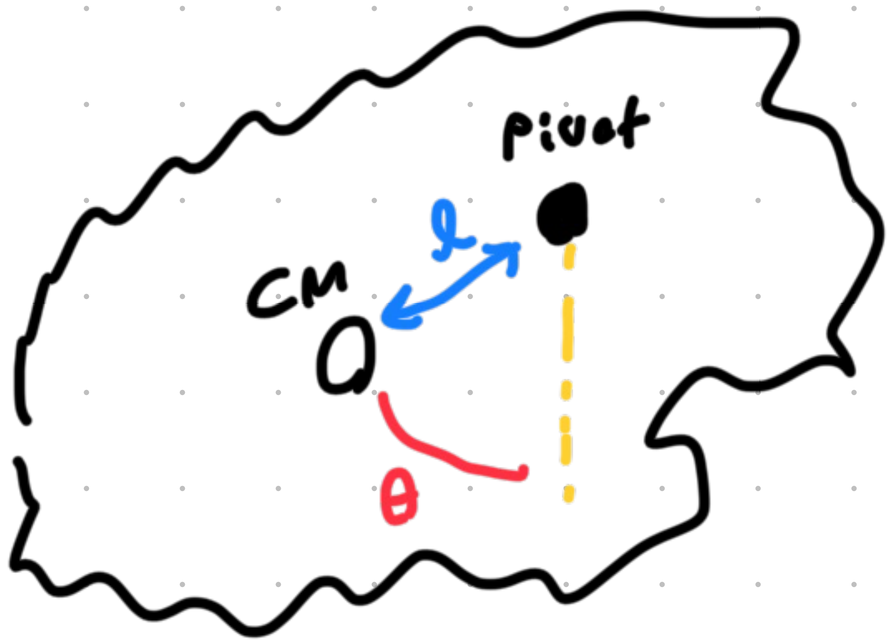
Physical Pendulums

if given m, I_{cm} and l, what is ω?



$$F(x) = -kx \quad \text{restoring force} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$F: \; m\ddot{x} = -kx \Leftrightarrow I\ddot{\theta} = -k\theta = \tau = I\alpha$$

$$I_{pivot} = I_{cm} + ml^2$$

$$\tau = I_{pivot}\,\alpha = -mgl\sin\theta$$

$$I_{pivot}\ddot{\theta} \simeq \boxed{-mgl\theta}$$ (small angle)

$$\ddot{\theta} \simeq -\frac{mgl}{I_{pivot}}\theta \quad \rightarrow \omega^2$$

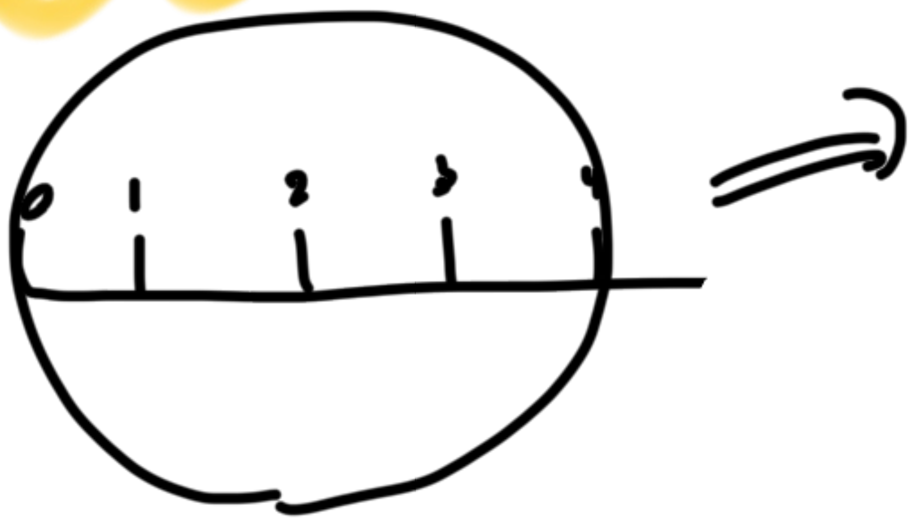
$$\omega = \sqrt{\frac{mgl}{I_{pivot}}}$$

