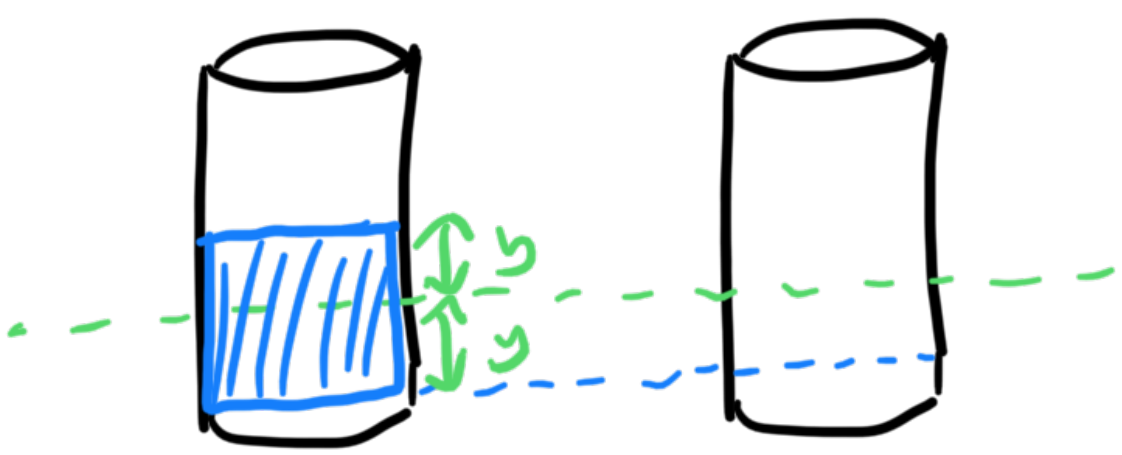
$\omega = ?$

$$F = m_{drop}\, g = 2\rho y A g = k y$$

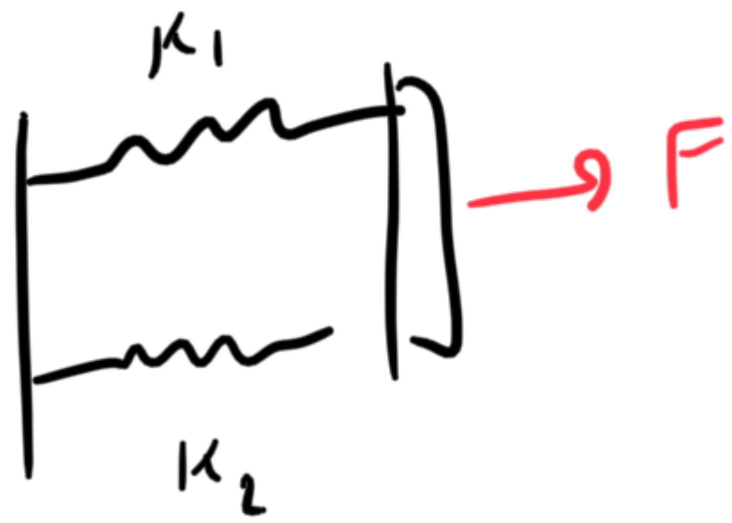
$$k = 2\rho A g$$

$$m = \rho L A$$

$$\boxed{\omega = \sqrt{k/m} = \sqrt{\frac{2\rho A g}{\rho L A}} = \sqrt{\frac{2g}{L}}}$$

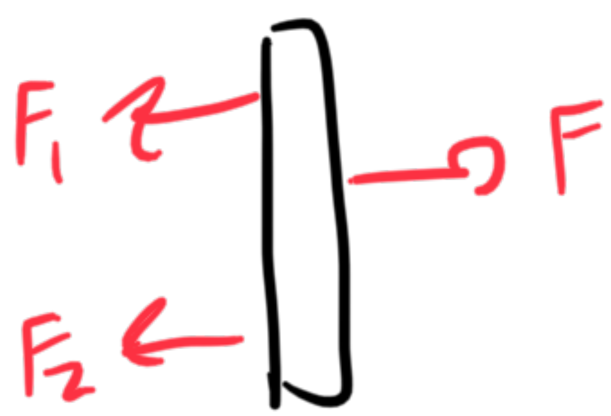


2018 B # 18



$$F = -kx$$

$$x = -\frac{F}{k}$$



$$F_1 = k_1 x$$

$$F_2 = k_2 x$$

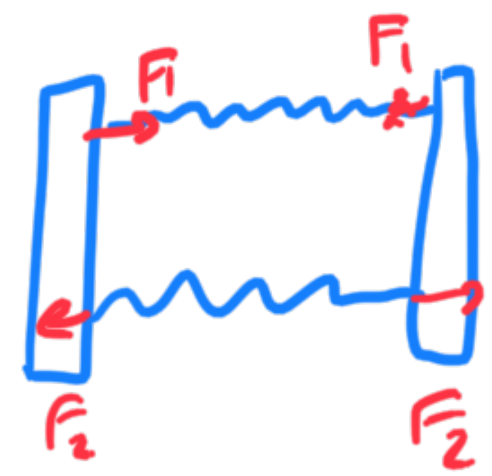
$$F = F_1 + F_2 = k_1 x + k_2 x$$
$$= (k_1 + k_2)x$$

