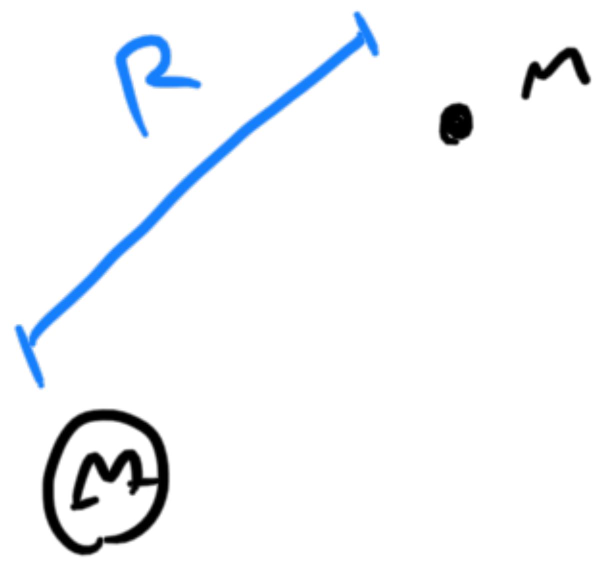
Orbits (Topic 7 in F=ma physics)



only force is gravity

what is the period of orbit?

$$m a_{cent} = \frac{mv^2}{R} = F_{\text{radially inwards}} = F_{grav} = \frac{GMm}{R^2}$$

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

each orbit is $2\pi R$ long so

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \boxed{2\pi\sqrt{\frac{R^3}{GM}} = T}$$

orbits are stable when governed by radially inwards forces which scale with R^{-2}.

$$F \propto R^{-2} \quad \text{or} \quad F = q R^{-2}$$

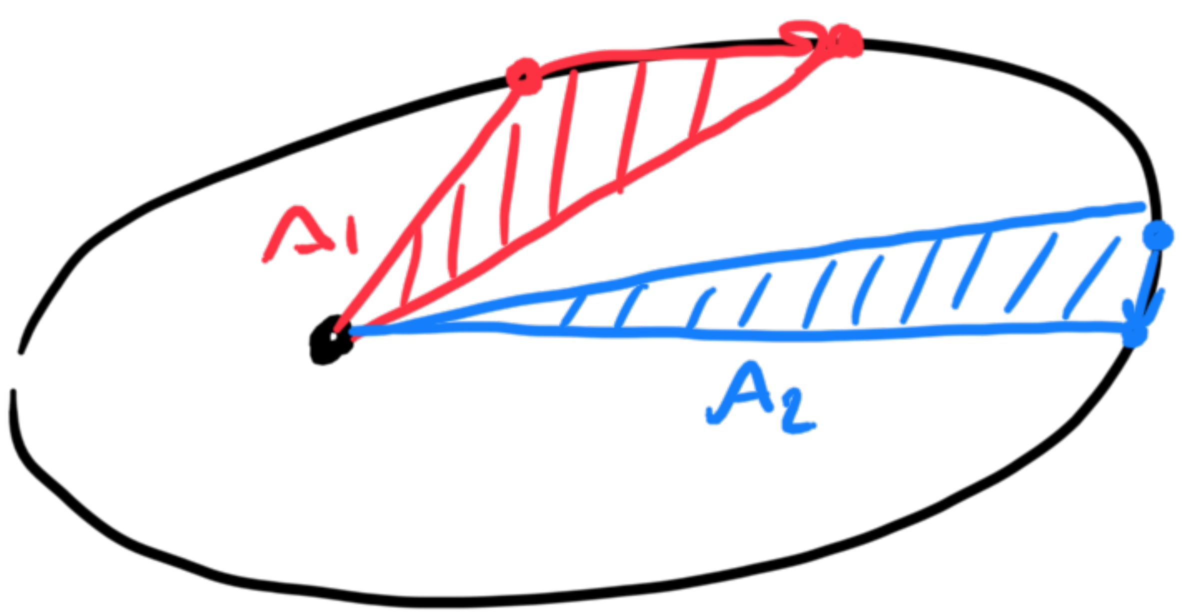
$$F = q R^{-2} = \frac{m v^2}{R} \Rightarrow v = \sqrt{\frac{q}{mR}}$$