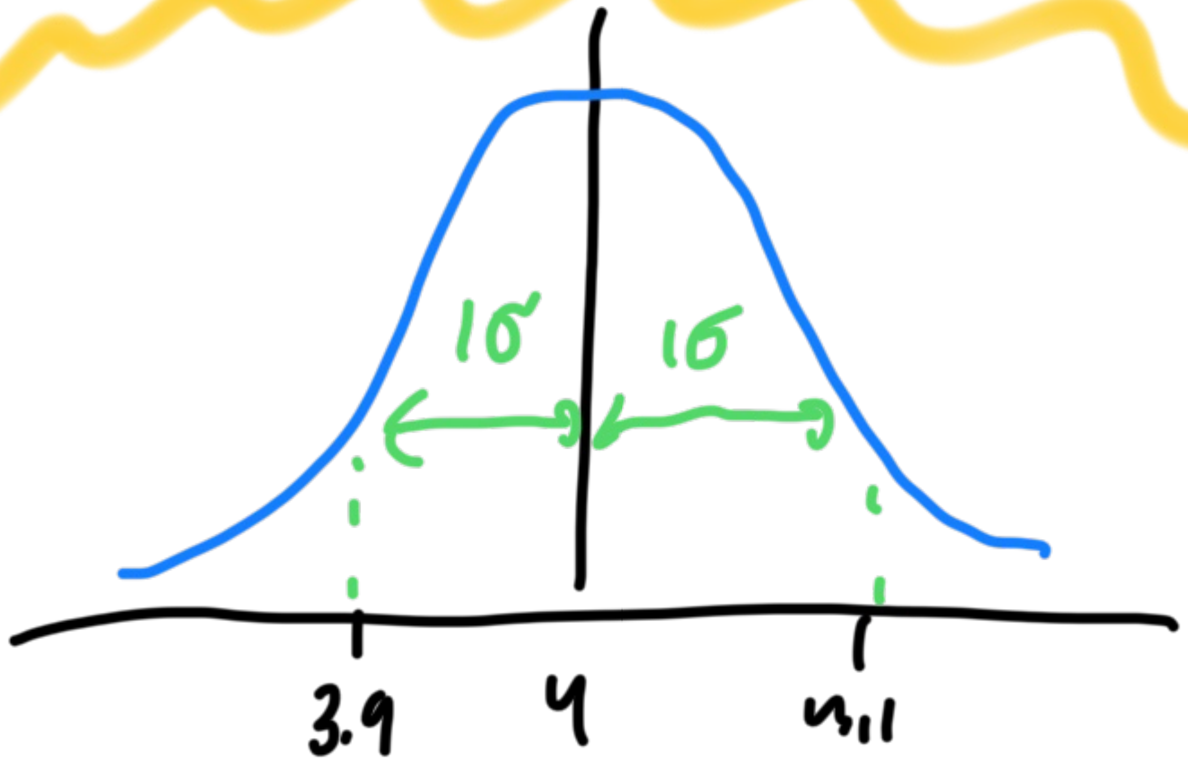
$$\boxed{\omega = \sqrt{\frac{mgl}{I_{cm} + ml^2}}}$$

Error Propagation

$x \pm \Delta x$ $\qquad$ $y \pm \Delta y$

(Gaussian uncertainties)





Gaussian
(bell curve)

$d \pm \Delta d = 4.0 \pm 0.1$ cm

Rules

$$\Delta(x+y) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta(xy) = \sqrt{(x\Delta y)^2 + (y\Delta x)^2}$$

$$\Delta(x^a) = |a|\, x^{a-1} (\Delta x)$$

Multiple Trials

$$\frac{x+2y}{3} \pm \underbrace{\Delta(x+2y)}_{\downarrow}$$

$$\frac{1}{\sqrt{3}}\sqrt{(\Delta x)^2 + 2(\Delta y)^2}$$

Relative uncertainties

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

when the quantities measured are multiplied or divided to compute the final result.

