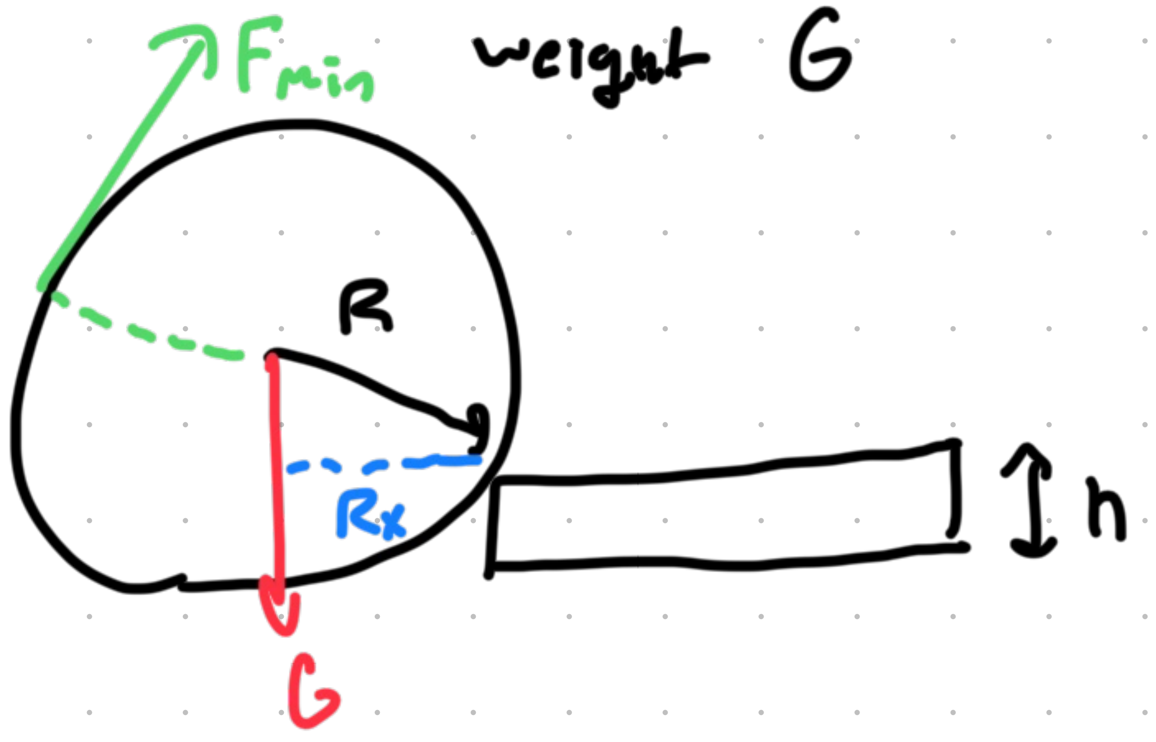
2019A #5



$F_{min} = ?$

$$\tau_F = \tau_G$$

best lever arm = $\boxed{2R}$

$$\tau_G = G \cdot R_x = G\sqrt{R^2-(R-h)^2}$$

$$R_x = \sqrt{R^2-(R-h)^2}$$

$$\tau_F = 2R F_{min}$$

$$F_{min} = \tau_G / 2R = G\frac{\sqrt{R^2-(R-h)^2}}{2R}$$

C

2019A #12

$V_f = \alpha V_i \qquad \alpha < 1$

$V_n = \alpha^n V_i$

$\tau_n = \frac{2v_n}{g} = \frac{2\alpha^n v_i}{g} = \alpha^n \tau_0$

$\tau_0 \equiv 2v_i/g$

... C?

ie if were τ vs n!

So, when does the n^{th} bounce occur?

$t_n = \sum_{k=1}^{n} \tau_k = \tau_0 \sum_{k=1}^{n} \alpha^k = \frac{\alpha \tau_0 (1-\alpha^{n-1})}{1-\alpha}$

$= \frac{\tau_1 - \tau_n}{1-\alpha}$

$t_n(1-\alpha) = \tau_1 - \tau_n$

$\tau_n = \tau_1 - t_n(1-\alpha)$ linearly decreasing

B

2019 A #16

measuring the depth of a well

minimum d where we can ignore finite speed of sound?

