Error Propagation

$x \pm \Delta x, \quad y \pm \Delta y$

Additive

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Multiplicative

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y \Delta x)^2}$$

Power

$$\Delta(x^a) = |a| \, x^{a-1} \Delta x$$

Multiple trials or weighted results

Xavier: $x \pm \Delta x$

Yvonne: $y \pm \Delta y$

Want to weight Yvonne's results twice as much

$$\Delta(x, 2y) = \frac{1}{\sqrt{3}} \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta y)^2}$$

$$= \frac{1}{\sqrt{3}} \sqrt{(\Delta x)^2 + 2(\Delta y)^2}$$

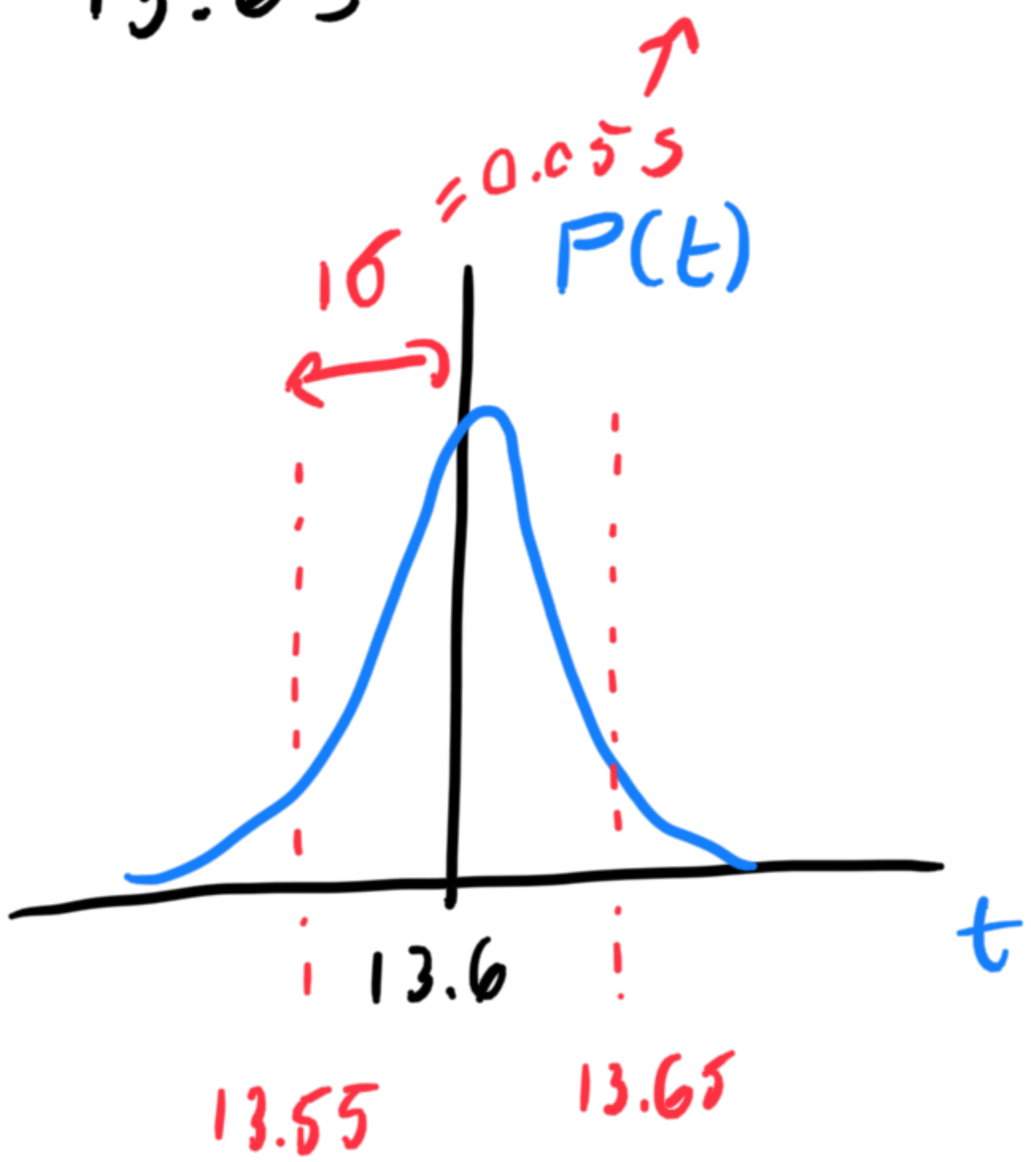
$t \pm \Delta t$

trial

$\boxed{13.6\,s}$

quote as

$13.6\,s \pm 0.05\,s$



Relative uncertainties

two uncertain quantities multiplied or divided,

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

Ex.

measuring g with a pendulum.

$L \pm \Delta L$, $\quad T \pm \Delta T$

$g \pm \Delta g = ?$

$g = 4\pi^2 L / T^2$

$$\Delta(T^{-2}) = |-2| \, T^{(-2-1)} \Delta T$$

$$= 2T^{-3} \Delta T$$

$$\Delta(LT^{-2}) = \sqrt{[L\Delta(T^{-2})]^2 + [(T^{-2})\Delta L]^2}$$

$$= \sqrt{[2LT^{-3}\Delta T]^2 + [(T^{-2})\Delta L]^2}$$

$$\Delta g = 4\pi^2 \Delta(LT^{-2})$$

$$\boxed{\Delta g = 4\pi^2 \sqrt{[2LT^{-3}\Delta T]^2 + [(T^{-2})\Delta L]^2}}$$