For gravity, $q = GMm \Rightarrow v = \sqrt{\frac{GM}{R}}$

Kepler's Laws

I. all bound orbits are ellipses with the orbited body at a focus.

II. a line joining the orbiter and orbited sweeps equal areas in equal times.



$A_1 = A_2$

$t_1 = t_2$

$\dot{\theta}_1 > \dot{\theta}_2$

III. $T^2 \propto R^3$

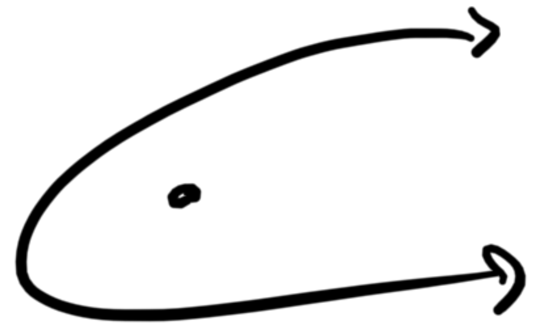
already derived for circular orbit above.

Non-circular orbits and unbound orbits.

$v/\sqrt{GM/R}$ (dimensionless)

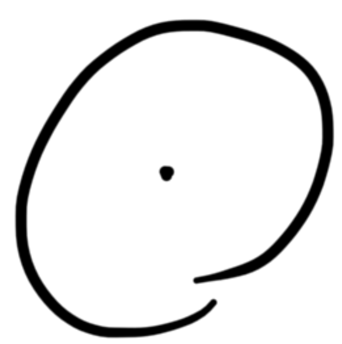
hyperbolic

$\sqrt{2}$ — parabolic orbit
"escape speed"