$d = gt^2/2$ $\qquad d_t = \dfrac{g(t+\Delta t)^2}{2}$

$\Delta d = \frac{1}{2} g\left[(t+\Delta t)^2 - t^2\right]$

$= \frac{1}{2} g\left[2t\Delta t + \cancel{(\Delta t)^2}\right]$ 0

second-order

$\Delta t = d/c$

$\Delta d \simeq gt\Delta t$

$= gt\, d/c$

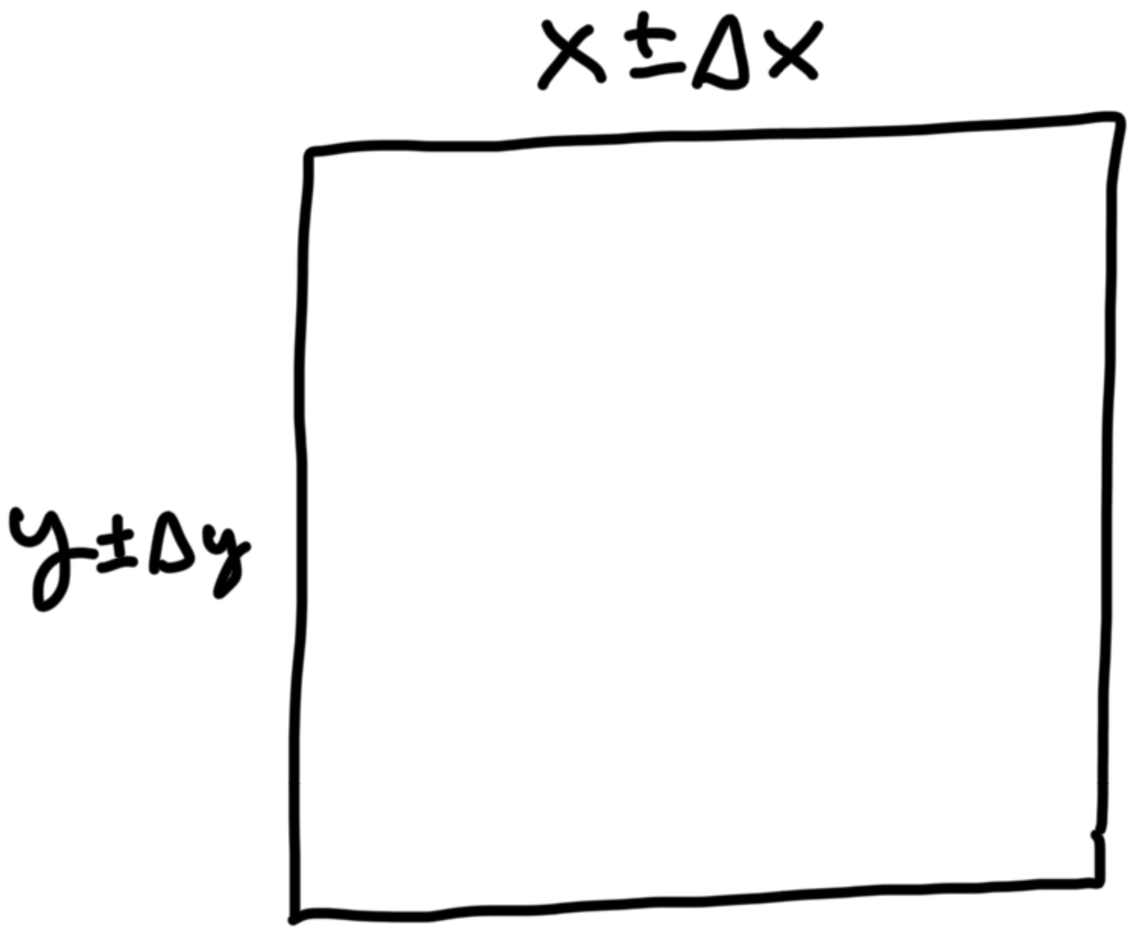
$\dfrac{\Delta d}{d} = \dfrac{gt}{c}$

$t = \sqrt{2d/g}$

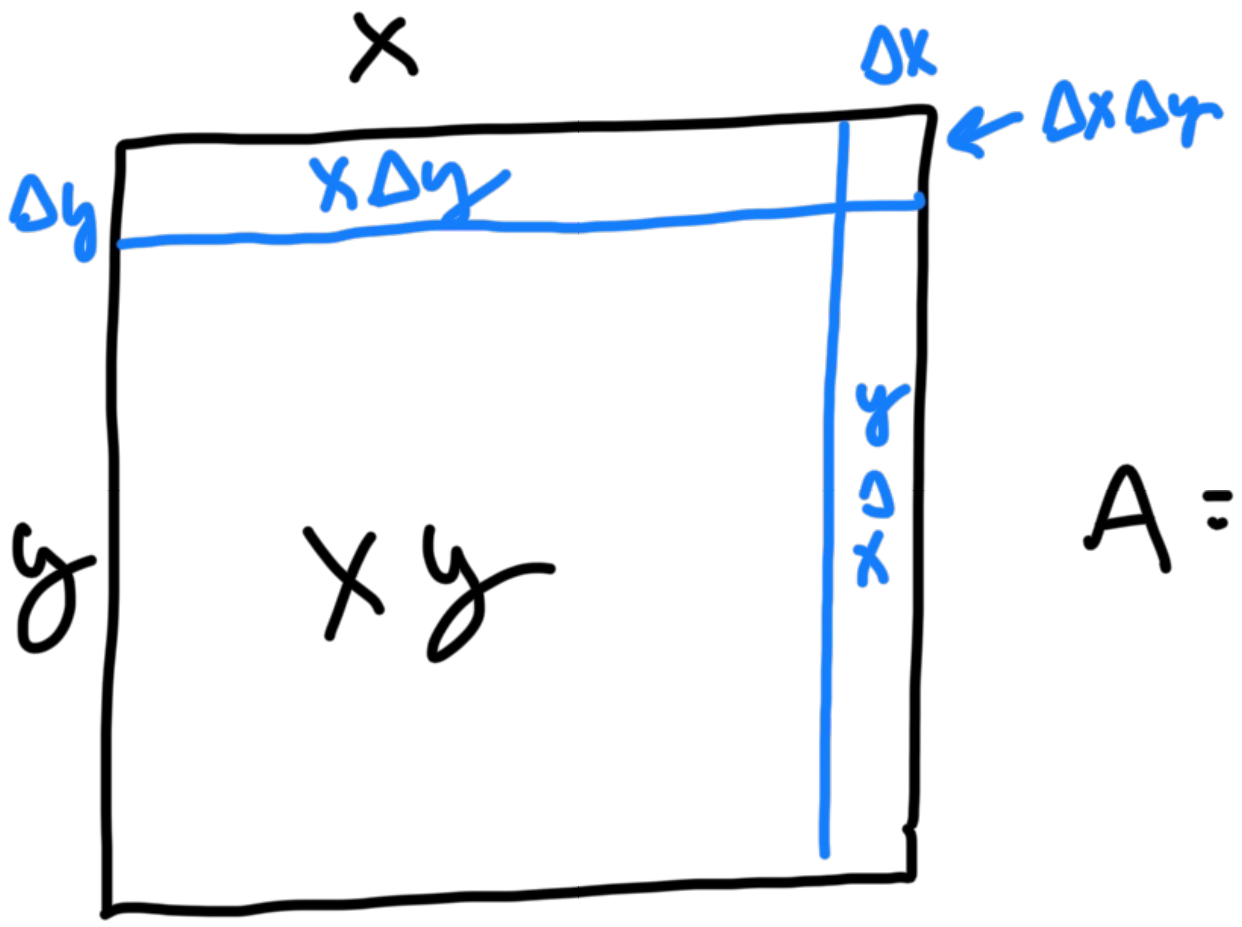
$\dfrac{\Delta d}{d} = \dfrac{g}{c}\sqrt{\dfrac{2d}{g}} = \dfrac{\sqrt{2dg}}{c} = 0.05$