Measure g using a pendulum

$L \pm \Delta L$ $T \pm \Delta T$

what is $g \pm \Delta g$?

$$g = 4\pi^2 L/T^2$$

$$\Delta g = ?$$

$$\Delta(T^{-2}) = |-2|\, T^{(-2-1)} \Delta T = 2T^{-3}\Delta T$$

$$\Delta(LT^{-2}) = \sqrt{[L\Delta(T^{-2})]^2 + [(T^{-2})\Delta L]^2}$$

$$= \sqrt{[2LT^{-3}\Delta T]^2 + [(T^{-2})\Delta L]^2}$$

$$\Delta g = 4\pi^2 \Delta(LT^{-2})$$

$$= 4\pi^2 \sqrt{[2LT^{-3}\Delta T]^2 + [(T^{-2})\Delta L]^2}$$

2018 B #19

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta s}{s}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} = 5.4\%$$

$$\Delta v = 0.054 \cdot 349 \text{ m/s} \approx 19 \text{ m/s}$$ E

2018 B #25

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

1. $\sqrt{(1\%)^2 + (1\%)^2} \simeq 1.4\%$

2. $\sqrt{(10\%)^2 + (0.1\%)^2} \simeq 10\%$

3. $\sqrt{(10\%)^2 + (1\%)^2} / \sqrt{10} \simeq 3.3\%$

D

Rotational Dynamics

rotation	relations	translation
$\vec{\tau}$	$\vec{\tau} = \vec{r} \times \vec{F} = r_\perp F = r F_\perp$	$\vec{F}$
$\vec{\alpha}$	$\alpha = \frac{a}{r}$ $\quad a = \alpha r$	$\vec{a}$
$\vec{\omega}$	$\omega = \frac{v}{r}$ $\quad v = \omega r$	$\vec{v}$
$\vec{\theta}$	$\theta = \frac{x}{r}$ $\quad x = \theta r$	$\vec{x}$
I	$I \propto mr^2$	m
K	$K = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$	K

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$F = ma$$

$$\tau = I\alpha$$

2018 B #5

Spin up to speed in 10 rev $\Rightarrow \alpha_1$

slow down in 50 rev $\Rightarrow \alpha_2$

$\alpha_1/\alpha_2 = ?$

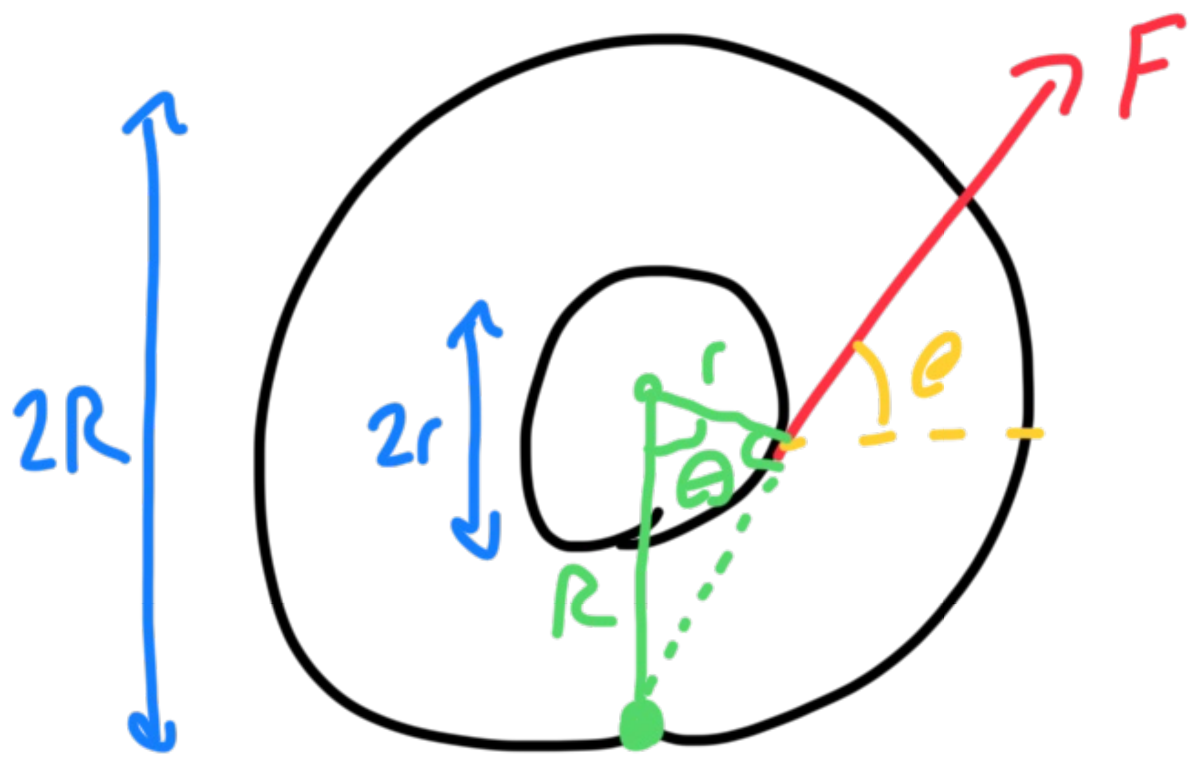
$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$