$$F_{net} = 0 = k_1(\ell - \ell_1) + k_2(\ell - \ell_2)$$

$$k_1\ell_1 + k_2\ell_2 = (k_1 + k_2)\ell$$

$$\ell = \frac{k_1\ell_1 + k_2\ell_2}{k_1 + k_2}$$

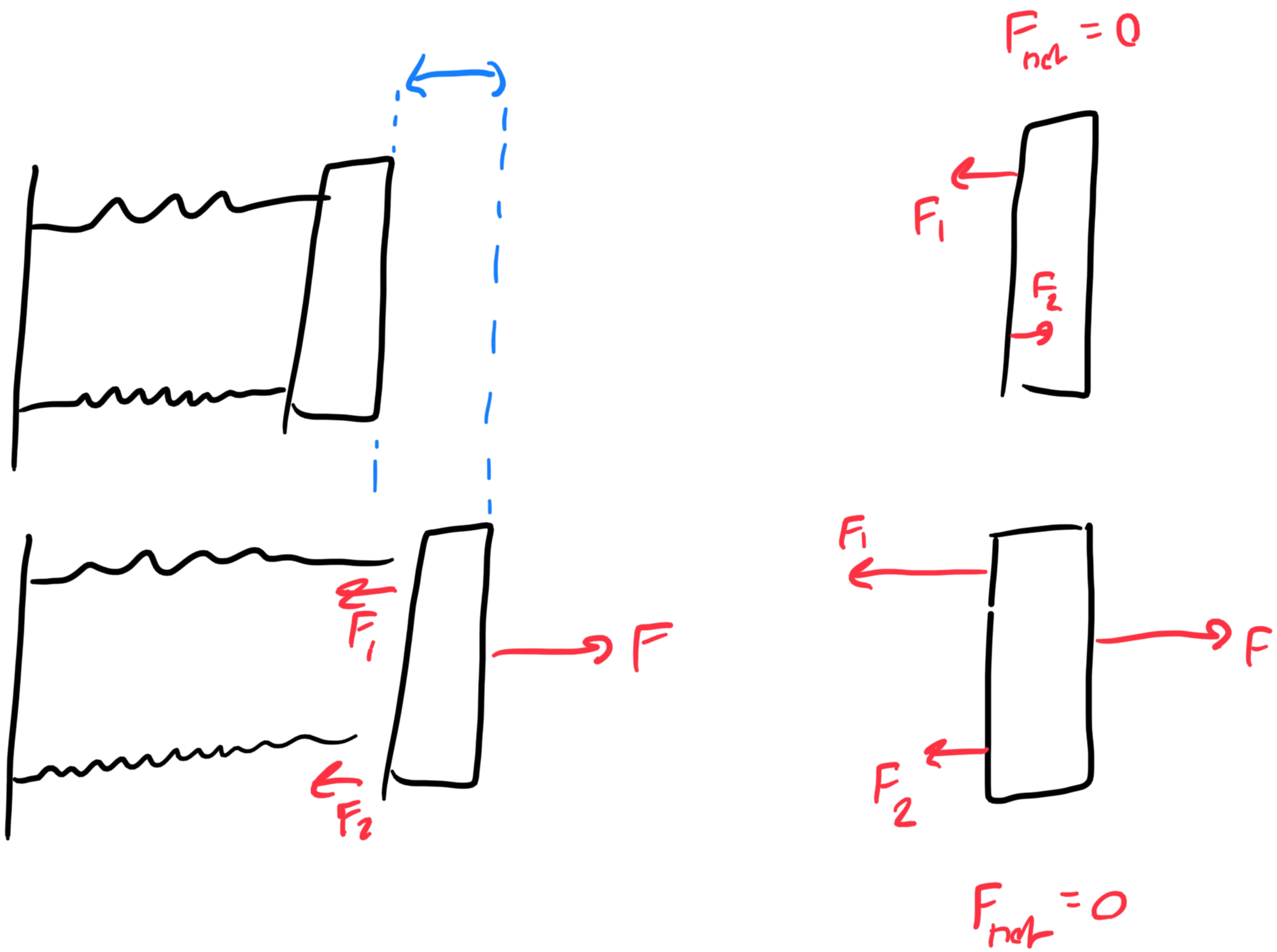
B



$$\vec{F}_1 + \vec{F}_2 = 0$$

$$\vec{F}_1 = k_1(\ell - \ell_1)$$

$$\vec{F}_2 = k_2(\ell - \ell_2)$$

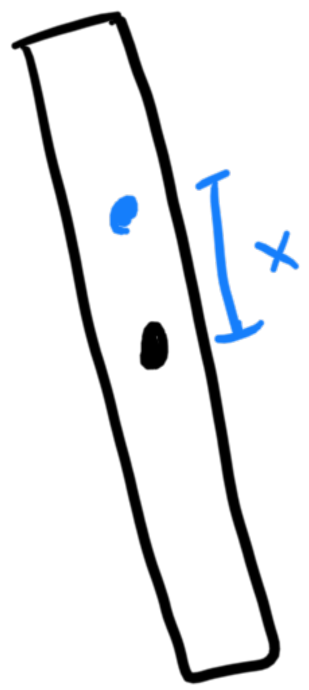


$$\Delta F_1 = -k_1 \Delta x$$

$$\Delta F_2 = -k_2 \Delta x$$

$$\Delta F = -(k_1 + k_2) \Delta x$$

2018B #21



same derivation

$$T = 2\pi\sqrt{\frac{I}{mgx}}$$