2018 B #19

$$\Delta v = \Delta\left(\frac{x}{t}\right)$$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

$x \pm \Delta x = 75 \pm 2.0$ cm

$t \pm \Delta t = 2.15 \pm 0.1$ ms

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{2}{75}\right)^2 + \left(\frac{0.1}{2.15}\right)^2} = 0.054$$

$$\Delta v = (349\ \text{m/s})(0.054)$$

$$= 19\ \text{m/s}$$

E

2018B #25

$R \pm \Delta R = 1 \pm 0.1\ \text{cm}$

$L \pm \Delta L = 1.00 \pm 0.01\ \text{m}$

$\frac{\Delta R}{R} = 10\%$

$\frac{\Delta L}{L} = 1\%$

(1) $\sqrt{(1\%)^2 + (1\%)^2} \simeq 1.4\%$

(2) $\sqrt{(10\%)^2 + (1\%)^2} \simeq 10\%$

(3) $\frac{1}{\sqrt{10}} \sqrt{(10\%)^2 + (1\%)^2} \simeq 3.3\%$

D

Rotations

rotation	relation	translation
$\vec{\tau}$	$\vec{\tau} = \vec{r} \times \vec{F}$, $\tau = r_\perp F = r F_\perp$	$\vec{F}$
$\vec{\alpha}$	$\alpha = \frac{a}{r}$ $\quad a = \alpha r$	$\vec{a}$
$\vec{\omega}$	$\omega = \frac{v}{r}$ $\quad v = \omega r$	$\vec{v}$
$\vec{\theta}$	$\theta = \frac{x}{r}$ $\quad x = \theta r$	$\vec{x}$
I	$I \propto mr^2$	m
K	$K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$	K

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ $\qquad$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