$\alpha \sim \frac{1}{\theta}$

$\frac{\alpha_1}{\alpha_2} = \frac{\theta_2}{\theta_1} = 5$ D

2018 B #14



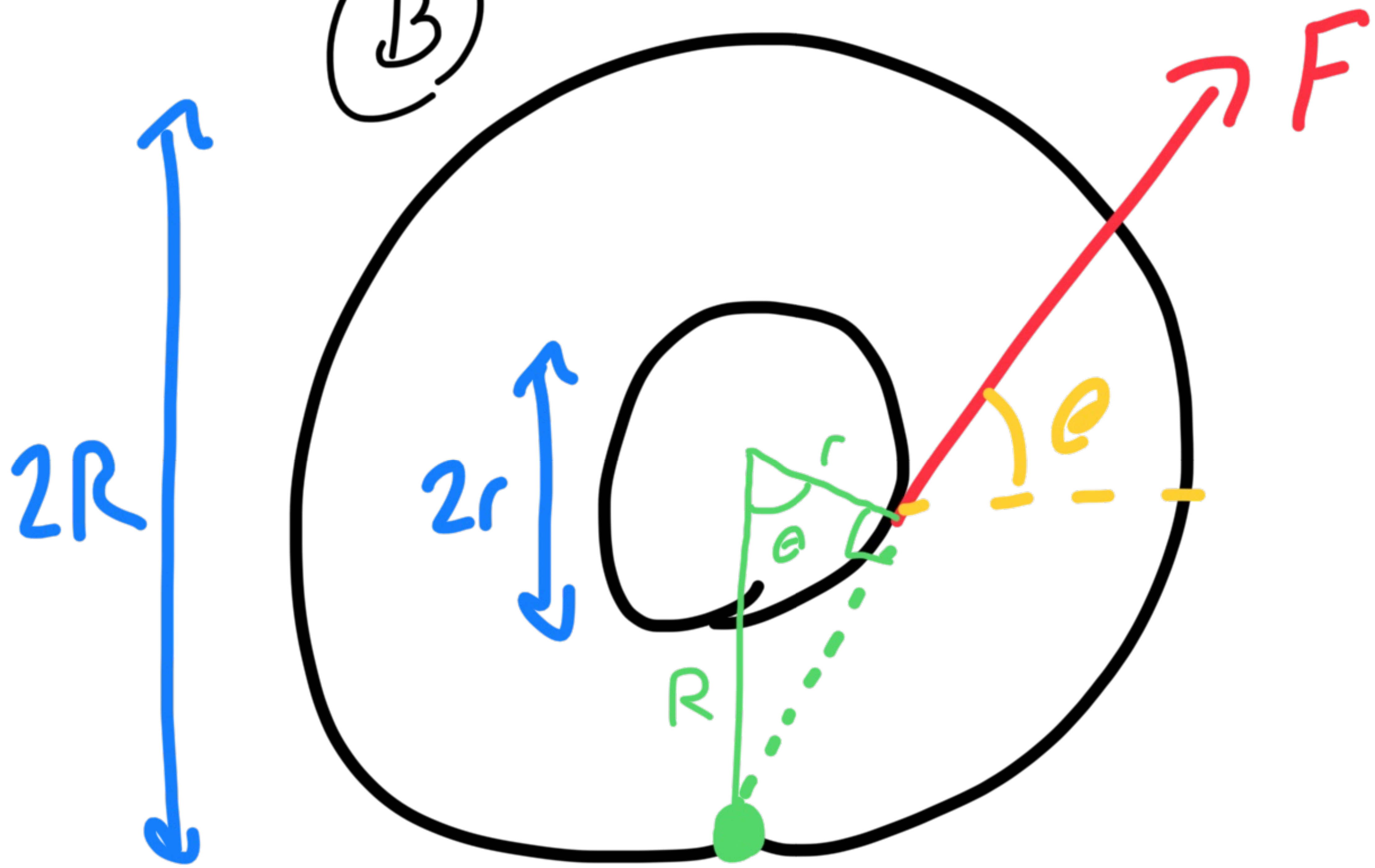
at what angle can we pull so the spool slips w/o rotating?

