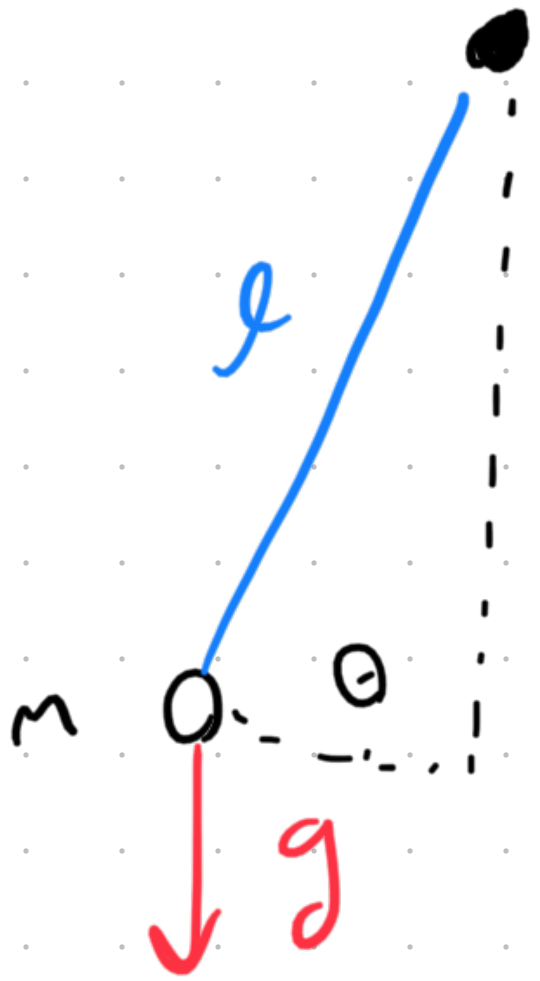
Dimensional Analysis

what is the period of this pendulum?



A $\sqrt{g/\ell}$

B $\sqrt{\ell/g}$

~~C~~ $\sqrt{mg/\ell}$

~~D~~ $\sqrt{\ell/mg}$

~~E~~ $\sqrt{\ell g}$

goal units $[s]$

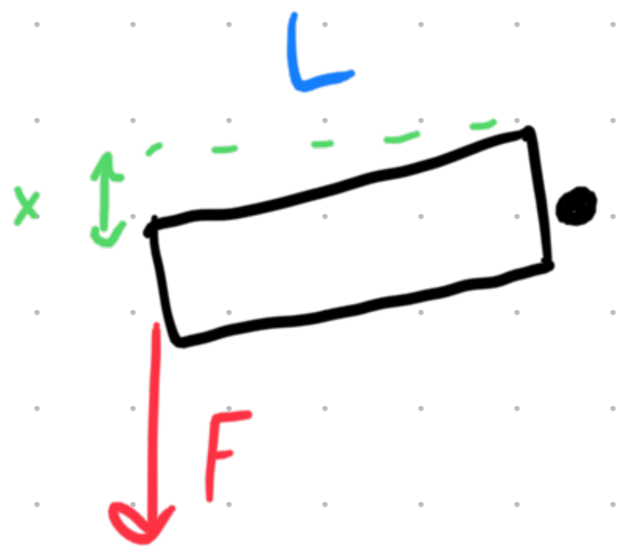
$[\ell] = m$

$[g] = \frac{m}{s^2}$

A $\left[\sqrt{\frac{g}{\ell}}\right] = \left(\frac{m}{s^2} \cdot \frac{1}{m}\right)^{\frac{1}{2}} = s^{-1}$

B $\left[\sqrt{\frac{\ell}{g}}\right] = \left(m \cdot \frac{s^2}{m}\right)^{\frac{1}{2}} = s$