2018B #5

$$\omega_f^2 - \omega_i^2 = 2\alpha\theta$$

$$v_f^2 - v_i^2 = 2ax$$

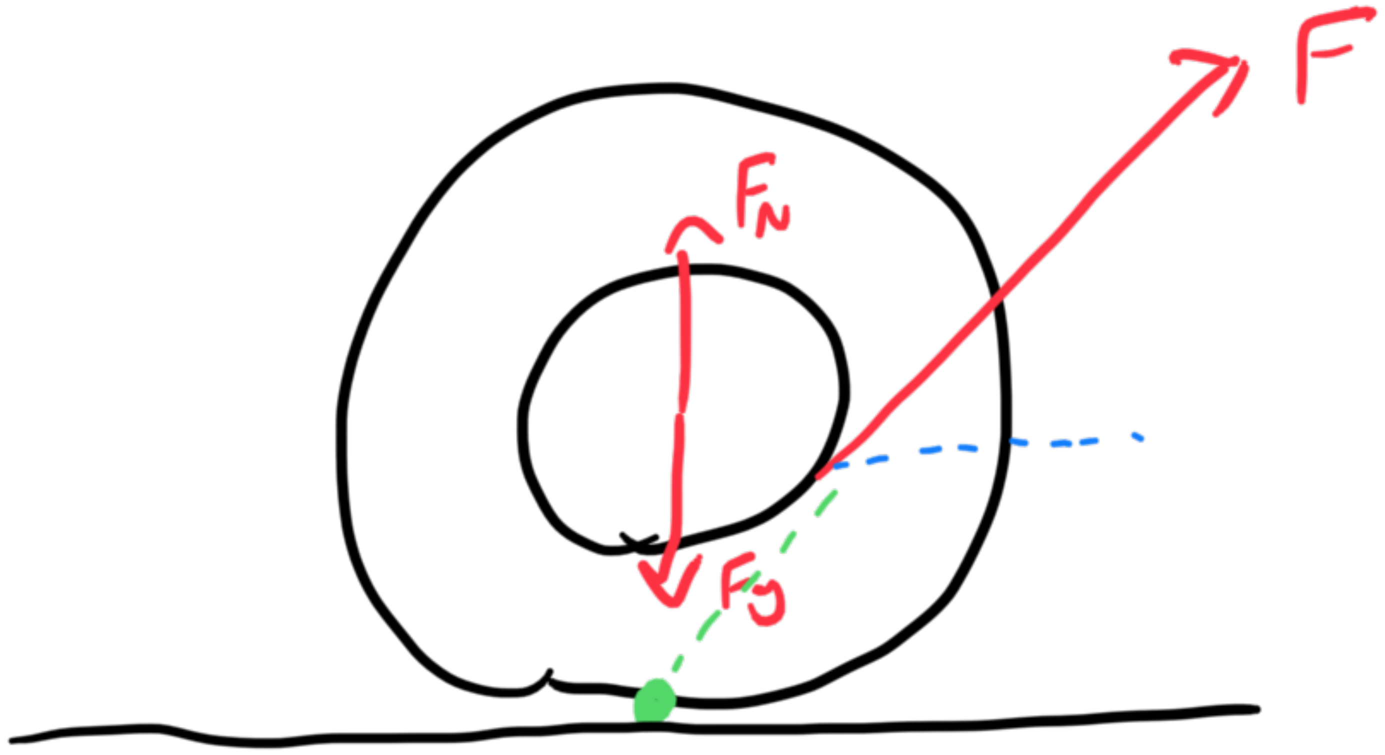
speed up slow down

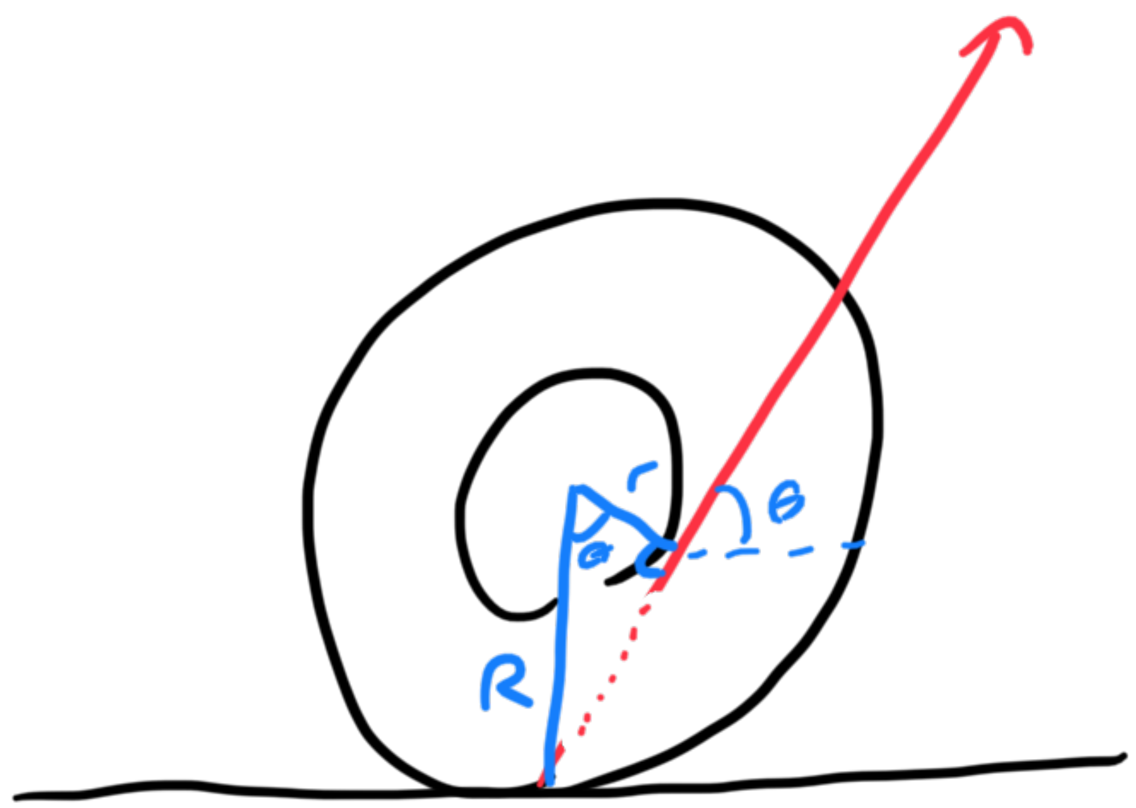
$$2\alpha_+\theta_+ = 2\alpha_-\theta_-$$