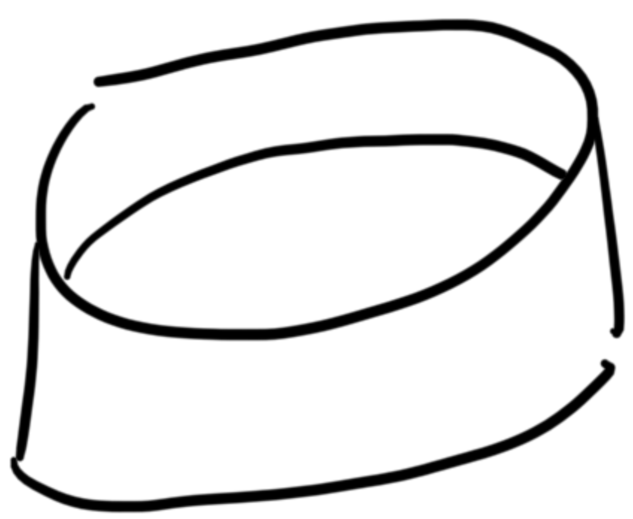
elliptical

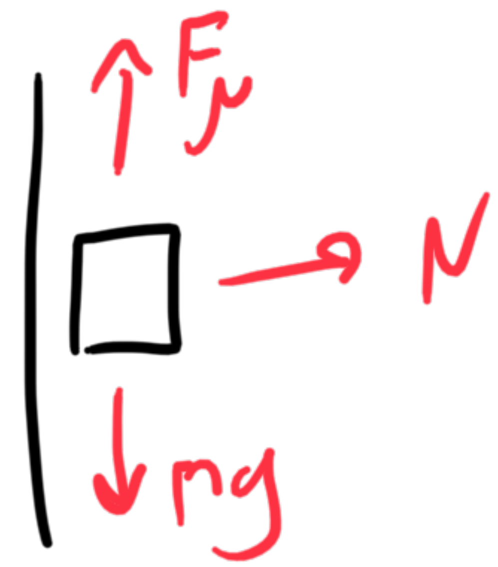
1 — circular orbit



elliptical

2017 #1

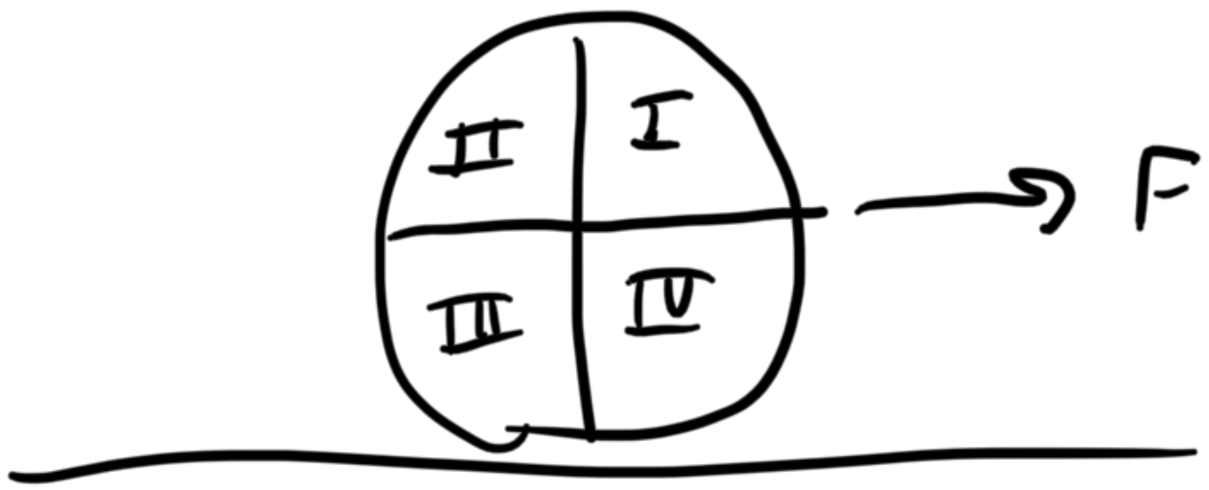




$$F_\mu = mg = \mu N = \mu \frac{ms^2}{R} \Rightarrow \mu \propto s^{-2}$$

$$N = F_{cent} = \frac{ms^2}{R}$$

2017 #18



3 accelerations

	IV
$\vec{a}_c$	↖
$\vec{a}_t$	↙
$\vec{a}_F$	→

any point moving with zero total acceleration?

can $||\vec{a}_c + \vec{a}_t|| = ||\vec{a}_F||$?

