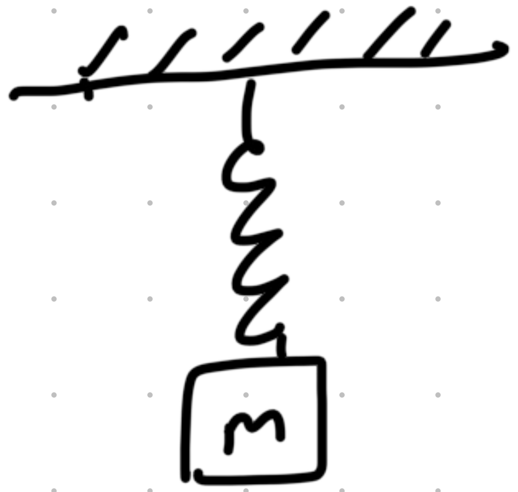
2019 A #4



$y_{max} = 5\,cm$

$T = ?$

$\frac{1}{2} m v^2 = m g y_{max}$

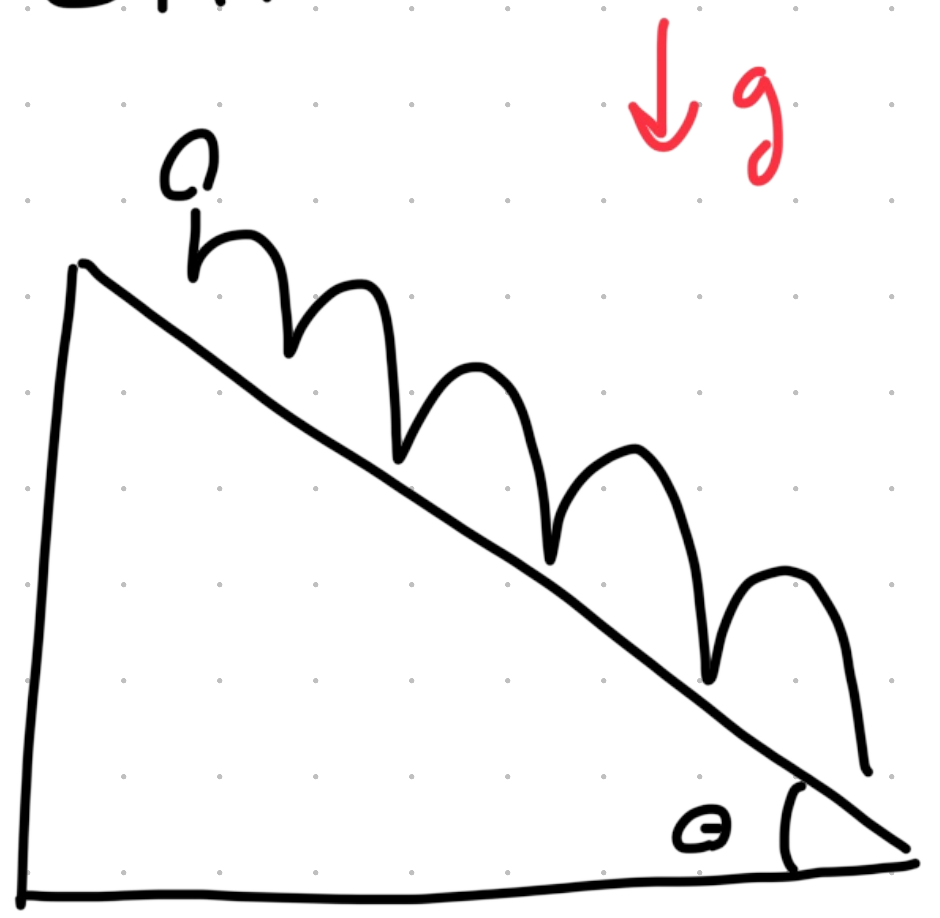
$T = 2\pi\sqrt{\frac{m}{k}}$

$E_0 = E_f \Rightarrow \frac{1}{2} k y_{max}^2 = m g y_{max}$

$\frac{m}{k} = \frac{y_{max}}{2g}$

$T = 2\pi\sqrt{\frac{y_{max}}{2g}} \simeq 0.31\,s$

2019 A #6



$\perp$, $\parallel$ coordinate system.

$\perp$: no losses because collisions are completely elastic, so y stays the same and thus t does not change.

A or E

$\parallel$: constant acceleration, $v_\parallel(t) = v_0 \sin\theta + gt\sin\theta$ which is increasing.