$$\frac{\theta_+}{\theta_-} = \frac{1}{5} \qquad \frac{\alpha_+}{\alpha_-} = 5$$

D

2018B #14



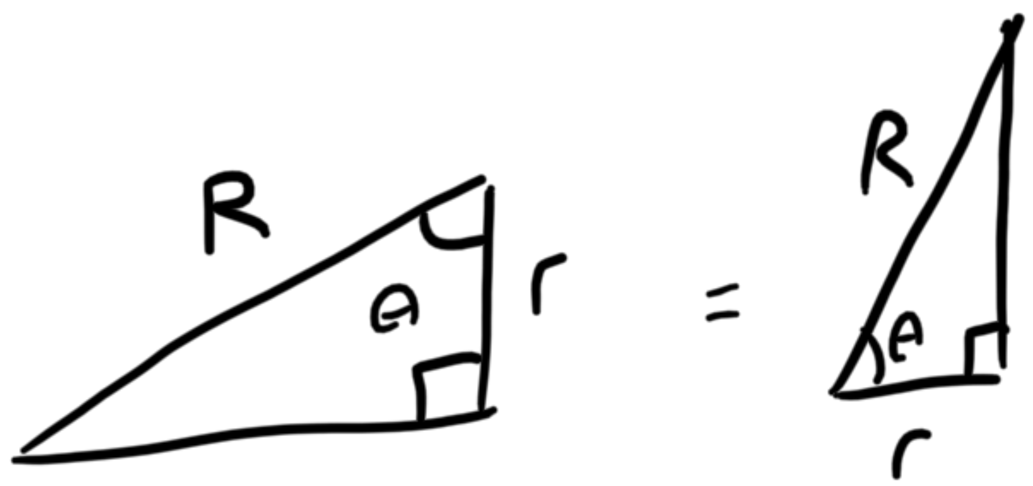


$\tau_g = \tau_N = 0$

$\tau_F = 0$ must be so there is no net torque.