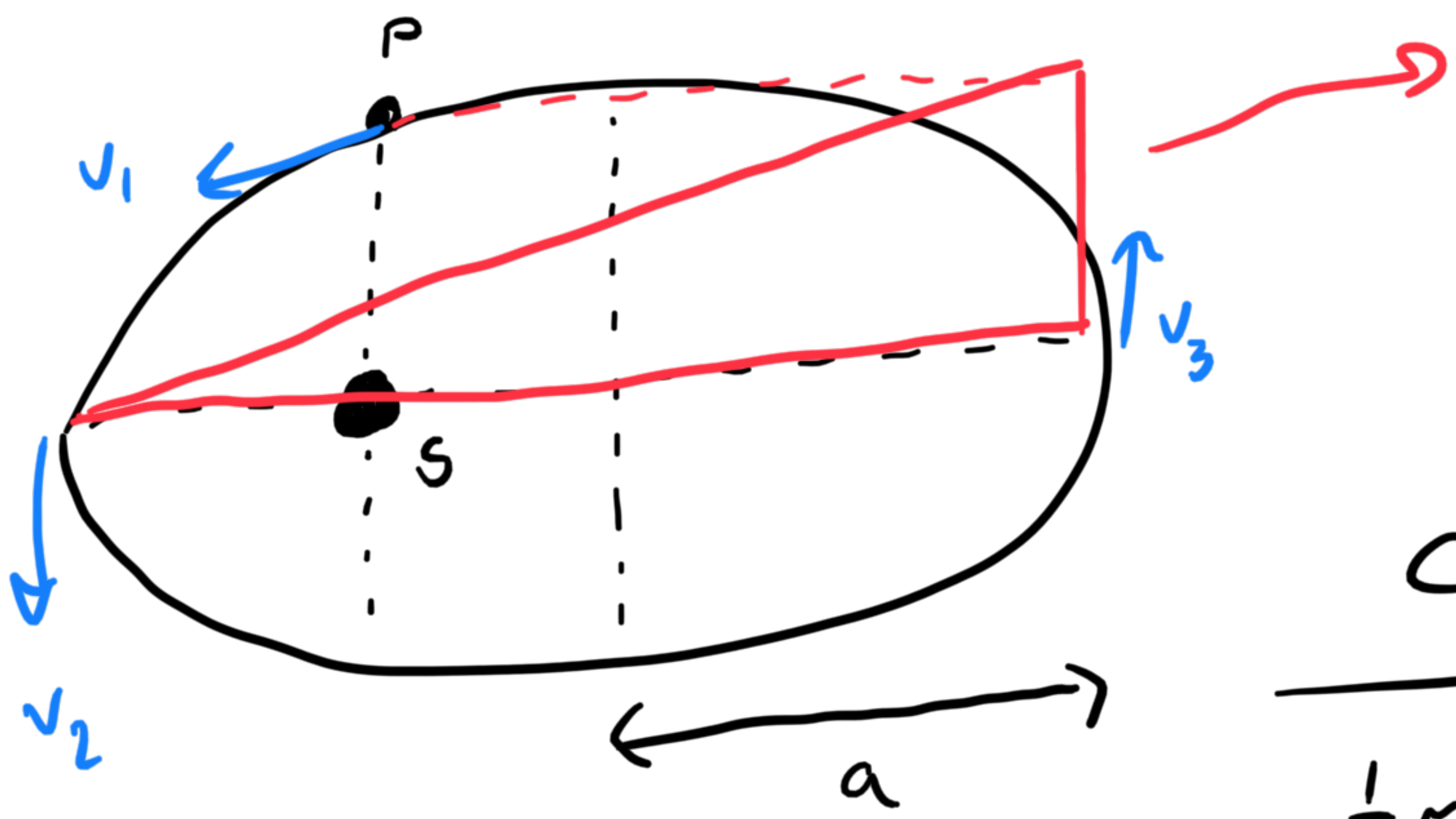
$$||\vec{a}_c + \vec{a}_t|| = \sqrt{a_c^2 + a_t^2} = r\sqrt{\omega^4 + \alpha^2}$$

$$||\vec{a}_F|| = \alpha R$$

IVT tells us that $r \in [0, R]$ s.t. $r\sqrt{\omega^4 + \alpha^2} = \alpha R$

so, magnitudes can be equal thus a point in IV exists!

2017 #25



$v_2 = ?$

How long is $\overline{SP}$?