A

2019A #12

intuitively ~~D~~ ~~E~~

from 2019A #6 logic, ~~A~~

$$v_n = \alpha^n v_i$$

$$\tau_n = \frac{2v_n}{g} = \frac{2\alpha^n v_i}{g} = \alpha\tau_0$$

$$\tau_0 \equiv 2v_i/g$$

total time

$$t_n = \sum_{j=0}^{n} \tau_j = \tau_0(\alpha+\alpha^2+\alpha^3+\dots+\alpha^n)$$

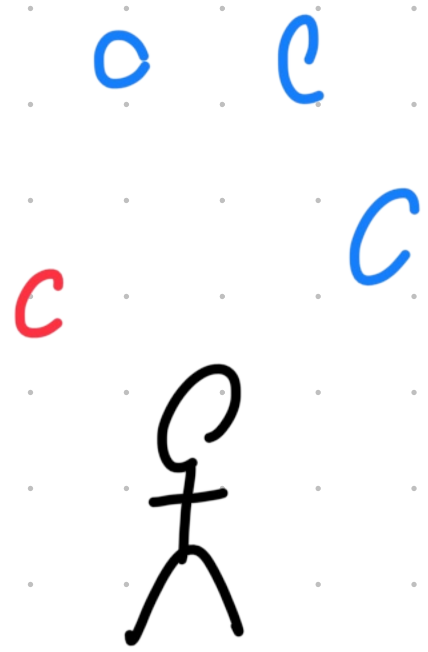
$$= \tau_0 \frac{\alpha(1-\alpha^{n-1})}{1-\alpha}$$

$$t_n = \frac{\tau_1 - \tau_n}{1-\alpha}$$

$\tau_1 - (1-\alpha)t_n = \tau_n$ which is linearly decreasing with τ

B

2019A#13



time between one specific ball getting tossed is NT

more velocity → higher T

more energy → more power

$E \propto v^2$

$[P] = \frac{[E]}{[T]}$

$NT = \frac{2v}{g}$

$v \propto N$

$P \propto E \propto v^2 \propto N^2$

C

2019A #14

coriolis effect

$\vec{a} = -2\vec{\omega} \times \vec{v}$

$|\vec{\omega}| = 2\pi \text{ rad/day}$

θ = latitude

L = distance

$d = \frac{at^2}{2} = \frac{2\omega v \sin\theta (L/v)^2}{2}$

$= \frac{L^2 \omega \sin\theta}{v} = 1.8 \text{ mm}$

cross of velocity and earth's rotation points west. negative sign means east.

D

2019A #21

KBL:

$$m\omega^2 r = G\frac{mM_{enc}}{r^2} \Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{enc}} = \frac{4\pi}{\rho G}$$

$$T_A = T_B$$

$$v_i = \omega r_i \qquad \omega \propto T \Rightarrow \omega_A = \omega_B$$

$$v_A > v_B$$

E

2019A#24

$U(x,y) = 9kx^2 + 16ky^2$

$U(z) = kz^2/2$ ... spring oscillator!

$T = 2\pi\sqrt{m/k}$

$k_x = 18k$ $k_y = 32k$

$T_x = 2\pi\sqrt{m/18k}$

$T_y = 2\pi\sqrt{m/32k} = \boxed{\frac{3}{4}T_x}$

minimum period

maximum period is the $\boxed{\text{LCM}}$ of both.

why?

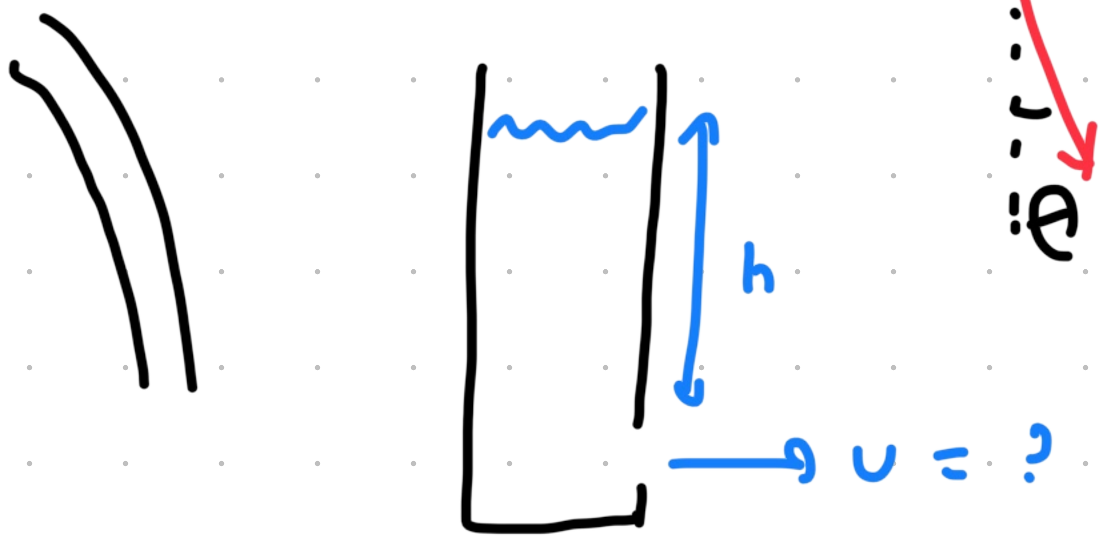
$T_{lcm} = 4T_y = 3T_x$

Ratio = 4 D

Lissajous curves!

2019 A #25

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$



$$\vec{g}_{eff} = \vec{a}_c + \vec{g} = \frac{v^2}{r}\hat{x} - g\hat{y}$$

$$h_{eff} = h\cos\theta$$

$$P = \rho g_{eff} h_{eff} = \rho g_{eff} h\cos\theta$$

$$= \rho g h$$

$$\rho g h = \rho v^2/2$$

$$v = \sqrt{2gh}$$

unchanged!

A