$d \leq (0.05)^2 c^2/2g = 13.6$ m

C

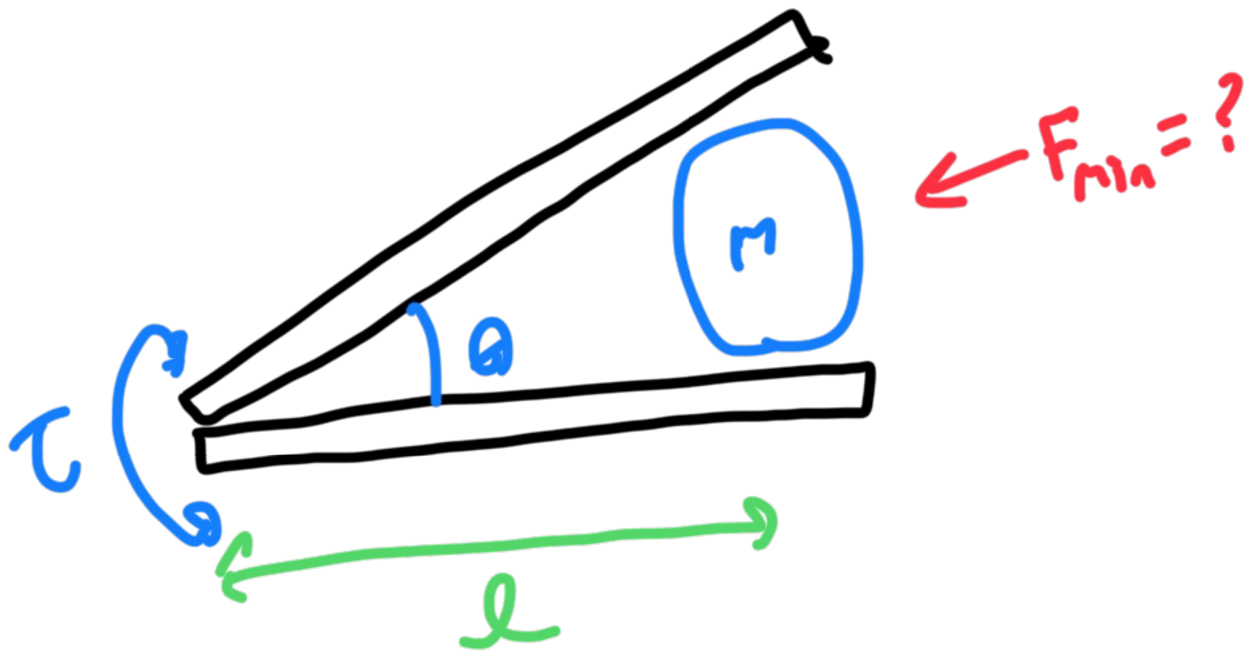


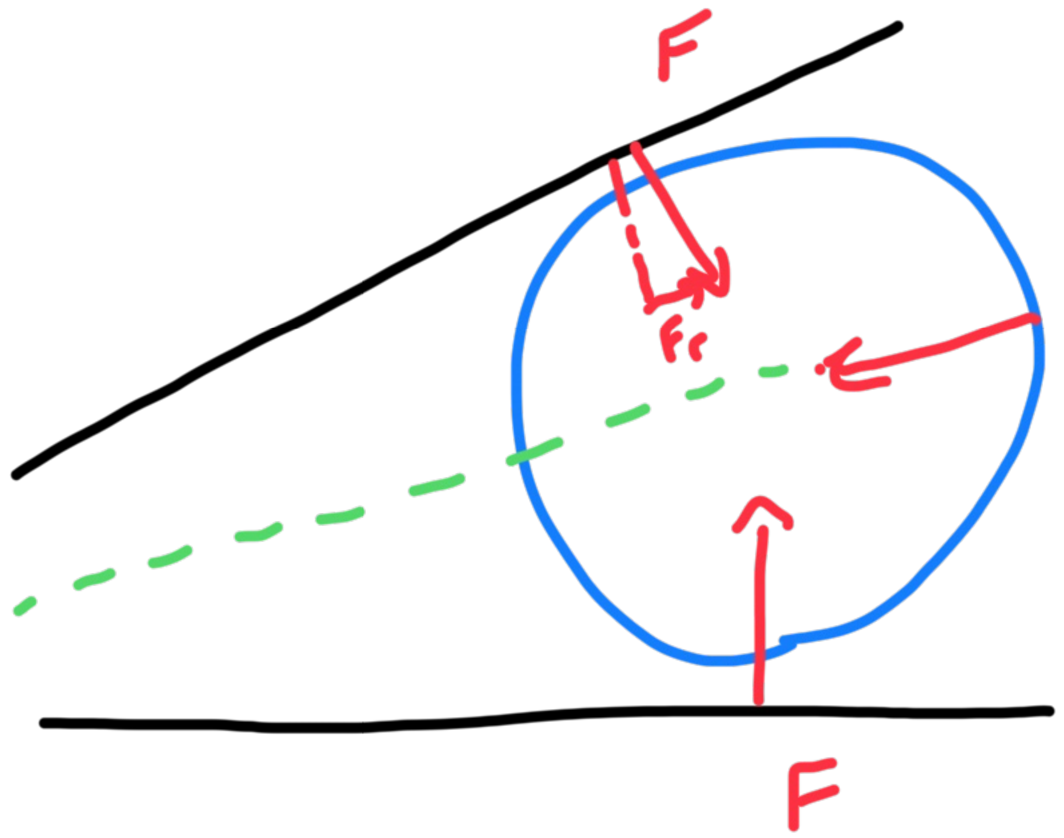
$$A \pm \Delta A = xy \pm \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$