$$\overline{SP} + \sqrt{\overline{SP}^2 + a^2} = 2a$$

$$\overline{SP} = 3a/4$$

Conservation of Energy

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_3^2 - \frac{GMm}{3a/2}$$

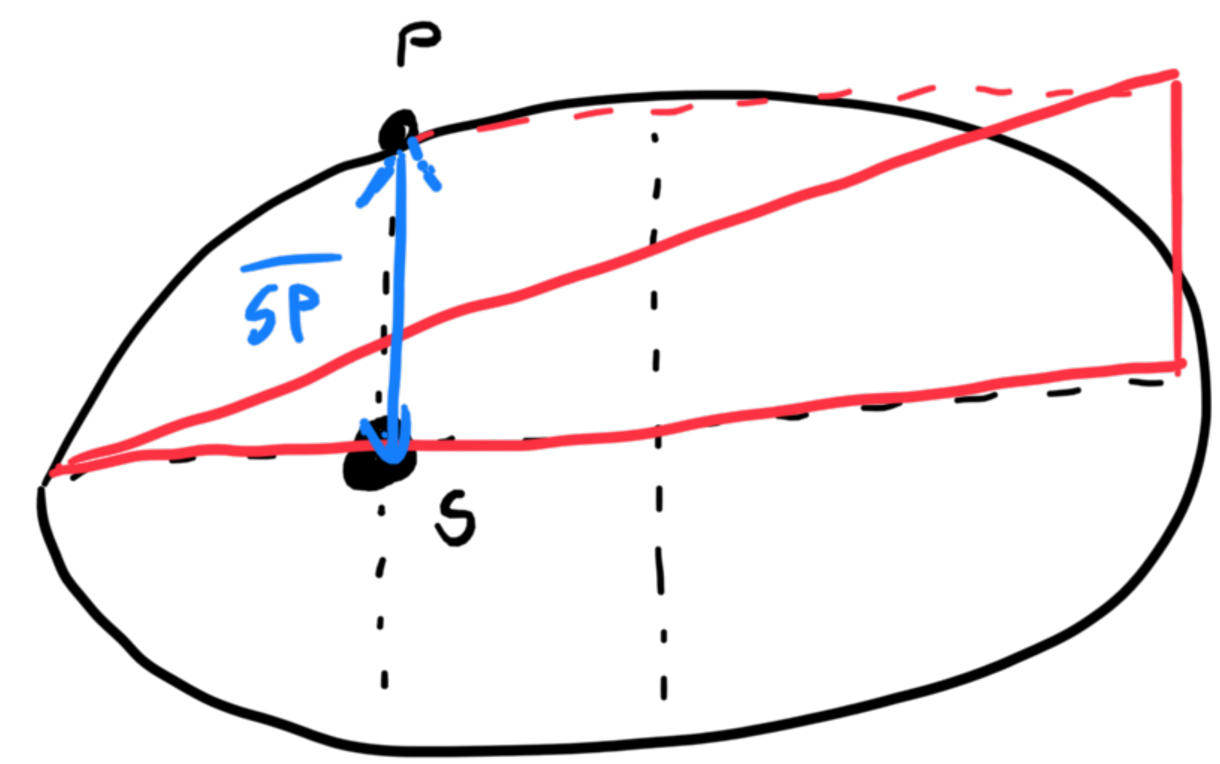
K2L: $\frac{m v_2 a}{2} = \frac{3 m v_3 a}{2} \Rightarrow v_3 = \frac{1}{3} v_2$

