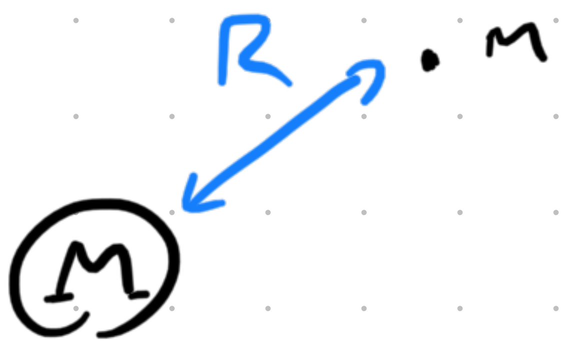
Orbits (Topic 7 in F=ma Physics)

only force is gravity



what is the period of orbit?

$$m a_{cent} = \frac{mv^2}{R} = F_{\text{radially inwards}} = F_{grav} = \frac{GMm}{R^2}$$

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

each orbit is $2\pi R$ long,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM/R}} = \boxed{2\pi\sqrt{R^3/GM} = T}$$

orbits are stable when governed by a radially inwards force that scales as R^{-2}

$F \propto R^{-2}$ $F = \phi R^{-2}$ ϕ = force constant.

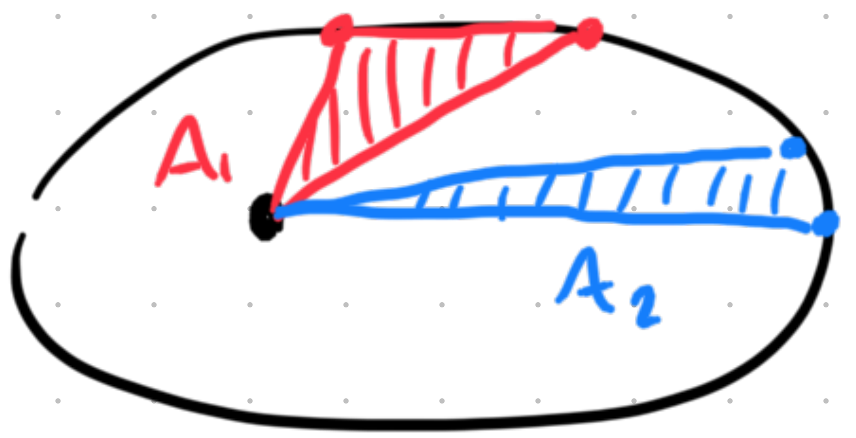
$$\phi R^{-2} = F = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{\phi}{mR}}$$

$\phi = GMm$ (Gravity) $v = \sqrt{\frac{GM}{R}}$

Kepler's Laws

I. all bound orbits are elliptical, with the orbited body at a focus.

II. a line joining the orbiter and orbited sweeps equal areas in equal time.



$A_1 = A_2$

$t_1 = t_2$

$\dot{\theta}_1 > \dot{\theta}_2$ aka $\omega_1 > \omega_2$ aka $v_1 > v_2$

III. $T^2 \propto R^3$ derivation above can be generalized to ellipses.

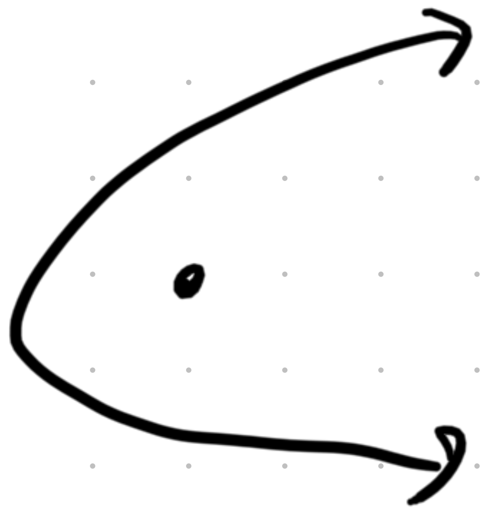
Non-circular and unbound orbits.