$$A = xy + x\Delta y + y\Delta x + \boxed{\cancelto{0}{\Delta x \Delta y}}$$

2019 A #17





$$F_r = F\sin\left(\frac{\theta}{2}\right)$$

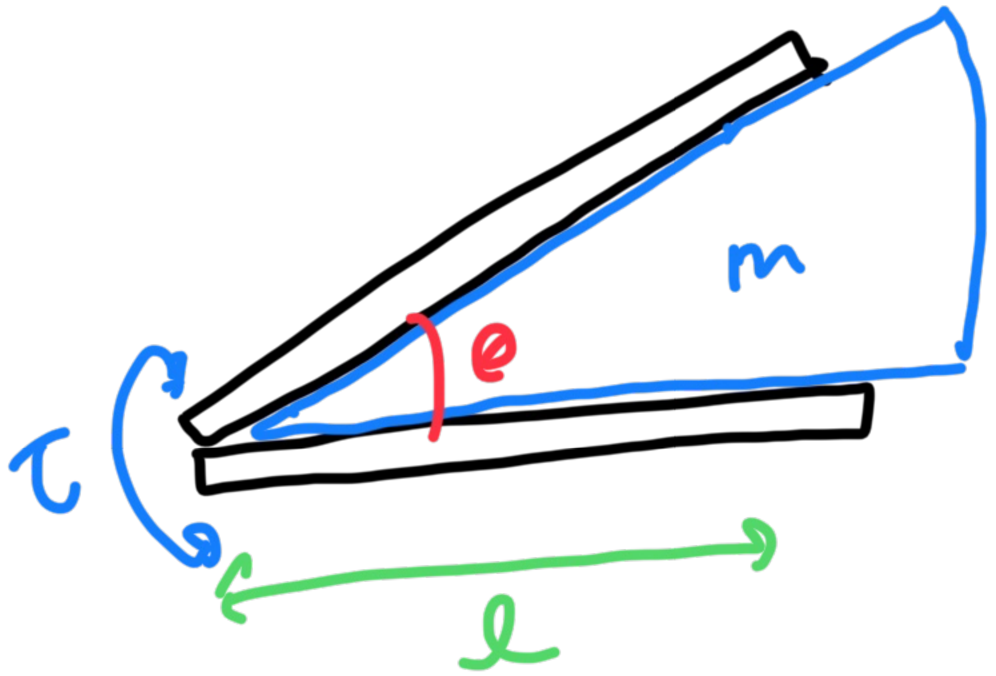
$$F = \tau/\ell$$

$$F_r = \frac{\tau}{\ell}\sin\left(\frac{\theta}{2}\right)$$

$$F_{app} = \frac{2\tau}{\ell}\sin(\theta/2)$$

A