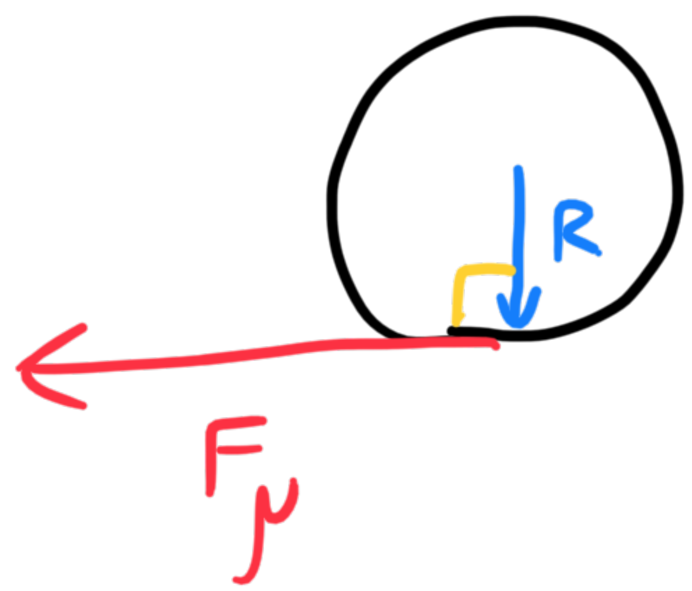
$\cos\theta = \frac{r}{R} \Rightarrow \theta = 41.4°$