$v/\sqrt{GM/R}$ (dimensionless)

hyperbolic

$\sqrt{2}$ — parabolic

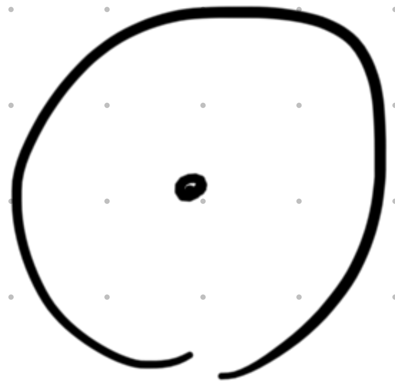
escape speed



elliptical

(R = perigee)

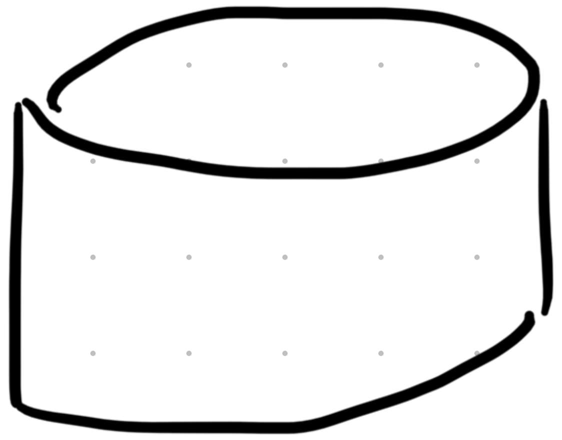
1 — circular

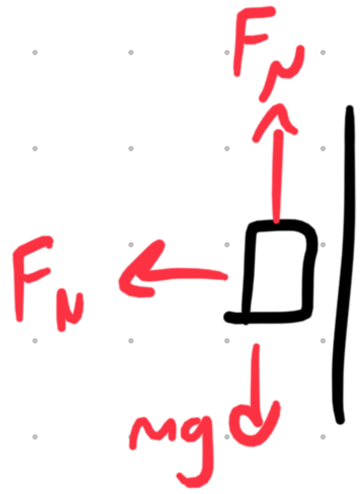


elliptical

(R = apogee)

2017 #1





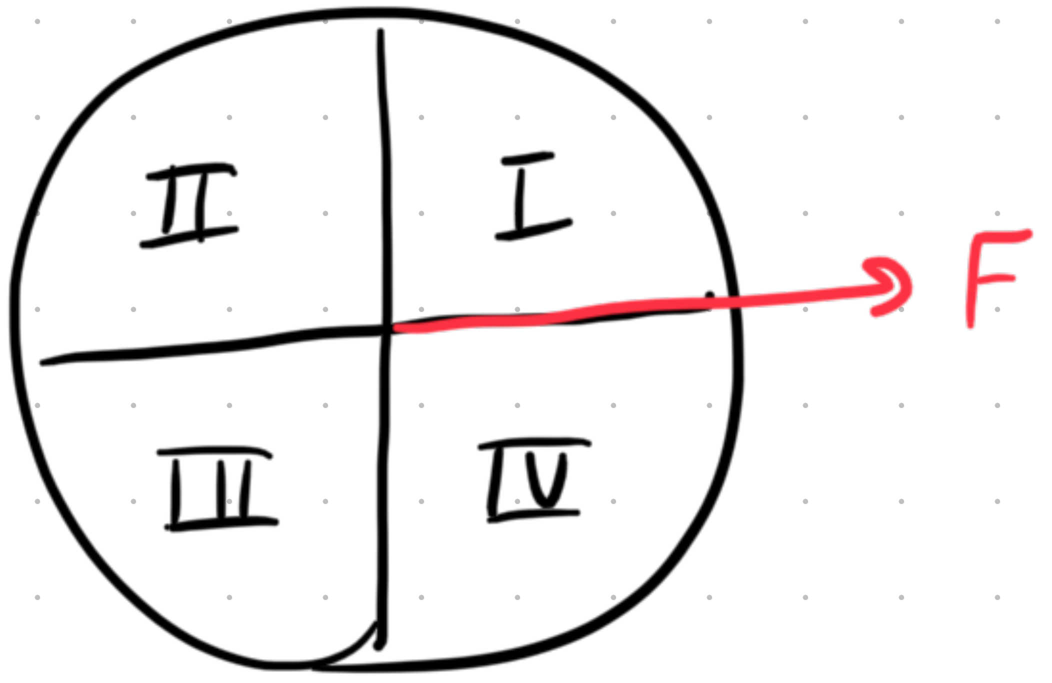
$$F_N = \frac{ms^2}{R}$$

$$F_\mu = mg = \mu \frac{ms^2}{R}$$

$$\mu \propto s^{-2}$$

D

2017 #18



any point that has zero acceleration?

