Dimensional Analysis.

What is the $\boxed{\text{period}}$ of a pendulum composed of a mass m hanging on a rope of length l experiencing acceleration due to gravity g?

A) $\sqrt{g/l}$

B) $\sqrt{l/g}$

~~C)~~ $\sqrt{mg/l}$

~~D)~~ $\sqrt{l/mg}$

~~E)~~ $\sqrt{gl}$

goal units $[s]$

$[l] = m$

$[g] = m/s^2$

$[m] = kg$

No mass!

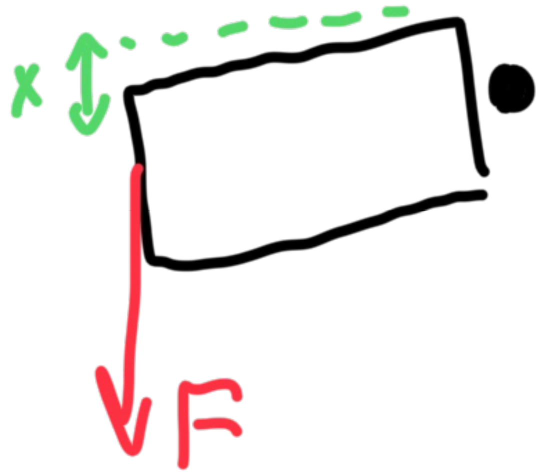
Want lengths to cancel!

$$\left[\sqrt{g/l}\right] = \sqrt{\frac{m/s^2}{m}} = s^{-1}$$

$$\left[\sqrt{l/g}\right] = \sqrt{\frac{m}{m/s^2}} = s$$

B

2018 B #6



$x \sim F$

$x \sim I^{-1}$ $\quad [I] = m^4$

$x \sim E^{?}$ $\quad [E] = N/m^2$

$x \sim L^{?}$

$$x = \frac{F}{I} E^{\alpha} L^{\beta}$$

So, to cancel N,

$\alpha = -1$

$$x = \frac{F}{IE} L^{\beta}$$

$$\left[\frac{F}{IE}\right] = \frac{N}{m^4 \frac{N}{m^2}} = m^{-2}$$

$\beta = 3$

D

Simple Harmonic Oscillators

$$\boxed{F(x) = -kx}$$

x = characteristic displacement from equilibrium.