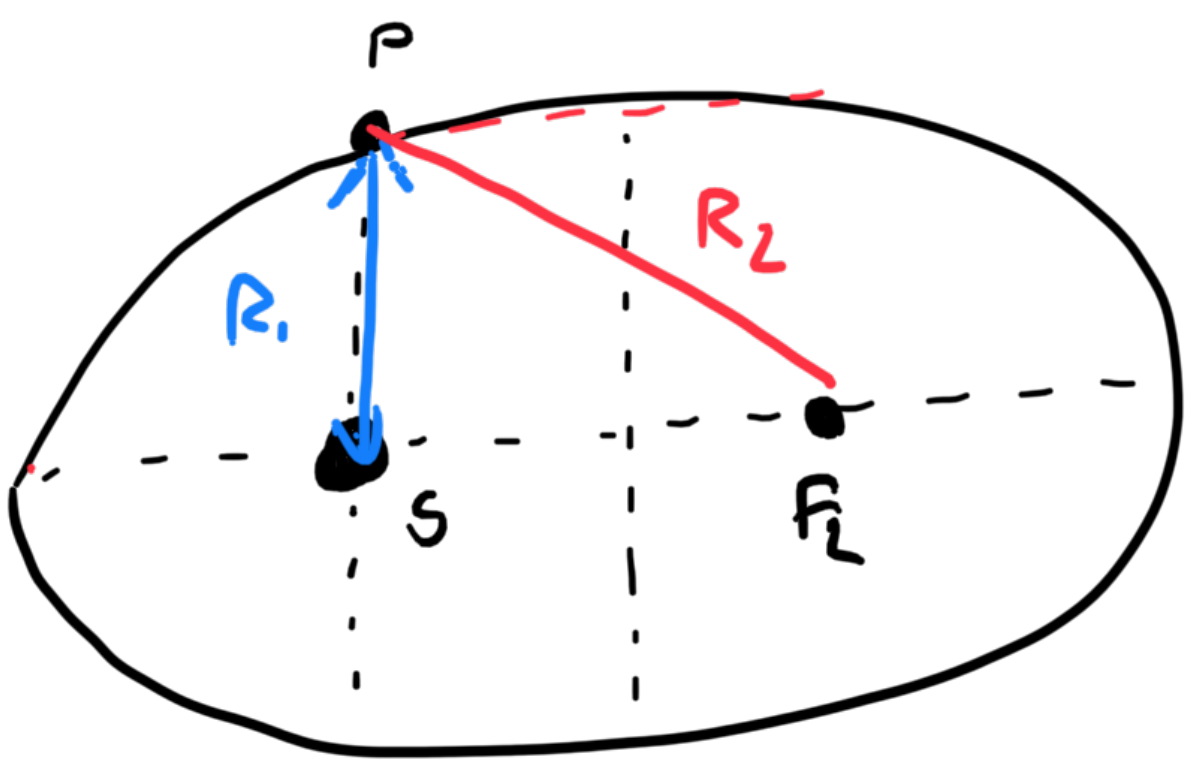
$$\frac{4}{9} m v_2^2 = \frac{4GMm}{3a} \Rightarrow v_2^2 = \frac{3GM}{a}$$

$$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_1^2 - \frac{GMm}{3a/4}$$

$$v_1^2 = v_2^2 - \frac{4GM}{a} + \frac{8GM}{3a} = \frac{5GM}{3a}$$

$$v_2^2/v_1^2 = 9/5 \Rightarrow v_2/v_1 = 3/\sqrt{5}$$





How to find $\overline{SP}$?

$$R_1 + R_2 = 2a$$

$$R_2^2 = R_1^2 + a^2$$

$$R_2 = \sqrt{R_1^2 + a^2}$$

$$R_1 + \sqrt{R_1^2 + a^2} = 2a$$

$$\sqrt{R_1^2 + a^2} = 2a - R_1$$

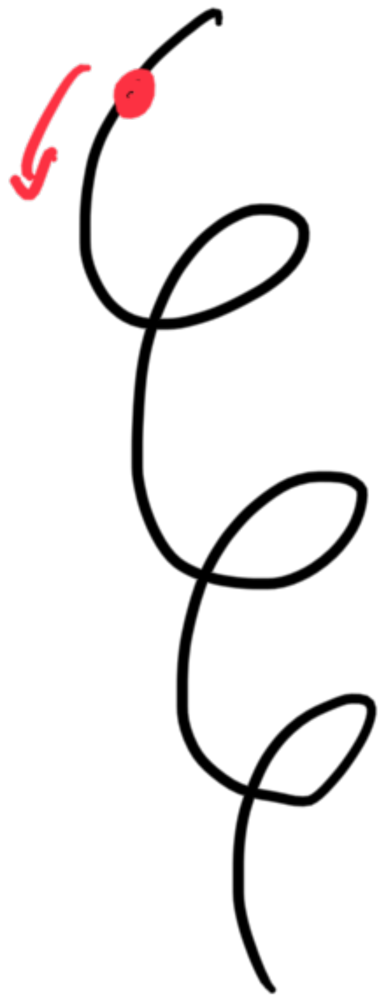
$$\cancel{R_1^2} + a^2 = 4a^2 - 4aR_1 + \cancel{R_1^2}$$

$$a^{\cancel{2}} = 4a^{\cancel{2}} - 4\cancel{a}R_1$$

$$a = 4a - 4R_1$$

$$4R_1 = 3a \Rightarrow R_1 = 3a/4 = \overline{SP}$$

2016 #4



eliminate A, B, C.

