Forces & Newton's 2nd Law

Considering that Newton's 2nd Law is literally the name of the exam, it's not out of place to say this might be the most useful concept to study.

First & foremost: $F = ma$.

- F: Force
- m: mass
- a: acceleration

We can use this for several types of problems, most dealing with force balance to find a net acceleration.

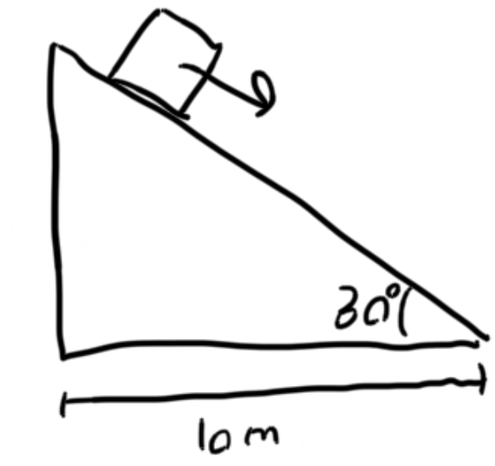
Ex. What is the gravitational force on a 20 kg object? ($a = g = 10\ m/s^2$)

$$F = ma = 20\,kg \cdot 10\ m/s^2 = 200\ kg\ m/s^2 = 200\ N.$$

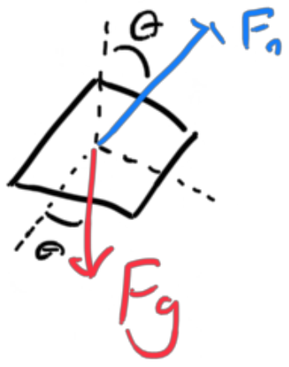
(Newton (unit))

Now for something slightly more involved:

Ex. How long does it take a 10 kg box with a frictionless bottom to roll down this ramp?

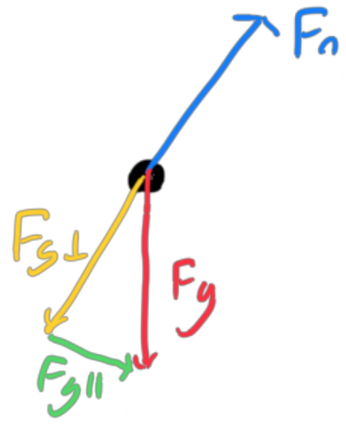


First, make a Free Body Diagram.



First off, geometrically, $\theta = 30°$.

Let's break F_g into $\perp$ & $\parallel$ components relative to F_n.



Vector equation to solve:

$$F_{net\perp} = 0 = \vec{F}_n + \vec{F}_{g\perp} \rightarrow F_n = -F_{g\perp}$$

(vectors) (scalars)

Geometrically: $\vec{F}_g = \vec{F}_{g\perp} + \vec{F}_{g\parallel} \longrightarrow F_{g\perp} = F_g \cos\theta$

Ex. continued

Pythagorean theorem: $F_g^2 = F_g^2 \cos^2\theta + F_{g\parallel}^2$

$$\Rightarrow F_{g\parallel}^2 = F_g^2 \sin^2\theta$$

$$\Rightarrow F_{g\parallel} = F_g \sin\theta$$

Numbers: $\theta = 30°$ $m = 10\,kg$

$$a_\parallel = F_{net\parallel}/m = F_{g\parallel}/m = \frac{F_g}{m}\sin\theta$$

$F_g = mg \Rightarrow a_\parallel = g\sin\theta = 10\,m/s^2 \cdot \sin 30° = 5\,m/s^2.$

Also, the base of the ramp is 10 m long, the surface is $10\,m/\cos 30° \simeq 11.54\,m$

Kinematics: $x_f = x_0 + \frac{1}{2}at^2$

$$11.54 = 0 + \frac{1}{2}\cdot 5 \cdot t^2 \Rightarrow t^2 = \frac{11.54}{2.5} = 4.62$$

$$\boxed{t = 2.15\,s}$$

Note: the mass didn't end up mattering!

Note 2: I effectively made a new coordinate system $(x,y) \to (\perp, \parallel)$. This is a very useful technique!