$$\vec{a}_c + \vec{a}_t + \vec{a}_F$$

for $\vec{a}_{total} = 0$, $\vec{a}_c + \vec{a}_t = -\vec{a}_F$

so due left.

~~A, B, C~~

r = point radius

R = disk radius

$\|\vec{a}_c + \vec{a}_t\| = \|\vec{a}_F\|$?

$\|\vec{a}_c + \vec{a}_t\| = \sqrt{a_c^2 + a_t^2} = r\sqrt{\omega^4 + \alpha^2}$

$\|\vec{a}_F\| = \alpha R$

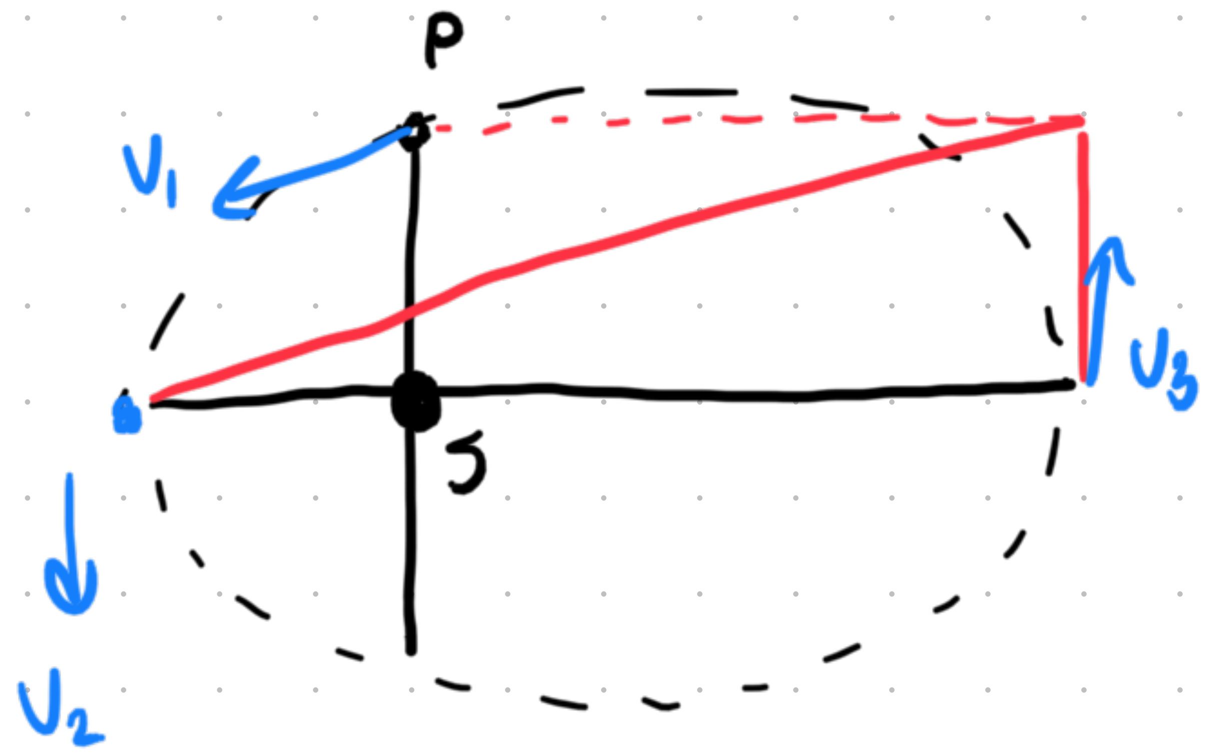
$\sqrt{\omega^4 + \alpha^2} > |\alpha|$

IVT says that $\exists\, r \in (0, R]$

s.t. $r\sqrt{\omega^4 + \alpha^2} = \alpha R$.