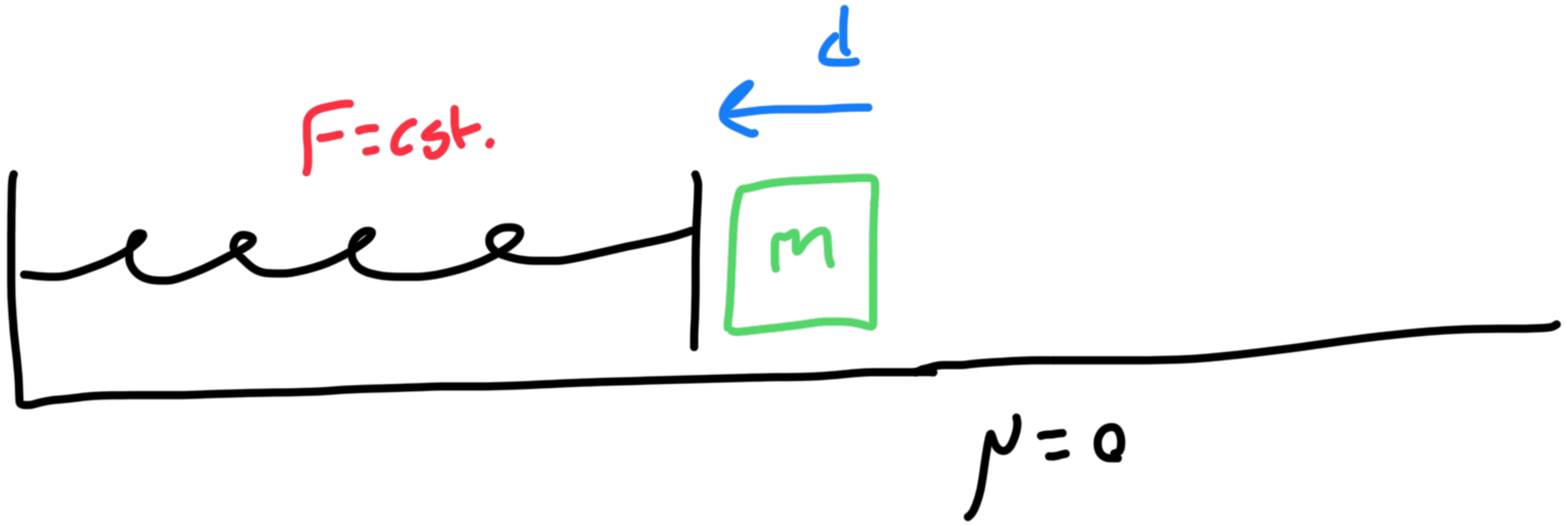
2019A #18



$V_{launch} = ?$

$E = \tau\theta$ (like Fd)

$\frac{1}{2}mv^2 = \tau\theta$ $\qquad v = \sqrt{\frac{2\tau\theta}{m}}$ D

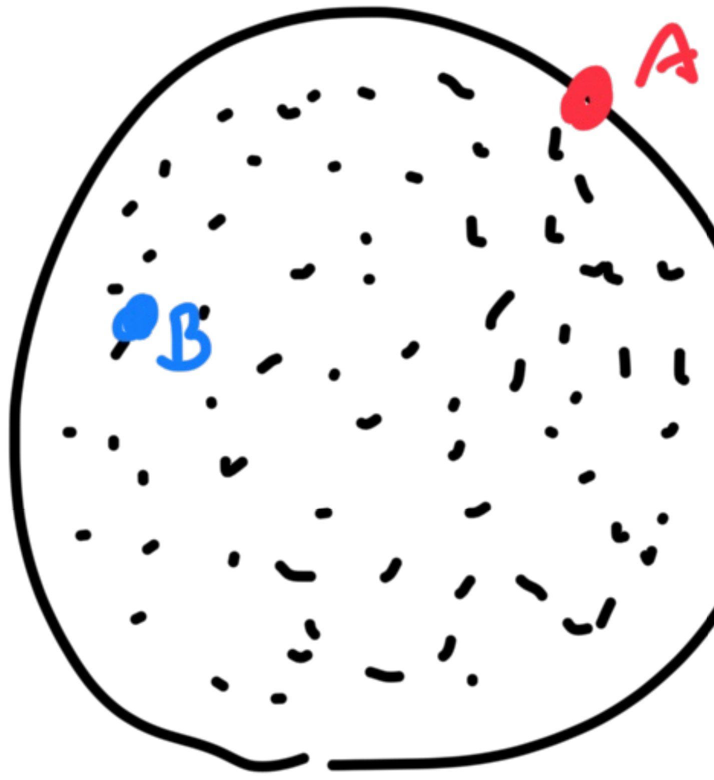


$$Fd = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2Fd}{m}}$$

<u>NOT</u> A SPRING LAUNCHER!