$a(t=0) \neq 0$ and $a(t) \neq$ cst.

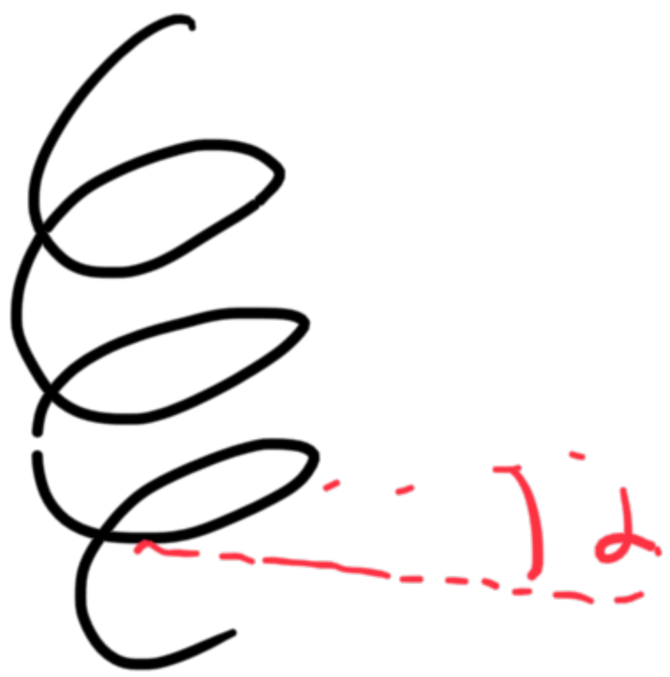
two components:

vertical, $a_v = g\sin^2\alpha$

horizontal, $a_h = g\sin\alpha\cos\alpha$

α = helix incline.

$a_w = \vec{a}_v + \vec{a}_h = g\sin\alpha$



$a = \sqrt{\vec{a}_w^{\,2} + \vec{a}_c^{\,2}}$

$a_c = \frac{v_h^2}{r} = \frac{(a_h t)^2}{r} = (g t \sin\alpha\cos\alpha)^2/r$

$a \sim \sqrt{p + at^4}$

it's not E.

D

2016 #8

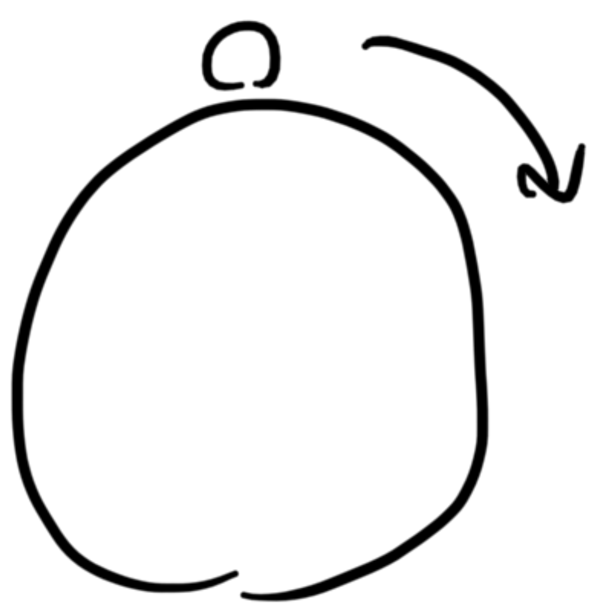
How do Kepler's laws change if $F_{grav} \sim R^{-3}$?

I, III depend on $F_{grav} \sim R^{-2}$

II is just conservation of angular momentum.

B

2016 #9



Velocity when bead leaves sphere?

Conservation of energy

$$mgh = \frac{1}{2}mv^2$$

Geometry

$$h = R(1-\cos\theta)$$

Contact forces

$$mg\cos\theta \geq mv^2/R$$

$$\cos\theta = v^2/gR$$

$$v = \sqrt{\frac{2gR}{3}} = \sqrt{gD/3}$$

E