So, yes! D

2017 #25



$v_2 = ?$

$\overline{SP} = ?$

$\overline{SP} + \sqrt{\overline{SP}^2 + a^2} = 2a$

$\overline{SP} = 3a/4$

Conservation of energy

$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_3^2 - \frac{GMm}{3a/2}$

K2L: $\frac{m v_2 a}{2} = \frac{3 m v_3 a}{2} \Rightarrow v_3 = \frac{1}{3} v_2$

$\frac{4}{9} m v_2^2 = \frac{GMm}{a/2} - \frac{GMm}{3a/2} = \frac{4GMm}{3a}$

$v_2^2 = \frac{3GM}{a}$

$\frac{1}{2} m v_2^2 - \frac{GMm}{a/2} = \frac{1}{2} m v_1^2 - \frac{GMm}{3a/4}$

$v_1^2 = v_2^2 - \frac{4GM}{a} + \frac{8GM}{3a} = \frac{5GM}{3a}$

$v_2^2/v_1^2 = 9/5 \Rightarrow v_2/v_1 = 3/\sqrt{5}$

A

2016 #4

radius r

angle α

$a_t = g\sin\alpha$

$a_c = \frac{v_h^2}{r}$

~~A, B, C~~

not constants

$a(t=0) \neq 0$

$a_{tv} = a_t \sin\alpha = g\sin^2\alpha$

$a_{th} = a_t \cos\alpha = g\sin\alpha\cos\alpha$

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{g^2\sin^2\alpha + \left(\frac{(g t \sin\alpha\cos\alpha)^2}{r}\right)^2}$

$= \sqrt{P + Qt^4}$

P, Q are positive constants

NOT LINEAR!

D

2016 #8

How would Kepler's Laws change if $F_{grav} \propto R^{-3}$ rather than $F_{grav} \propto R^{-2}$?

III changes because we used gravity to prove it.

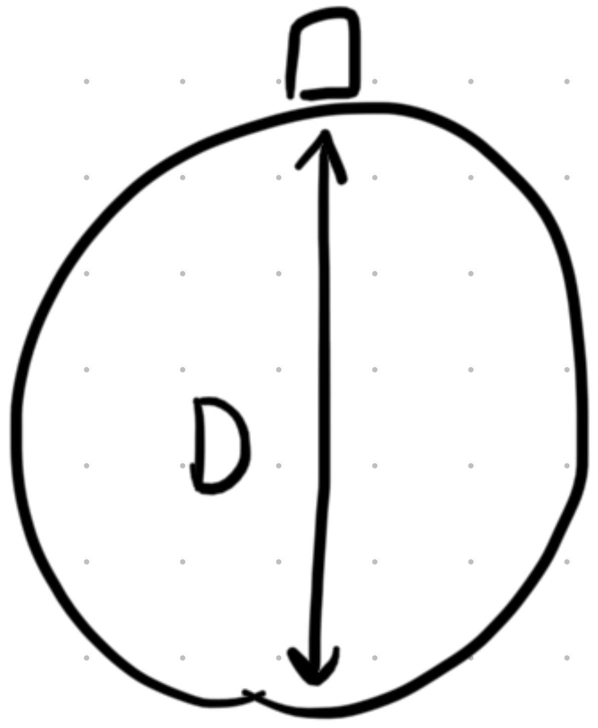
~~C D~~

I is also a consequence of gravity too.

II? that's just angular momentum conservation.

B

2016 #9



v when the bead falls off?

$$mgh = \frac{1}{2}mv^2$$ conservation of energy

$$h = R(1-\cos\theta)$$ geometry of the problem

$$mg\cos\theta \geq \frac{mv^2}{R}$$ contact forces

$$\cos\theta = v^2/gR$$

$$v = \sqrt{\frac{2gR}{3}} = \sqrt{\frac{gD}{3}}$$

E