2018 B #6



$$X = \frac{F}{I} E^{\alpha} L^{\beta}$$

$[F] = N$

$[I] = m^4$

$[E] = N m^{-2}$ ⇝ Newtons cancel $\alpha = -1$

$[L] = m$

$$X = \frac{F}{IE} L^{\beta}$$

$$\left[\frac{F}{IE}\right] = m^{-2}$$

$$[L^{\beta}] = m^3$$

$$\Rightarrow \beta = 3$$

thus, $X \propto L^3$

D

Simple Harmonic Oscillations

$$F(x) = -kx$$

$$ma = -kx$$

$$m\ddot{x} = -kx$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

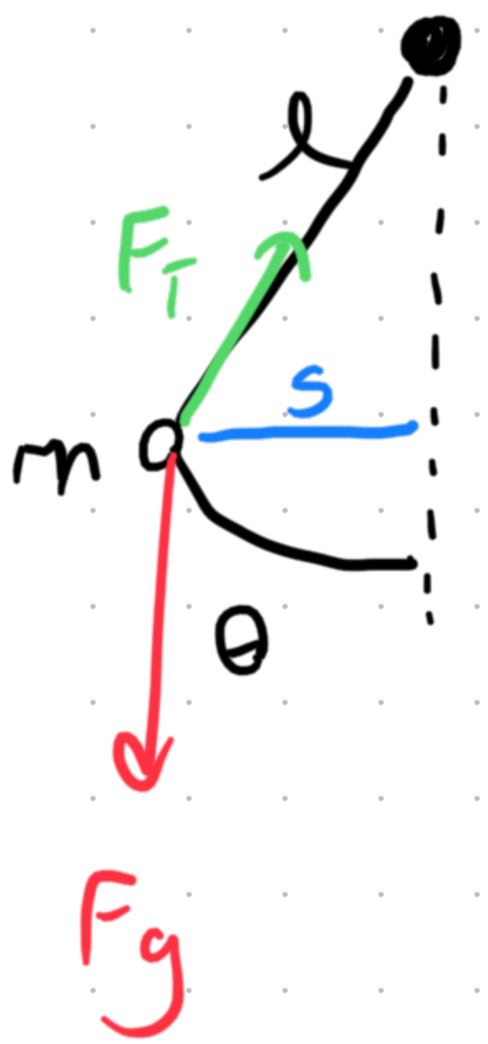
$$x = A\cos(\omega t + \phi)$$

A = max. displacement

$$\omega = \sqrt{k/m}$$

$$E_{osc} = \frac{1}{2} k A^2$$

Pendulums & small angle approximation.



natural oscillation frequency

$\omega = ?$

$F_{T_\theta} = F_T \sin\theta \simeq F_T \theta$

$F_T \simeq F_g$

$F_{T_\theta} \simeq -F_g \theta = -F_g \frac{s}{\ell}$

$F(s) \propto -s$

Restoring Force!