2019A #21



constant ρ

How do speeds and orbital periods compare?

Kepler's 3rd Law

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_{enc}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{enc}} = \frac{4\pi}{\rho G}$$

$\Rightarrow$ T is indep. of r.

$$T_A = T_B$$

(that's enough)

E

$$\Rightarrow \quad \omega_A = \omega_B$$

$$V_A = \omega_A R_A > V_B = \omega_B R_B$$

2019A #24

$U(x,y) = 9kx^2 + 16ky^2$

given $U(z) = \frac{1}{2}kz^2$, $T = 2\pi\sqrt{\frac{m}{k}}$

$K_x = 18k$ $\quad K_y = 32k$

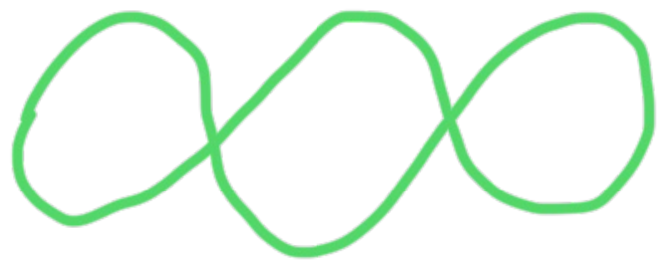
$T_x = 2\pi\sqrt{\frac{m}{18k}}$ $\quad T_y = 2\pi\sqrt{\frac{m}{32k}} = \frac{3}{4}T_x$

minimum period T_y

no x motion

max period is the LCM.

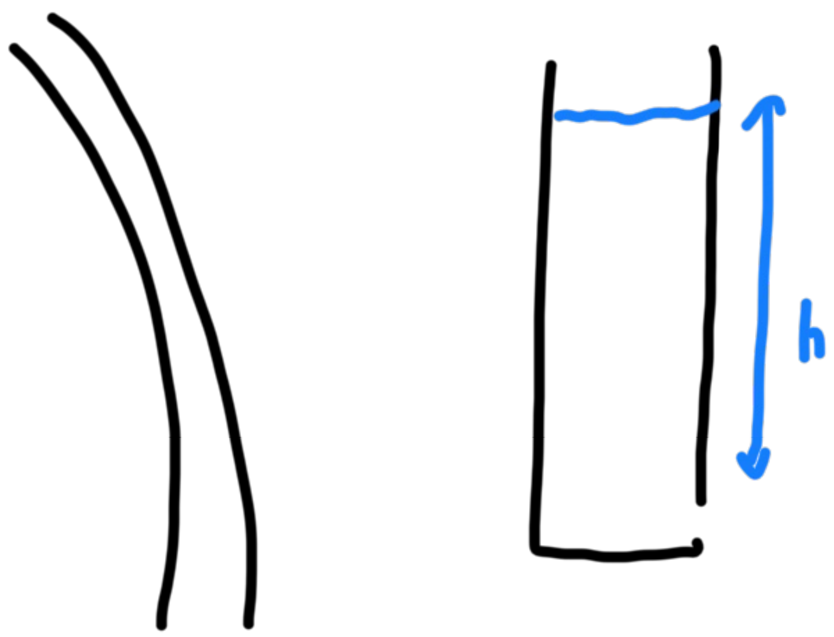
Lissajous Curves



$T_{lcm} = 4T_y = 3T_x$

D

2019A #25



$\theta = \tan^{-1}\left(v^2/gr\right)$

centripetal accel.

$h_{eff} = h\cos\theta$

$g_{eff}\cos\theta = g$

$$P = \rho g_{eff} h_{eff}$$

$$= \rho \, g/\cancel{\cos\theta} \; h \cancel{\cos\theta}$$

$$= \rho g h$$

$$\rho g h = \frac{\rho v^2}{2}$$

$$v = \sqrt{2gh}$$

A