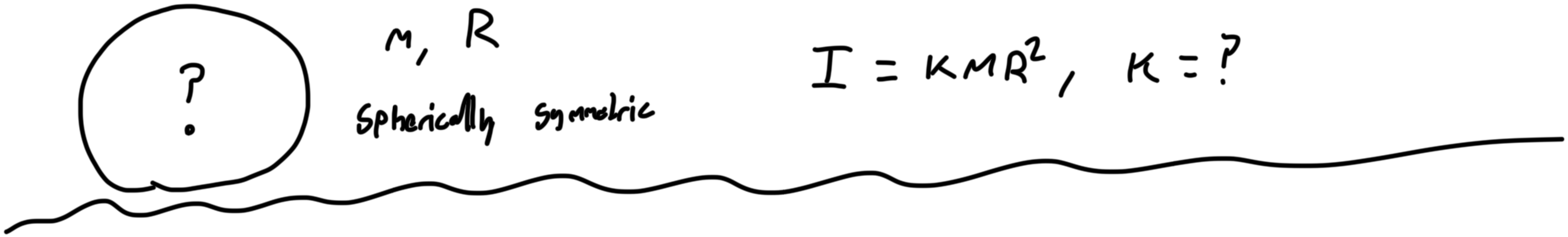
$\Sigma\tau = 0$ $\tau = r_\perp F = r F_\perp$

Force and lever arm parallel ⇒ no torque!

$\cos\theta = \frac{r}{R}$ $\theta = \cos^{-1}\left(\frac{r}{R}\right) = 41.4°$

(B)





$I = kMR^2, \quad k = ?$

if no energy is lost, then we can find k!

$$E_0 = \frac{1}{2} k m R^2 \omega_0^2$$

$$v = \omega R$$

$$E_f = \frac{1}{2} m v^2 + \frac{1}{2} k m R^2 \omega^2 = \frac{1}{2}\left(mR^2\omega^2 + kmR^2\omega^2\right) = \frac{1}{2}(1+k) m R^2 \omega^2$$

$$E_f = E_c$$

$$\frac{1}{2}(1+k)mR^2\omega^2 = \frac{1}{2}kMR^2\omega_0^2$$

$$(1+k)\omega^2 = k\omega_0^2$$

$$\omega^2 = k(\omega_0^2-\omega^2)$$

$$k = \frac{\omega^2}{\omega_0^2-\omega^2} = \boxed{\frac{v^2}{\omega_0^2R^2-v^2}}$$