Homework from October 20^{th}.

2018 A # 11 - derive spring constant of cut spring

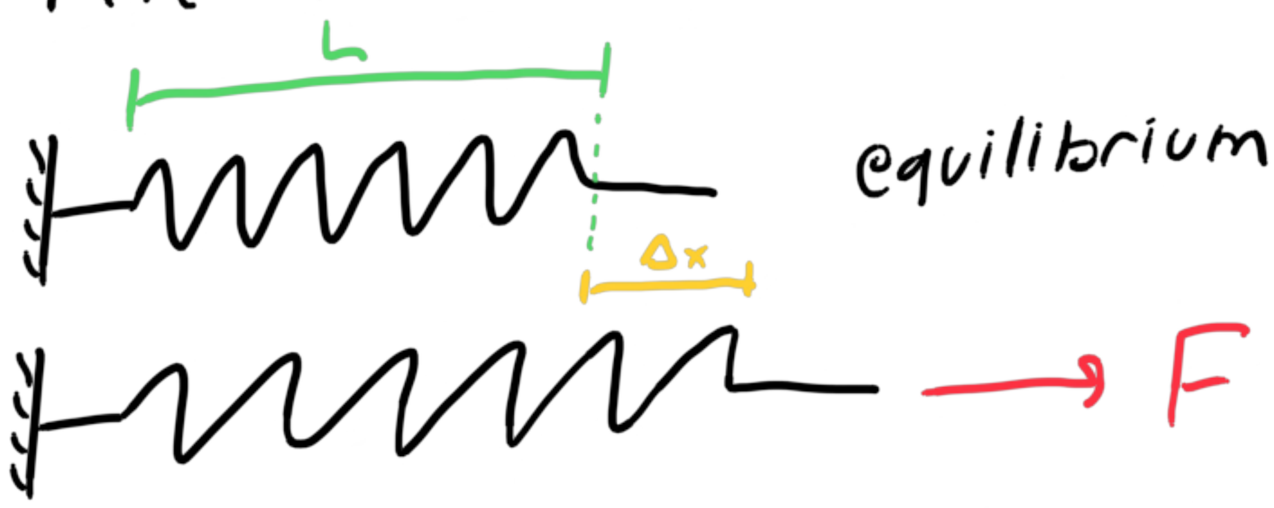
14 - solve the general case

15 - fully solve

11.

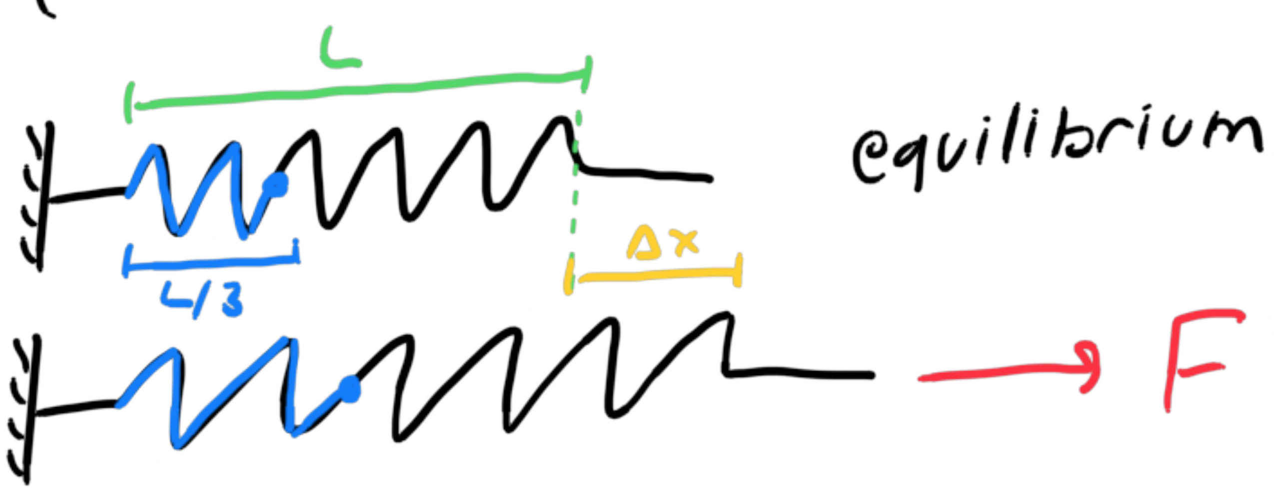
First, the spring constant k is defined as $k \equiv F/\Delta x$.