$$ma = -kx$$

$$m\ddot{x} = -kx$$

$$x = A\cos(\omega t + \phi)$$

A = amplitude

ϕ = phase shift

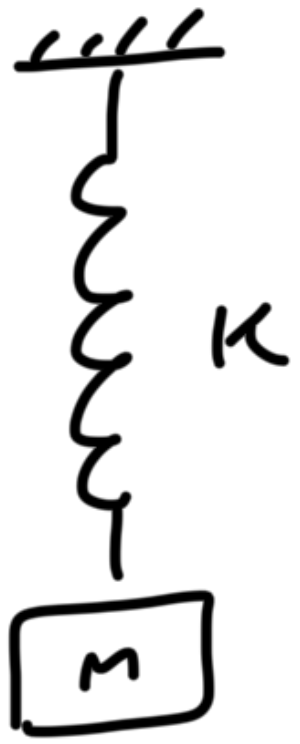
ω = natural oscillation frequency

$$\omega = \sqrt{k/m}$$

$$E_{osc} = \frac{1}{2}kA^2$$

$$\omega = \sqrt{K/M}$$

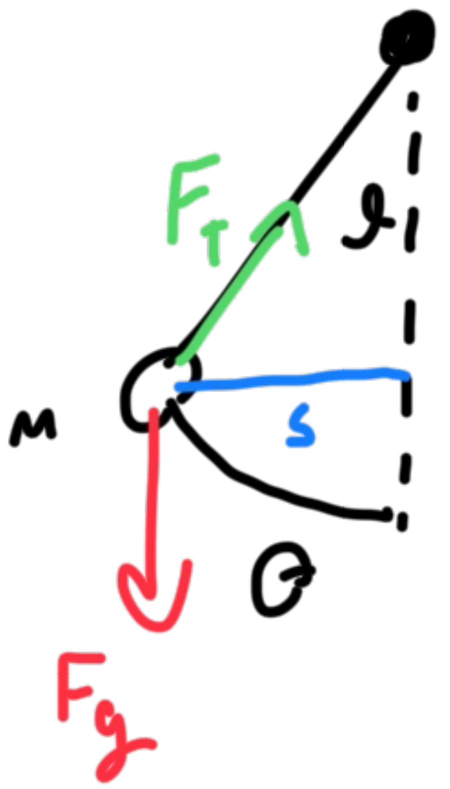
$\Rightarrow$



$$\omega = \sqrt{K/M}$$

Pendulum

$\omega = ?$



$\sin\theta \simeq \theta$

$$F_{T_\theta} \simeq F_T \sin\theta \simeq F_T\,\theta$$

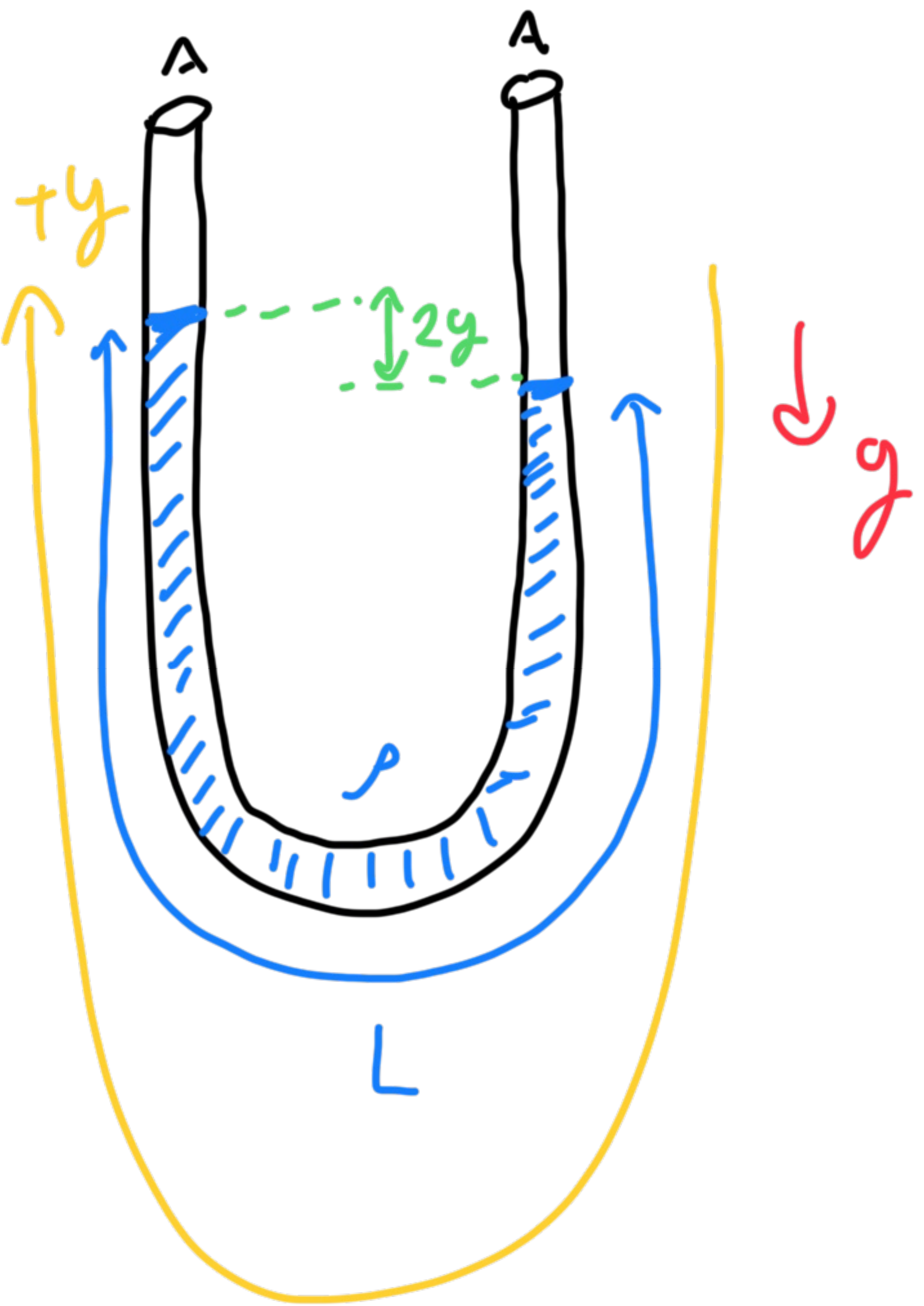
$$F_T \simeq F_g$$

$$F_{T_\theta} \simeq -F_g\,\theta = \boxed{-F_g \frac{s}{\ell}} \propto -s$$

Restoring Force!

$$\boxed{F = -Kx} \qquad K = \frac{Mg}{\ell}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{\frac{Mg}{\ell}}{M}} = \boxed{\sqrt{\frac{g}{\ell}} = \omega}$$



$\omega = ?$ $\qquad \rho = \text{density}$

Extra $2yA$ of water on one side.

$$F = mg = -2\rho y A g = \boxed{-k}y$$

$$\omega = \sqrt{\frac{2\cancel{\rho}\cancel{A}g}{\cancel{\rho}L\cancel{A}}} = \sqrt{\frac{2g}{L}}$$

2018 B #21

Uniform bar of length L & mass M pivoted a distance x from center. The bar is released from rest at horizontal. When is the period of oscillation minimal?

$$\boxed{\omega = \sqrt{\frac{mgx}{I}}}$$

DERIVE!

physical pendulum

I about the axis of rotation.

$$T = 2\pi\sqrt{I/mgx}$$

$$I = I_c + Mx^2 = \frac{1}{12}ML^2 + Mx^2$$

$$\frac{I}{Mx} = x + \left(\frac{1}{12}L^2\right)x^{-1}$$

$c(y + 1/y) = \min$ with $c = L/2\sqrt{3}$, $y = x/c$

$y + 1/y = \min$ of 2

$y = 1$ $x = c = L/2\sqrt{3}$ B