$\omega = \sqrt{k/m}$

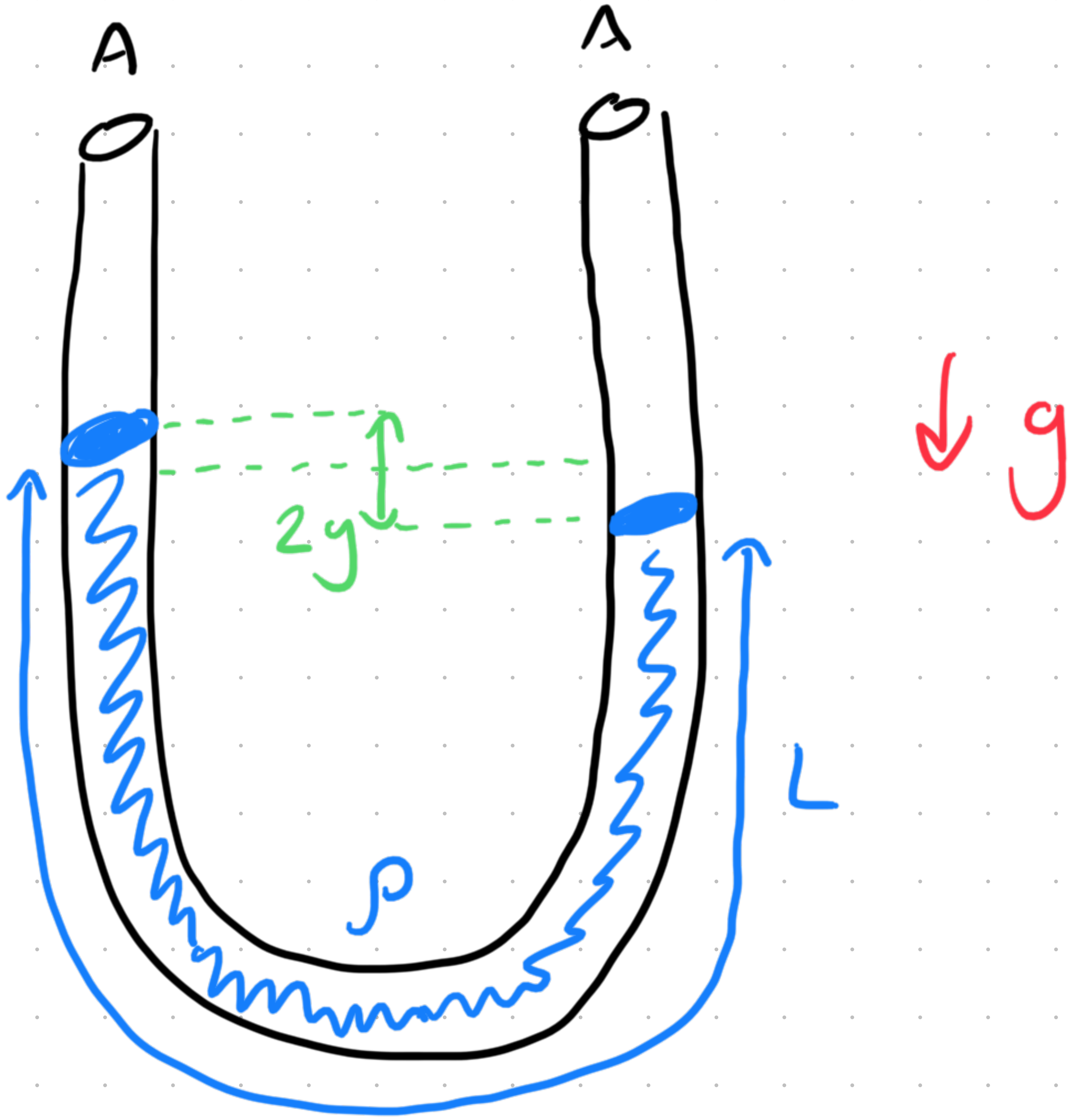
$F(s) = -k s$

$k = F_g/\ell$

$= mg/\ell$

$\omega = \sqrt{\frac{mg/\ell}{m}}$

$\boxed{\omega = \sqrt{g/\ell}}$



$$\omega = ?$$

$$F_{disp} = m_{disp}\, g = -2\rho g y A$$

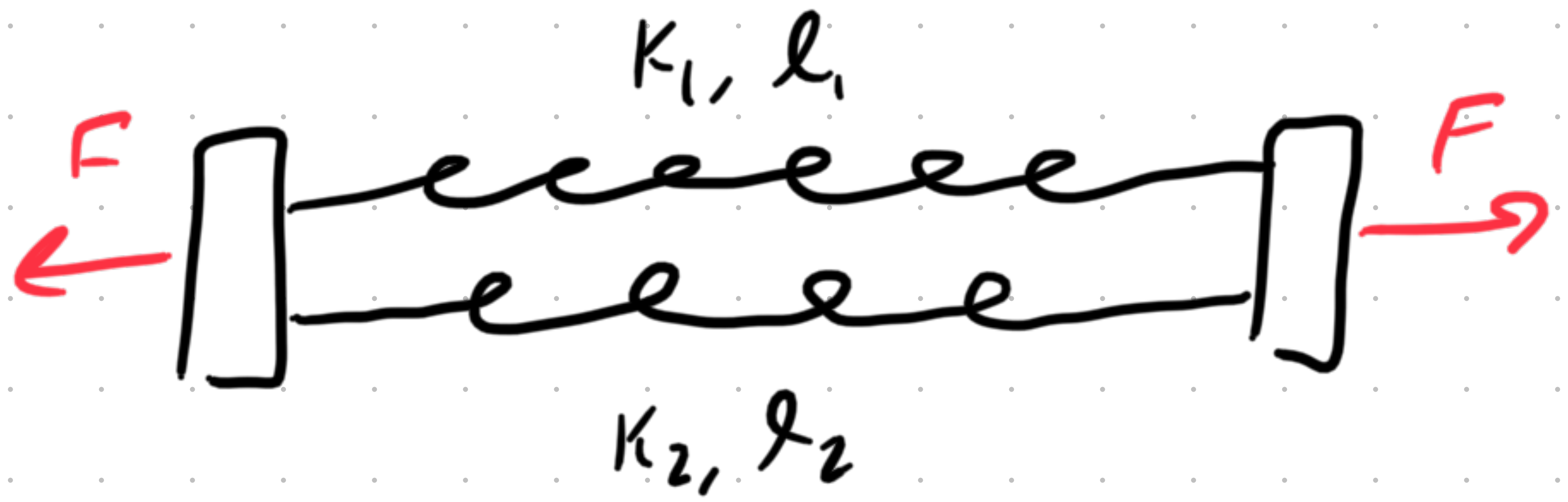
$$= -ky$$

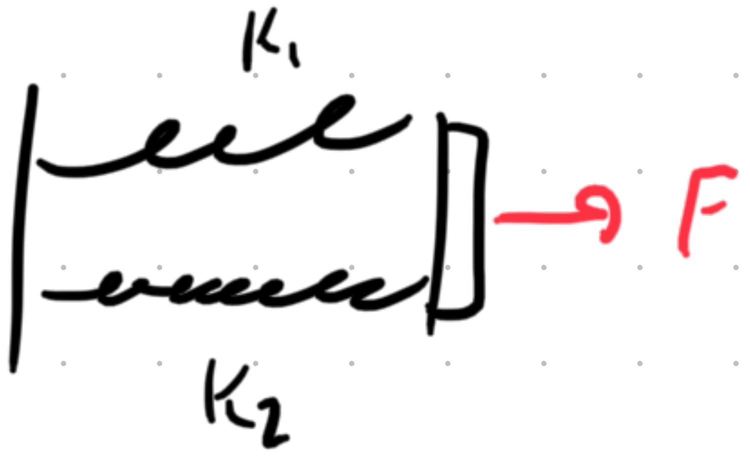
$$k = 2\rho A g$$

$$m = \rho L A$$

$$\omega = \sqrt{k/m} = \sqrt{\frac{2\rho A g}{\rho L A}} = \sqrt{\frac{2g}{L}}$$

2018 B #18





$$F = -kx$$

$$x = \frac{-F}{k}$$

$$F_1 = k_1 x$$

$$F_2 = k_2 x$$

$$F = F_1 + F_2 = k_1 x + k_2 x = (k_1 + k_2)x$$

$$k_{eq} = k_1 + k_2$$

$F_{net} = 0$ for equilibrium

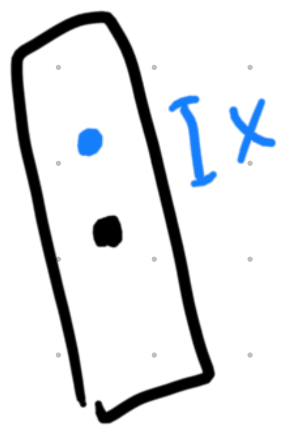
$$= k_1(\ell - \ell_1) + k_2(\ell - \ell_2)$$