The initial problem:



now, let's cut it into two pieces, $\frac{1}{N}L$ & $\frac{N-1}{N}L$.

(illustrated with $N=3$)



with force F, both parts have the same F acting on each, since the dot • isn't accelerating.

So, on the blue segment,

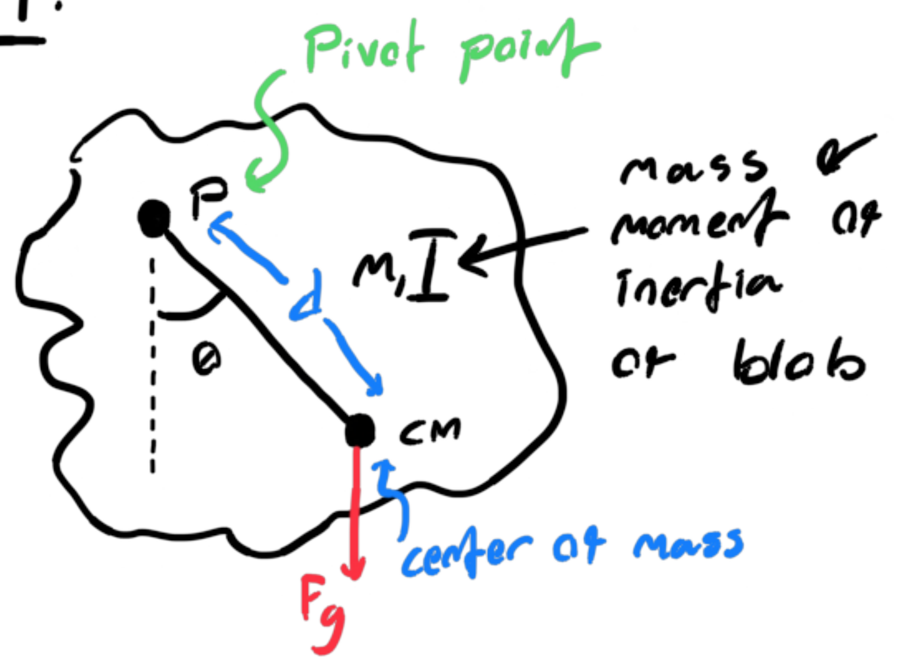
$$F_b = F$$

$$\Delta x_b = \Delta x \cdot \frac{1}{N}$$

$$k_b = F_b/\Delta x_b = \frac{F}{\Delta x \cdot \frac{1}{N}} = \frac{k}{1/N} = \boxed{Nk}$$

Knowing this result will be very useful for any F=ma or similar level test, and this is a classic derivation regardless!

14.



We can treat the whole blob as a point of mass m at the center of mass of the blob.

Using angular F=ma,

$$\tau = I\alpha = -mgd\sin\theta.$$

So, in the small angle approximation, $I\alpha = -mgd\theta$.

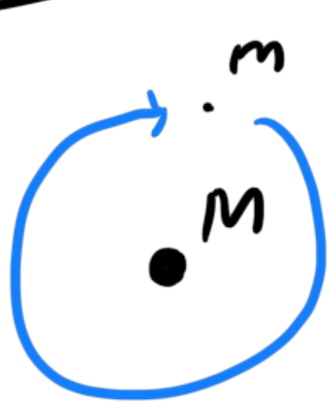
This looks just like a regular pendulum, (see Pendulums notes) so Hooke's Law tells us

$$F = -kx \Rightarrow \omega = \sqrt{k/m} = \boxed{\sqrt{mgd/I}}$$

Using analogies

$$\tau = I\alpha,\quad F = ma \Rightarrow I\alpha = -mgd\theta \Rightarrow ma = -mgd\theta.$$

15.



With $M \gg m$, $a = v^2/r$, $a = \frac{GM}{r^2}$ so

$v = \sqrt{GM/r}$. Then,

$$K = \frac{mv^2}{2} = \frac{m}{2}\cdot\frac{GM}{r} = -\frac{1}{2}U.$$

So, $\Delta K + \Delta U = -1J = -\Delta K$

$$\Rightarrow \boxed{\Delta K = +1J}$$