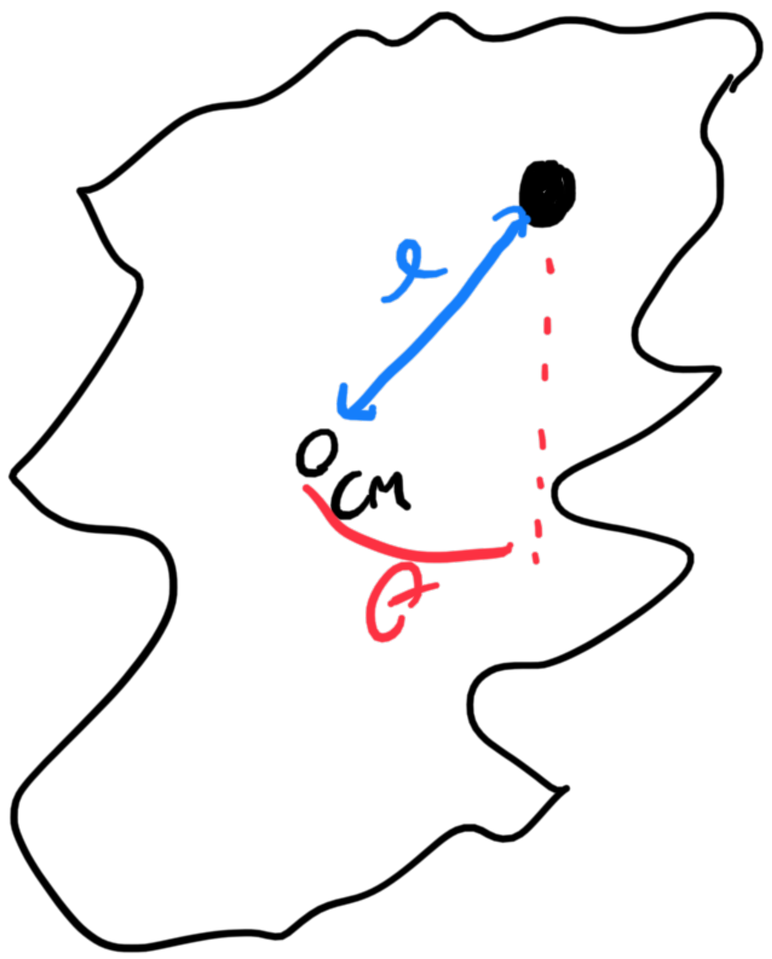
$$I = \frac{1}{12}mL^2 + mx^2$$

minimize $$I/mx = x + \left(\frac{L^2}{12}\right)x^{-1}$$

$c\left(z+\frac{1}{z}\right)$ minimized when $z=1$

$$z = \frac{x}{L/(2\sqrt{3})}$$

$$x = L/2\sqrt{3}$$



I_{cm} given

$I_{rot} = I_{cm} + Ml^2$

$\tau = I_{rot}\alpha = -F_g l \sin\theta$

$= -F_g l \theta$

$= -mgl\theta$

$ma = -kx$

$m\ddot{x} = -kx$

$I\alpha = -\kappa\theta$

$I\ddot{\theta} = -\kappa\theta$

$I_{rot}\alpha = -mgl\theta$

$\omega = \sqrt{\frac{mgl}{I_{rot}}}$