$$k_1 \ell_1 + k_2 \ell_2 = (k_1 + k_2)\ell$$

$$\ell = \frac{k_1 \ell_1 + k_2 \ell_2}{k_1 + k_2}$$

B

2018 B #21



$$\boxed{T = 2\pi\sqrt{\frac{I}{mgx}}}$$

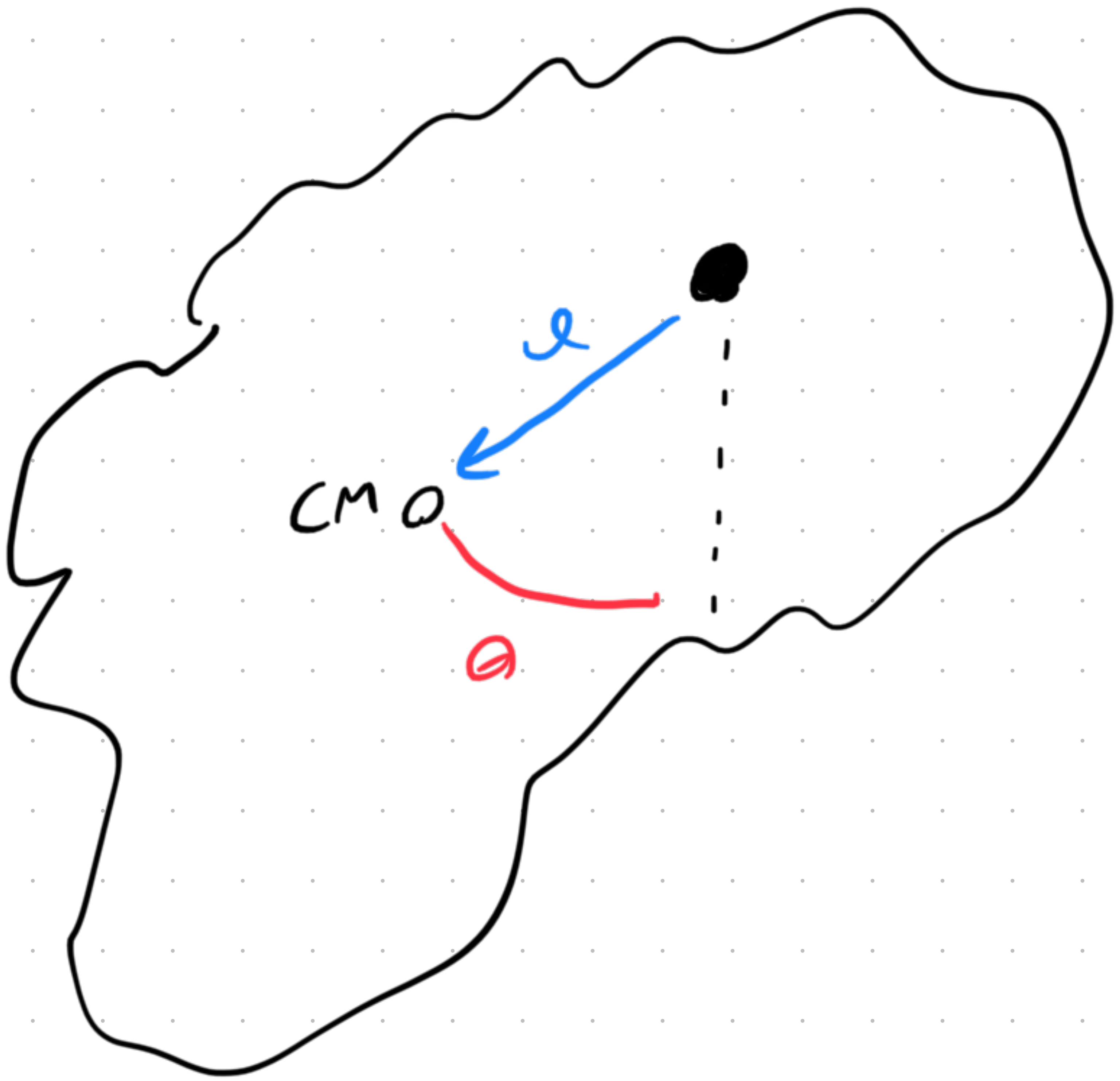
$$I = \frac{1}{12}mL^2 + mx^2$$

minimize $\frac{I}{mx} = x + \left(\frac{L^2}{12}\right)x^{-1}$

$c\left(z + \frac{1}{z}\right)$ minimized when $z = 1$

$$z = \frac{x}{L/(2\sqrt{3})} \Rightarrow x = \frac{L}{2\sqrt{3}}$$

B



I_{cm} given

$I_{rot} = I_{cm} + m\ell^2$ parallel axis thm

$\tau = I_{rot}\,\alpha = -F_g \ell \sin\theta$

$= -F_g \ell\, \theta$ small angle approx

$= -mg\ell\theta$

$ma = -kx$

$m\ddot{x} = -kx$

$I\alpha = -k\theta$

$I\ddot{\theta} = -k\theta$

$$\boxed{\omega = \sqrt{\frac{k}{I}} = \sqrt{\frac{mg\ell}{I_{rot}}}}$$