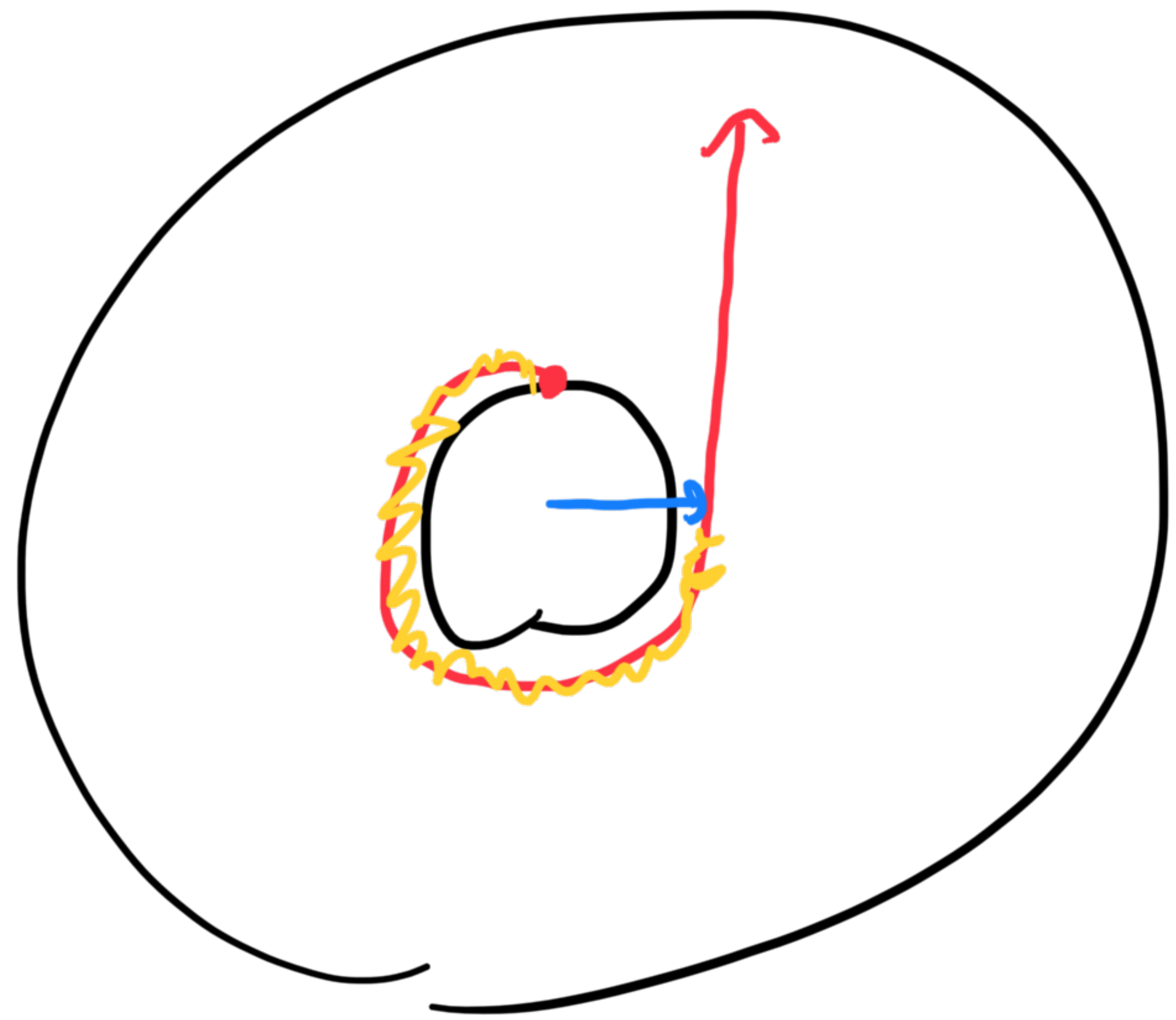
B



$r = R\cos\theta$

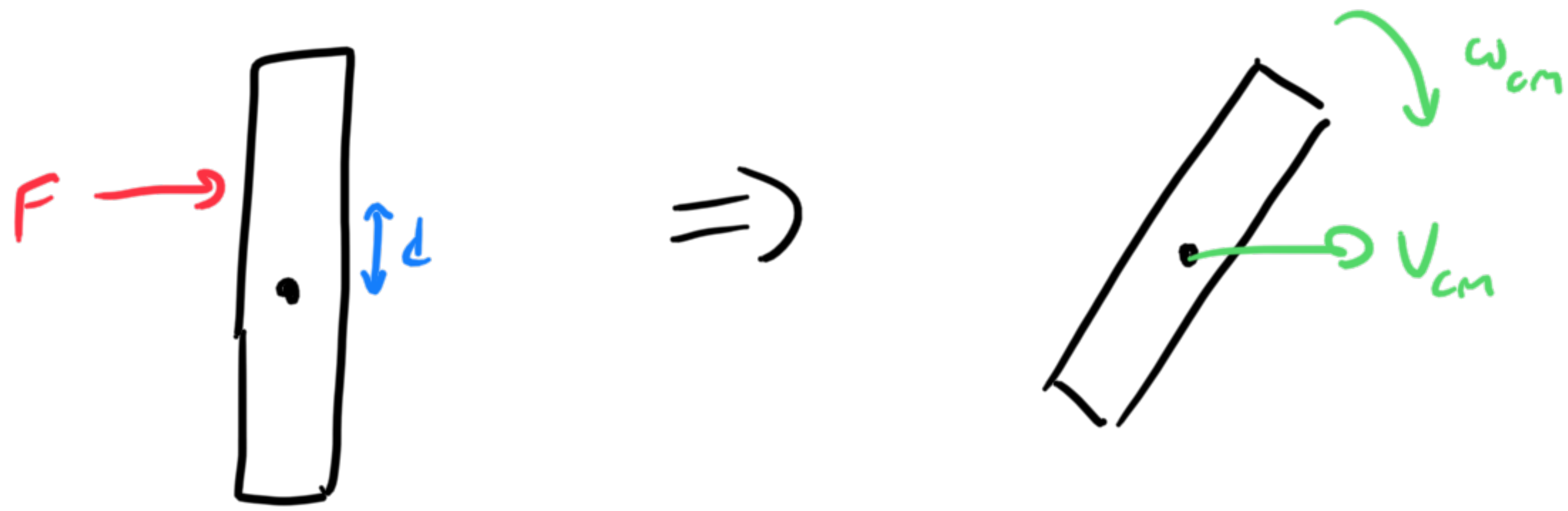
$\cos\theta = \frac{r}{R}$





See 2018A #8

Week 3 (2024)



Force applied for a fixed Δt.

$$F = ma = \frac{\Delta p}{\Delta t} \Rightarrow F\Delta t = J = \Delta p = mv_{cm}$$

$$\boxed{v_{cm} = F\Delta t/m}$$

$$J_\tau = F\Delta t\, d = I\alpha\Delta t = I\omega_{cm}$$

$$= \frac{1}{12}ML^2\omega_{cm}$$

$$\boxed{\omega_{cm} = 12\frac{Jd}{ML^2} = 12\frac{F\Delta t\, d}{ML^2}}$$

$$E = F\Delta y = J\frac{\Delta y}{\Delta t}$$

Δt small, $\Delta y \simeq d\Delta\theta + \Delta x$

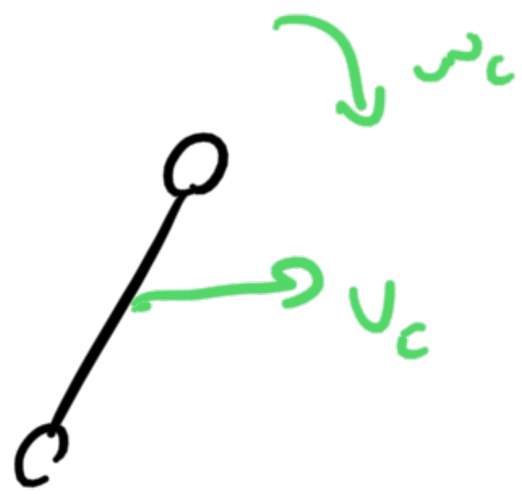
$$\alpha = \frac{Jd}{I\Delta t} \qquad a = \frac{J}{M\Delta t}$$

$$\Delta\theta = \frac{1}{2}\alpha\Delta t^2 = \frac{Jd\Delta t}{2I}$$

$$\Delta x = \frac{1}{2}a\Delta t^2 = \frac{J\Delta t}{2M}$$

$$\Delta y = \frac{J\Delta t}{2M}\left(1 + \frac{d^2}{L^2}\right) \Rightarrow \Delta E = \frac{J^3}{2M}\left(1 + \frac{d^3}{L^2}\right)$$

2018B #23



$$P = (m_1+m_2)v_c \Rightarrow v_c = \frac{m_1 v}{m_1+m_2}$$

$$r_1 = \frac{m_2 L}{m_1+m_2} \qquad r_2 = \frac{m_1 L}{m_1+m_2}$$

$$v = v_c + r_1 \omega = \frac{m_1 v}{m_1+m_2} + \frac{m_2}{m_1+m_2} L\omega$$

initially $v_{m_2} = 0$

so $v_c + \vec{\omega} \times \vec{r_2} = 0$

$$|r_2 \omega| = |v_c|$$

$v_2 = 0$ when a whole rotation has been completed.

$$t = \frac{2\pi}{\omega} = 2\pi L / v$$

A